Multi-modal nonmonotonic logics of minimal knowledge

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Abstract

In this paper we introduce multi-modal logics of minimal knowledge. Such a family of logics constitutes the first proposal in the field of epistemic nonmonotonic logic in which the three following aspects are simultaneously addressed: (i) the possibility of formalizing multiple agents through multiple modal operators; (ii) the possibility of using first-order quantification in the modal language; (iii) the possibility of formalizing nonmonotonic reasoning abilities for the agents modeled, based on the principle of minimal knowledge. We illustrate the expressive capabilities of multi-modal logics of minimal knowledge to provide a formal semantics to peer-to-peer data integration systems, which constitute one of the most recent and complex architectures for distributed information systems.

Introduction

Nonmonotonic modal logics Research in the formalization of commonsense reasoning has pointed out the need of formalizing agents able to reason introspectively about their own knowledge and ignorance (Moore 1985a; Levesque 1990). Modal epistemic logics have thus been proposed, in which modalities are interpreted in terms of knowledge or belief. Generally speaking, the conclusions an introspective agent is able to draw depend on both what she knows and what she *does not* know. Hence, any such conclusion may be retracted when new facts are added to the agent's knowledge. For this reason, many *nonmonotonic* modal formalisms have been proposed in order to characterize the reasoning abilities of an introspective agent: among them, we recall the nonmonotonic modal logics originally proposed by McDermott and Doyle (McDermott & Doyle 1980; McDermott 1982; Marek & Truszczyński 1993), Moore's autoepistemic logic (Moore 1985b), Lifschitz's logic of minimal knowledge and negation as failure MKNF/MBNF (Lifschitz 1991; 1994), Levesque's logic of only knowing (Levesque 1990), and ground nonmonotonic modal logics (Halpern & Moses 1985; Donini, Nardi, & Rosati 1997; Tiomkin & Kaminski 1990).

Modal logics of minimal knowledge Among the nonmonotonic modal logics proposed in the literature, some are based on the so-called principle of *minimal knowledge*. In particular, Halpern and Moses in (Halpern & Moses 1985) defined an epistemic logic, based on the modal system S5, for modeling knowledge and ignorance of processes in a distributed computer system, which is based on a very intuitive semantics: consider only the models of the knowledge base (i.e. the epistemic states of the agent modeled) in which knowledge is minimal (i.e. the ignorance of the agent is maximal). Hence, this formalism is also known as logic of minimal epistemic states, and constitutes the basis of several nonmonotonic modal formalisms proposed in the literature, among which (Lifschitz 1991; Lin & Shoham 1992; Meyer & van der Hoek 1995; Shoham 1987). In particular, Lifschitz (Lifschitz 1991; 1994) has proposed a bimodal logic, known as MKNF,[1] combining the minimal knowledge paradigm with the notion of negation as failure in logic programming.

Notably, it was shown that the logic of minimal epistemic states can be given a fixpoint characterization (Tiomkin & Kaminski 1990) which actually defines a whole family of logics of minimal knowledge states, the so-called *ground nonmonotonic modal logics* (Truszczyński 1991; Schwarz 1992a), obtained by varying the underlying modal system. Hence, such family of logics can be considered as obtained through a generalization of the notion of minimal knowledge, according to the different modal system chosen.

MKNF has been used in order to give a declarative semantics to very general classes of logic programs (Lifschitz & Woo 1992; Schwarz & Lifschitz 1993; Inoue & Sakama 1994), which generalize the stable model semantics of negation as failure in logic programming (Gelfond & Lifschitz 1988; 1990; 1991). Also, MKNF can be viewed as an extension of the theory of epistemic queries to databases (Reiter 1990), which deals with the problem of querying a first-order database about its own knowledge. Due to its ability of expressing many features of nonmonotonic logics (Lifschitz 1994; Schwarz & Lifschitz 1993; Rosati 1999), MKNF is generally considered as a unifying

[1] Actually, Lifschitz in (Lifschitz 1991) defined the logic MKNF, while in (Lifschitz 1994) he presented the logic MBNF, which slightly differs from MKNF.

framework for several nonmonotonic formalisms, including default logic, autoepistemic logic, circumscription, epistemic queries, and logic programming.

Limitations of current proposals Let us now point out two limitations of the research in nonmonotonic modal logics:

- The vast majority of the studies in nonmonotonic modal logics in the literature deal with *propositional* modal logics, while there are very few proposals concerning nonmonotonic extensions of first-order logic.
- Almost all the modal approaches nonmonotonic logic use a single modality, i.e., they lack the ability of expressing the knowledge of many agents. In particular, none of the nonmonotonic formalisms based on the principle of minimal knowledge which have been proposed in the literature is *multi-modal*, i.e., is able to express the different epistemic states of a set of agents. On the other hand, this aspect has been extensively studied in the monotonic multi-modal systems for knowledge and belief $\mathsf{K}_n, \mathsf{T}_n, \mathsf{S4}_n, \mathsf{K45}_n, \mathsf{KD45}_n, \mathsf{S5}_n$ (see, e.g., (Halpern & Moses 1992)).

On the other hand, recent developments in the field of distributed information systems have outlined the need for a multi-modal, first-order, nonmonotonic logic. In particular, several recent studies in the formalization of peer-to-peer distributed systems (Bernstein *et al.* 2002; Calvanese *et al.* 2003; Franconi *et al.* 2003; Calvanese *et al.* 2004; 2005) have clearly pointed out that the intended semantics of information in this kind of applications is naturally captured by an epistemic logic approach based on the principle of minimal knowledge, in which each system is modeled as an autonomous epistemic agent, and the exchange of information in the system is represented by epistemic sentences that express the relationships among the epistemic states of the different agents. Moreover, such studies highlight that, in order to fully represent the peculiar aspects of such application scenarios, we need a logic with first-order quantification abilities that is also able to formalize typical nonmonotonic reasoning features of the epistemic agents (Calvanese *et al.* 2005).

Our contribution The aim of this paper is to provide a first proposal of multi-modal nonmonotonic logics. In particular, we define the family of logics $\mathcal{S}_n^A$, which has the following characteristics:

- each logic in the family $\mathcal{S}_n^A$ is a nonmonotonic logic based on the principle of minimal knowledge, in particular it can be viewed as a generalization of Lifschitz's logic MKNF;
- each such logic is a multi-modal logic, since it can be viewed as a nonmonotonic extension of the family of multi-modal logics $\mathcal{S}_n$ (Halpern & Moses 1992);
- each such logic is a modal first-order logic, i.e., it allows for first-order quantification.

In particular, we point out that the semantic definition of $\mathcal{S}_n^A$ is based on a preference order on possible-world structures, following the studies on a model theory for nonomotonic modal logics (Shoham 1987; Schwarz 1992b; Schwarz & Truszczyński 1994; Donini, Nardi, & Rosati 1997). Also, it can be seen as a generalization of the possible-world semantics of MKNF and MBNF (Lifschitz 1991; 1994).

Then, to show an example of the representational abilities of the logic $\mathcal{S}_n^A$, we use the logic $\mathsf{K45}_n^A$ to formalize the behaviour of knowledge in distributed, peer-to-peer information systems. To our purposes, this kind of application is of particular interest, since it requires all the three main ingredients of the logics $\mathcal{S}_n^A$, namely, multiple modal operators, first-order quantification, and nonmonotonic abilities.

In the next section, we recall standard (monotonic) multi-modal logics. Then, we define syntax and semantics of the nonmonotonic multi-modal logics $\mathcal{S}_n^A$, and analyze the relationship between the family $\mathcal{S}_n^A$ and nonmonotonic modal logics previously defined. In the subsequent section we illustrate the representation abilities of one of these logics ($\mathsf{K45}_n^A$) in the formalization of knowledge in distributed, peer-to-peer information systems. Finally, we draw some conclusions.

Multi-modal logics

In this section we recall multi-modal epistemic logics (Halpern & Moses 1992). We assume that the reader is familiar with the basics of modal logic (Chellas 1980).

The language $\mathcal{L}_k$ is the usual function-free first-order multi-modal language, i.e., it is obtained from function-free first-order logic by adding a set $\mathbf{K_1}, \ldots, \mathbf{K_n}$ of modal operators, for the forming rule: if ϕ is a (possibly open) formula, then also $\mathbf{K_i}\phi$ is so, for $1 \leq i \leq n$ for a fixed n. We use ψ_c^x to denote the formula obtained from ψ by substituting each free occurrence of the variable x with the constant c.

To define the semantics, we start from first-order interpretations. In particular, we restrict our attention to first-order interpretations that share a fixed infinite domain Δ. We further assume that for each domain element $d \in \Delta$, we have a unique constant $c_d \in \Gamma$ that denotes exactly d, and, vice versa, that every constant $c_d \in \Gamma$ denotes exactly one domain element $d \in \Delta$. In other words, the constants in Γ act as *standard names* (Levesque & Lakemeyer 2001).

Formulas of $\mathcal{L}_k$ are interpreted over $\mathcal{S}_n$-structures. Given a modal system $\mathcal{S}$, where $\mathcal{S} \in \{\mathsf{K}, \mathsf{T}, \mathsf{K4}, \mathsf{K45}, \mathsf{KD45}, \mathsf{S4}, \mathsf{S5}\}$, a $\mathcal{S}_n$*-structure* is a Kripke structure E of the form $(W, \{R_1, \ldots R_n\}, V)$, where:

- W is a set whose elements are called *possible worlds*;
- V is a function assigning to each $w \in W$ a first-order interpretation $V(w)$;
- each R_i, called the *accessibility relation* for the modality $\mathbf{K_i}$, is a binary relation over W that satisfies the conditions for the modal system $\mathcal{S}_n$ described below.

Different multi-modal logics are obtained by imposing different conditions that each accessibility relation R_i has to satisfy: in particular,

- when $\mathcal{S}_n = \mathsf{T}_n$, each R_i is reflexive;

- when $\mathcal{S}_n = \mathsf{K4}_n$, each R_i is transitive;
- when $\mathcal{S}_n = \mathsf{K45}_n$, each R_i is transitive and euclidean;
- when $\mathcal{S}_n = \mathsf{KD45}_n$, each R_i is serial, transitive and euclidean;
- when $\mathcal{S}_n = \mathsf{S4}_n$, each R_i is reflexive and transitive;
- when $\mathcal{S}_n = \mathsf{S5}_n$, each R_i is reflexive, transitive and euclidean.

It is well-known that the above four conditions on the accessibility relation (serial, reflexive, transitive, euclidean) of $\mathcal{S}_n$-structures correspond respectively to impose validity of the following axiom schemas:

$\mathbf{K_i}\phi \supset \neg\mathbf{K_i}\neg\phi$	axiom schema D
$\mathbf{K_i}\phi \supset \phi$	axiom schema T
$\mathbf{K_i}\phi \supset \mathbf{K_i}\mathbf{K_i}\phi$	axiom schema 4
$\neg\mathbf{K_i}\phi \supset \mathbf{K_i}\neg\mathbf{K_i}\phi$	axiom schema 5

A $\mathcal{S}_n$*-interpretation* is a pair (E, w), where $E = (W, \{R_1, \ldots R_n\}, V)$ is a $\mathcal{S}_n$-structure, and w is a world in W. A sentence (i.e., a closed formula) ϕ *is true in an interpretation* (E, w) (or, is true on world $w \in W$ in E), written $E, w \models \phi$ iff:

- $E, w \models P(c_1, \ldots, c_n)$ iff $V(w) \models P(c_1, \ldots, c_n$
- $E, w \models \phi_1 \wedge \phi_2$ iff $E, w \models \phi_1$ and $E, w \models \phi_2$
- $E, w \models \neg\phi$ iff $E, w \not\models \phi$
- $E, w \models \exists x. \psi$ iff $E, w \models \psi^x_c$ for some constant c
- $E, w \models \mathbf{K_i}\phi$ iff $E, w' \models \phi$ for every w' such that $(w, w') \in R_i$

A $\mathcal{S}_n$*-model* for ϕ is a $\mathcal{S}_n$-interpretation E, w such that $E, w \models \phi$.

We say that a sentence ϕ is $\mathcal{S}_n$*-satisfiable* if there exists a $\mathcal{S}_n$-model for ϕ, *unsatisfiable* otherwise. A $\mathcal{S}_n$-model for a set Σ of sentences is a $\mathcal{S}_n$-model for every sentence in Σ. A sentence ϕ is $\mathcal{S}_n$*-entailed* by a set Σ of sentences, written $\Sigma \models_{\mathcal{S}_n} \phi$, if and only if $E, w \models \phi$ in every $\mathcal{S}_n$-model E, w of Σ.

Multi-modal logics of minimal knowledge and negation as failure

In this section we introduce a nonmonotonic extension of the multi-modal logics recalled in the previous section. Informally, such an extension is obtained by adding a new set of modal operators $\mathbf{A_1}, \ldots, \mathbf{A_n}$ to the modal language. Then, following (and generalizing) the semantic construction of the logic MKNF (Lifschitz 1991), the modal operators $\mathbf{K_1}, \ldots, \mathbf{K_n}$ are interpreted as epistemic operators of minimal knowledge, and the modal operators $\mathbf{A_1}, \ldots, \mathbf{A_n}$ are interpreted as epistemic operators of *justified assumption* (Lin & Shoham 1992), which corresponds to the well-known notion of *negation as failure* (Lifschitz 1994).

Adding modal operators of negation as failure

First, we introduce the language $\mathcal{L}^A_k$, which is an extension of $\mathcal{L}_k$ obtained by adding to the first-order modal language a new set of modal operators, $\mathbf{A_1}, \ldots, \mathbf{A_n}$.

The semantics of $\mathcal{L}^A_k$ sentences is formally defined as follows. A $\mathcal{S}^A_n$-structure E is a tuple $(W, \{R_1, \ldots, R_n, R^a_1, \ldots, R^a_n\}, V)$, where:

- W is a set of worlds;
- each R_i and each R^a_i are binary relations over W satisfying the conditions imposed by the system $\mathcal{S}_n$ (described in the previous section);
- V is a function mapping worlds to first-order interpretations.

Therefore, with respect to $\mathcal{S}_n$-structures, $\mathcal{S}^A_n$-structures have n additional accessibility relations $R^a_1, \ldots, R^a_n$. Such relations account for the additional modal operators $\mathbf{A_1}, \ldots, \mathbf{A_n}$.

The notion of truth of a $\mathcal{L}^A_k$ sentence in a world of a $\mathcal{S}^A_n$-structure is analogous to the notion given in Section for $\mathcal{L}_k$, with the addition of the following rule:

- $E, w \models \mathbf{A_i}\phi$ iff $E, w' \models \phi$ for each w' such that $(w, w') \in R^a_i$

Nonmonotonic semantics

So far, the family of logics $\mathcal{S}^A_n$ do not appear as a significant extension of the logics $\mathcal{S}_n$: indeed, according to the above notion of truth, the new modal operators $\mathbf{A_i}$ are treated just like any $\mathbf{K_i}$ operator in $\mathcal{S}_n$, so there is no apparent reason to distinguish the $\mathbf{A_i}$'s operators from the $\mathbf{K_i}$'s.

Actually, for each logic $\mathcal{S}^A_n$, the different (nonmonotonic) meaning of the two sets of modal operators in $\mathcal{S}^A_n$ with respect to $\mathcal{S}_n$ is due to the following notion of $\mathcal{S}^A_n$-model for a sentence ϕ, which differs from the (classical) notion of $\mathcal{S}_n$-model, and is obtained by imposing a preference order over $\mathcal{S}^A_n$-structures satisfying ϕ.

Definition 1 *Let* $E = (W, \{R_1, \ldots, R_n, R^a_1, \ldots, R^a_n\}, V)$ *and* $E' = (W', \{R'_1, \ldots, R'_n, R^a_1, \ldots, R^a_n\}, V')$ *be* $\mathcal{S}^A_n$*-structures. We say that* E' is preferred to E *if the following conditions hold:*

1. $W' \supseteq W$ *and* $V'(w) = V(w)$ *for every* $w \in W$,
2. $R'_i \supseteq R_i$, *for all* $i \in \{1, \ldots, n\}$,
3. *there exist* $w_1 \in W$, $w_2 \in W'$, $i \in \{1, \ldots, n\}$ *such that* $(w_1, w_2) \in R'_i - R_i$ *and there exists no* $w' \in W$ *such that* $(w_1, w') \in R_i$ *and* $V(w') = V'(w_2)$.

Intuitively, E' is preferred to E if E' is a structure "larger" than E (conditions 1 and 2) and there exists a world w_1 which is connected in E' (through the relation R'_i) to a larger set of possible worlds than in E (condition 3), which means that w_1 in E has "less objective knowledge" than in E' with respect to the modality K_i, since adding possible worlds in a structure reduces the knowledge represented the accessibility relations interpreting the $\mathbf{K_i}$'s operators.

For instance, it can be immediately verified that, if E' is preferred to E, then, for each first-order sentence ϕ and for each $w \in W$, if $E', w \models \mathbf{K_i}\phi$ then $E, w \models \mathbf{K_i}\phi$, but not vice-versa.

Definition 2 *Let* $\phi \in \mathcal{L}^A_k$, *let* $E = (W, R_1, \ldots, R_n, R^a_1, \ldots, R^a_n, V)$ *be a* $\mathcal{S}^A_n$*-structure, and let* $w \in W$. (E, w) *is a* $\mathcal{S}^A_n$-model for ϕ *if the following conditions hold:*

1. *$E, w \models \phi$;*
2. *$R_i = R_i^a$ for each $i \in \{1, \ldots, n\}$;*
3. *there exists no $\mathcal{S}_n^A$-structure $E' = (W', \{R'_1, \ldots, R'_n, R_1^a, \ldots, R_n^a\}, V')$ such that E' is preferred to E, and $E', w \models \phi$.*

A $\mathcal{S}_n^A$-model for a set Σ of sentences is a $\mathcal{S}_n^A$-model for every sentence in Σ. A sentence ϕ is $\mathcal{S}_n^A$-*entailed* by a set Σ of sentences, written $\Sigma \models_{\mathcal{S}_n^A} \phi$, if and only if $E, w \models \phi$ in every $\mathcal{S}_n^A$-model E, w of Σ.

Let us now try to provide an intuition for the semantics of the logics in the family $\mathcal{S}_n^A$. The above semantics formalizes the idea of selecting $\mathcal{S}_n^A$-structures that satisfy two intuitive principles:

1. *knowledge is minimal*, which is realized through the notion of preference between structures;
2. *assumptions are justified by knowledge*, which is realized by the fact that, for each i, the meaning of the operators $\mathbf{A_i}$ and $\mathbf{K_i}$ is the same, since $R_i = R_i^a$.

Such semantic principles of minimal knowledge and justified assumptions are well-known in nonmonotonic reasoning (Lin & Shoham 1992; Lifschitz 1994; Rosati 1999). In particular, we recall that the principle of justified assumption exactly corresponds to the semantics of the modal operator in Moore's autoepistemic logic (Rosati 1999). Moreover, as illustrated in (Lifschitz 1991; 1994; Lin & Shoham 1992), the justified assumption operator exactly formalizes the notion of *negation as failure* in logic programming under the stable model semantics.

Remark. From the technical viewpoint, the above preference semantics for the logics $\mathcal{S}_n^A$ is a non-trivial extension of analogous semantic constructions underlying other nonmonotonic modal logics. The main difference with respect to such previous constructions is that here, due to the presence of multiple modal operators, we cannot impose the condition that the preferred models of a theory always correspond to structures in which each accessibility relation is total (which have a syntactic counterpart in the so-called *stable sets* of modal formulas (Stalnaker 1993)). Consequently, minimality of knowledge in the preferred models is imposed via a different, although simple, condition (formally stated by Definition 1), which can be seen as a generalization of analogous minimality criteria in previous, simpler nonmonotonic modal formalisms like MKNF (Lifschitz 1991) or ground nonmonotonic modal logics (Donini, Nardi, & Rosati 1997).

The logics $\mathcal{S}_n^A$ vs. nonmonotonic modal logics

We now analyze more in detail the relationship between the family of logics $\mathcal{S}_n^A$ and previous nonmonotonic modal logics. In particular, we want to point out the following deep correspondences between the logics $\mathcal{S}_n^A$ and some well-known nonmonotonic modal logics:

- *Correspondence between MKNF and* $\mathsf{S5}_n^A$. First, we analyze the relationship between the logic $\mathsf{S5}_n^A$ and Lifschitz's logic MKNF. More precisely, we start by recalling that the language of MKNF makes use of two modal operators K and *not*. Now, given an MKNF theory Σ, it can be proved that the MKNF-models of Σ coincide with the $\mathsf{S5}_n^A$-models of the theory Σ' obtained from Σ by replacing each occurrence of the modal operator K with the modality $\mathbf{K_1}$, and replacing each occurrence of the modal operator *not* with the modality $\neg\mathbf{A_1}$. Therefore, the logic $\mathsf{S5}_n^A$ can be viewed as the multi-modal generalization of MKNF, and, more generally, the whole family of logics $\mathcal{S}_n^A$ can be seen as a generalization of the semantic construction underlying the logic MKNF.
- *Correspondence between autoepistemic logic and* $\mathsf{S5}_n^A$. As a consequence of the previous correspondence, and since in turn MKNF constitutes a generalization of Moore's autoepistemic logic (Rosati 1999), it follows that an analogous precise correspondence holds between the logic $\mathsf{S5}_n^A$ and Moore's autoepistemic logic, which allows us to also interpret $\mathsf{S5}_n^A$ as a multi-modal generalization of Moore's autoepistemic logic.
- *Correspondence between ground logic* $\mathsf{S5}_G$ *and* $\mathsf{S5}_n^A$. The family of *ground nonmonotonic modal logics* studied in (Tiomkin & Kaminski 1990; Truszczyński 1991; Schwarz 1992a; Donini, Nardi, & Rosati 1997) is also deeply related to the logics $\mathcal{S}_n^A$. More precisely, it can be shown that the ground nonmonotonic modal logic based on the modal system S5 and known as $\mathsf{S5}_G$ (Donini, Nardi, & Rosati 1997) corresponds to the logic $\mathsf{S5}_n^A$, in the sense that, given a unimodal theory Σ, the $\mathsf{S5}_G$-models of Σ coincide with the $\mathsf{S5}_n^A$-models of the theory Σ' obtained from Σ by replacing each occurrence of the modal operator with the modality $\mathbf{K_1}$.

Modeling knowledge in a P2P system

In this section we show the representational abilities of the multi-modal logics $\mathcal{S}_n^A$. In particular, we use one of such logics, $\mathsf{K45}_n^A$, to provide a formal semantics to *peer-to-peer (P2P) data integration systems*, which constitute one of the most recent and complex architectures in the field of distributed information systems.

For a detailed introduction to P2P data integration systems, we refer the reader to (Halevy *et al.* 2003), and for more details on the formalization presented in this section, we refer to (Calvanese *et al.* 2005). In the following, we assume that the reader is familiar with the basics of relational database theory (Abiteboul, Hull, & Vianu 1995).

P2P data integration systems

We refer to a fixed, infinite, denumerable set Γ of constants. Such constants are shared by all peers, and denote the data items managed by the P2P data integration system (denoted by P2PDIS in the following). Moreover, given a relational alphabet A, we denote with $\mathcal{L}_A$ the set of function-free first-order logic (FOL) formulas whose relation symbols are in A and whose constants are in Γ.

A *P2P data integration system* $\mathcal{P} = \{P_1, \ldots, P_n\}$ is constituted by a set of n peers. Each peer $P_i \in \mathcal{P}$ (cf. (Halevy *et al.* 2003)) is defined as a tuple $P_i = (id, G, S, L, M, \mathcal{L})$, where:

- id is a symbol that identifies the peer P_i within $\mathcal{P}$, called the identifier of P_i.
- G is the *schema* of P_i, which is a finite set of formulas of $\mathcal{L}_{A_G}$ (representing local integrity constraints), where A_G is a relational alphabet (disjoint from the other alphabets in $\mathcal{P}$) called the *alphabet* of P_i. We assume that the language $\mathcal{L}_{A_G}$ of peer P_i includes the special sentence $\bot_i$ that is false in every interpretation for $\mathcal{L}_{A_G}$. Intuitively, the peer schema provides an intensional view of the information managed by the peer.
- S is the *(local) source schema* of P_i, which is simply a finite relational alphabet (again disjoint from the other alphabets in $\mathcal{P}$), called the *local alphabet* of P_i. Intuitively, the source schema describes the structure of the data sources of the peer (possibly obtained by wrapping physical sources), i.e., the sources where the real data managed by the peer are stored.
- L is a set of *(local) mapping assertions* between G and S. Each local mapping assertion is an expression of the form $cq_S \rightsquigarrow cq_G$, where cq_S and cq_G are two conjunctive queries of the same arity, respectively over the source schema S and over the peer schema G. The local mapping assertions establish the connection between the elements of the source schema and those of the peer schema in P_i. In particular, an assertion of the form $cq_S \rightsquigarrow cq_G$ specifies that all the data satisfying the query cq_S over the sources also satisfy the concept in the peer schema represented by the query cq_G. In the terminology used in data integration, the combination of peer schema, source schema, and local mapping assertions constitutes a GLAV *data integration system* (Lenzerini 2002) managing a set of sound data sources S defined in terms of a (virtual) global schema G.
- M is a set of *P2P mapping assertions*, which specify the semantic relationships that the peer P_i has with the other peers. Each assertion in M is an expression of the form $cq' \rightsquigarrow cq$, where cq, called the *head* of the assertion, is a conjunctive query over the peer (schema of) P_i, while cq', called the *tail* of the assertion, is a conjunctive query of the same arity as cq over (the schema of) one of the other peers in $\mathcal{P}$. A P2P mapping assertion $cq' \rightsquigarrow cq$ from peer P_j to peer P_i expresses the fact that the P_j-concept represented by cq' is mapped to the P_i-concept represented by cq. From an extensional point of view, the assertion specifies that every tuple that can be retrieved from P_j by issuing query cq' satisfies cq in P_i.
- $\mathcal{L}$ is a relational query language specifying the class of queries that the peer P_i can process. We assume that $\mathcal{L}$ is any fragment of FOL that accepts at least conjunctive queries and the sentence $\bot_i$. We say that the queries in $\mathcal{L}$ are those *accepted by* P_i. Notice that this implies that, for each P2P mapping assertion $cq' \rightsquigarrow cq$ from another peer P_j to peer P_i in M, we have that cq' is accepted by P_j.

An *extension* for a P2PDIS $\mathcal{P} = \{P_1, \ldots, P_2\}$ is a set $\mathcal{D} = \{D_1, \ldots, D_n\}$, where each D_i is an extension of the predicates in the local source schema of peer P_i.

A P2PDIS, together with an extension, is intended to be queried by external users. A user enquires the whole system by accessing any peer P of $\mathcal{P}$, and by issuing a *query* q to P. The query q is processed by P if and only if q is expressed over the schema of P and is accepted by P.

Example 3 Let us consider the P2PDIS in Figure 1, in which we have 4 peers P_1, P_2, P_3, and P_4 (in the following, we assume that each peer P_i is identified by i).

The global schema of peer P_1 is formed by a relation schema $\mathsf{Person}_1(\underline{\mathsf{name}}, \mathsf{livesIn}, \mathsf{citizenship})$, where name is the key (we underline the key of a relation). P_1 contains a local source $\mathsf{S}_1(\mathsf{name}, \mathsf{livesIn})$, mapped to the global view by the assertion $\{x, y \mid \mathsf{S}_1(x, y)\} \rightsquigarrow \{x, y \mid \exists z. \mathsf{Person}_1(x, y, z)\}$. Moreover, it has a P2P mapping assertion $\{x, z \mid \exists y. \mathsf{Citizen}_2(x, y, z)\} \rightsquigarrow \{x, z \mid \exists y. \mathsf{Person}_1(x, y, z)\}$ relating information in peer P_2 to those in peer P_1.

P_2 has $\mathsf{Citizen}_2(\underline{\mathsf{name}}, \mathsf{birthDate}, \mathsf{citizenship})$ as global schema, and a local source $\mathsf{S}_2(\mathsf{name}, \mathsf{birthDate}, \mathsf{citizenship})$ mapped to the global schema through the local mapping $\{x, y, z \mid \mathsf{S}_2(x, y, z)\} \rightsquigarrow \{x, y, z \mid \mathsf{Citizen}_2(x, y, z)\}$. P_2 has no P2P mappings.

P_3 has $\mathsf{Person}_3(\underline{\mathsf{name}}, \mathsf{livesIn}, \mathsf{citizenship})$ as global schema, contains no local sources, and has a P2P mapping $\{x, y, z \mid \mathsf{Person}_1(x, y, z)\} \rightsquigarrow \{x, y, z \mid \mathsf{Person}_3(x, y, z)\}$ with P_1, and a P2P mapping $\{x, y, z \mid \mathsf{Citizen}_4(x, y, z)\} \rightsquigarrow \{x, y, z \mid \mathsf{Person}_3(x, y, z)\}$ with P_4.

P_4 has $\mathsf{Citizen}_4(\underline{\mathsf{name}}, \mathsf{livesIn}, \mathsf{citizenship})$ as global schema, and a local source $\mathsf{S}_4(\mathsf{name}, \mathsf{livesIn}, \mathsf{citizenship})$ mapped to the global schema through the local mapping $\{x, y, z \mid \mathsf{S}_4(x, y, z)\} \rightsquigarrow \{x, y, z \mid \mathsf{Citizen}_4(x, y, z)\}$. P_4 has no P2P mappings.

Finally, Figure 1 shows also an extension of the P2P data integration system, which includes S_1("Joe", "Rome"), S_2("Joe", "24/12/70", "Canadian"), and S_4("Joe", "Rome", "Italian"). ■

Formalization of P2P systems in $\mathsf{K45}_n^A$

In order to logically formalize a P2PDIS, several aspects of the intended meaning of information in such a system must be taken into account. Due to lack of space,[2] here we only focus on *inconsistency tolerance*, which is the characteristic that enforces the need of a nonmonotonic logic for the above purpose. Informally, inconsistency tolerance corresponds to the ability of providing a semantics to the system even in the presence of contradicting information (e.g., data contradicting integrity constraints on the peer schemas).

More specifically, we want the P2PDIS to be inconsistency-tolerant in the following sense:

1. When a peer is *locally inconsistent*, i.e., data at the sources in P_i contradict, via the local mapping, the peer schema, making the whole peer inconsistent, the P2PDIS should be equivalent to the one obtained by eliminating

[2] For a detailed description of the intended semantics of information in a P2PDIS we refer to (Halevy *et al.* 2003; Calvanese *et al.* 2005).

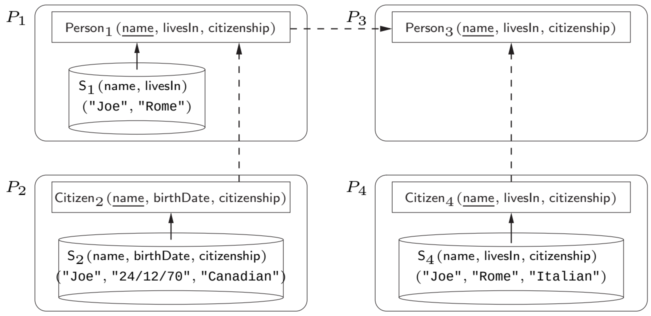

Figure 1: A P2P data integration system

the peer P_i from the system. In other words, an inconsistent peer should be "isolated" from the other peers: in this way, a local inconsistency does not affect the overall consistency (and meaning) of the system. The choice of isolating locally inconsistent peers is motivated by the modularity of P2PDISs pursued by our approach, in which each peer is considered as a black box. Of course, the study of inconsistency might be also interesting in an alternative setting not focused on modularity. However, this is outside the scope of the present paper.

2. In the presence of *P2P inconsistency*, i.e., when in a peer P_i the data coming from another peer P_j (through a P2P mapping) contradict the local data of P_i (or the data coming to P_i from another peer P_k), the peer P_i should not reach an inconsistent state: rather, it should discard a *minimal* amount of the data retrieved from the other peers in order to preserve consistency.

Due to the characteristics mentioned above, $\mathsf{K45}_n^A$ is well-suited to formalize P2PDISs. Let $\mathcal{P} = \{P_1, \ldots, P_n\}$ be a P2PDIS in which each peer P_i has identifier i. We use the modal operators $\mathbf{K_i}$ and $\mathbf{A_i}$ to model the peer i. More precisely, for each peer $P_i = (i, G, S, L, M, \mathcal{L})$ we define the theory $\mathcal{T}_K(P_i)$ in $\mathsf{K45}_n^A$ as the union of the following sentences:

- Global schema G of P_i: for each sentence ϕ in G, we have
$$\mathbf{K_i}\phi$$
Observe that ϕ is a first-order sentence expressed in the alphabet of P_i, which is disjoint from the alphabets of all the other peers in $\mathcal{P}$.
- Local mapping assertions L between G and the local source schema S: for each mapping assertion $\{\mathbf{x} \mid \exists\mathbf{y}.\, body_{cq_S}(\mathbf{x},\mathbf{y})\} \leadsto \{\mathbf{x} \mid \exists\mathbf{z}.\, body_{cq_G}(\mathbf{x},\mathbf{z})\}$ in L, we have
$$\mathbf{K_i}(\forall\mathbf{x}.\, \exists\mathbf{y}.\, body_{cq_S}(\mathbf{x},\mathbf{y}) \supset \exists\mathbf{z}.\, body_{cq_G}(\mathbf{x},\mathbf{z}))$$
- P2P mapping assertions M: for each P2P mapping assertion $\{\mathbf{x} \mid \exists\mathbf{y}.\, body_{cq_j}(\mathbf{x},\mathbf{y})\} \leadsto \{\mathbf{x} \mid \exists\mathbf{z}.\, body_{cq_i}(\mathbf{x},\mathbf{z})\}$ between the peer j and the peer i in M, we have
$$\begin{array}{l}\forall\mathbf{x}.\, \neg\mathbf{A_j}\bot_j \wedge \mathbf{K_j}(\exists\mathbf{y}.\, body_{cq_j}(\mathbf{x},\mathbf{y}))\wedge \\ \neg\mathbf{A_i}(\neg\exists\mathbf{z}.\, body_{cq_i}(\mathbf{x},\mathbf{z})) \supset \\ \mathbf{K_i}(\exists\mathbf{z}.\, body_{cq_i}(\mathbf{x},\mathbf{z}))\end{array} \quad (1)$$
Informally, the above sentence specifies the following rule: for each tuple of values $\mathbf{t}$, if peer j is *locally consistent* and knows the sentence $\exists\mathbf{y}.\, body_{cq_j}(\mathbf{t},\mathbf{y})$, and the sentence $\exists\mathbf{z}.\, body_{cq_i}(\mathbf{t},\mathbf{z})$ *is consistent with what peer i knows*, then peer i knows the sentence $\exists\mathbf{z}.\, body_{cq_i}(\mathbf{t},\mathbf{z})$. In other words, information flows from peer j to peer i through a P2P mapping assertion only if j is locally consistent and if adding such information to peer i does not give rise to a P2P inconsistency in peer i. More precisely, the meaning of the above sentence in $\mathsf{K45}_n^A$ is that exactly a *maximal* amount of information (i.e., a maximal set of tuples) consistent with peer i flows from peer j to peer i through the P2P mapping assertion. Moreover, under such a formalization the P2PDIS is tolerant to local inconsistency, in the sense that the peers that are locally inconsistent are "isolated" from the rest of the system (i.e., thay cannot propagate information).

We denote by $\mathcal{T}_K(\mathcal{P})$ the theory corresponding to the P2PDIS $\mathcal{P}$, i.e., $\mathcal{T}_K(\mathcal{P}) = \bigcup_{i=1,\ldots,n} \mathcal{T}_K(P_i)$.

The extension $\mathcal{D} = \{D_1, \ldots, D_n\}$ of a P2PDIS $\mathcal{P}$ is modeled as a sentence constituted by the conjunction of all facts corresponding to the tuples stored in the sources, i.e., $DB(\mathcal{D}) = \bigwedge_{i=1}^n DB(D_i)$ where $DB(D_i) = \mathbf{K_i}(\bigwedge_{t \in r^{D_i}} r(t))$.

A client of the P2PDIS interacts with one of the peers, say peer P_i, posing a *query* to it. A query q is an open formula $q(\mathbf{x})$ with free variables $\mathbf{x}$ expressed in the language accepted by the peer P_i (we recall that such a language is a subset of first-order logic). The semantics of a query $q \in \mathcal{L}$ posed to a peer $P_i = (i, G, S, L, M, \mathcal{L})$ of $\mathcal{P}$ with respect to an extension $\mathcal{D}$ is defined as the set of tuples $\{\mathbf{t} \mid \mathcal{T}_K(\mathcal{P}) \cup DB(\mathcal{D}) \models_{\mathsf{K45}_n^A} \mathbf{K_i}q(\mathbf{t})\}$, where $q(\mathbf{t})$ denotes the sentence obtained from the open formula $q(\mathbf{x})$ by replacing all occurrences of the free variables in $\mathbf{x}$ with the corresponding constants in $\mathbf{t}$.

Example 4 We are now able to provide the formalization of the P2PDIS of Example 3. The theories $\mathcal{T}_K(P_1), \ldots, \mathcal{T}_K(P_4)$ modeling the four peers are reported in Figure 2. ∎

It can be shown (see (Calvanese *et al.* 2005) for details) that the above formalization in $\mathsf{K45}_n^A$ provides a formal se-

$$\mathbf{K_1}(\forall x, y, y', z, z'.\, \mathsf{Person}_1(x, y, z) \wedge \mathsf{Person}_1(x, y', z') \supset y = y' \wedge z = z')$$
$$\mathbf{K_1}(\forall x, y.\, \mathsf{S}_1(x, y) \supset \exists z.\, \mathsf{Person}_1(x, y, z))$$
$$\forall x, z.\, \neg\mathbf{A_2}\bot_2 \wedge \mathbf{K_2}(\exists y.\, \mathsf{Citizen}_2(x, y, z)) \wedge \neg\mathbf{A_1}\neg(\exists y.\, \mathsf{Person}_1(x, y, z)) \supset \mathbf{K_1}(\exists y.\, \mathsf{Person}_1(x, y, z))$$

theory $\mathcal{T}_K(P_1)$

$$\mathbf{K_2}(\forall x, y, y', z, z'.\, \mathsf{Citizen}_2(x, y, z) \wedge \mathsf{Citizen}_2(x, y', z') \supset y = y' \wedge z = z')$$
$$\mathbf{K_2}(\forall x, y, z.\, \mathsf{S}_2(x, y, z) \supset \mathsf{Citizen}_2(x, y, z))$$

theory $\mathcal{T}_K(P_2)$

$$\mathbf{K_3}(\forall x, y, y', z, z'.\, \mathsf{Person}_3(x, y, z) \wedge \mathsf{Person}_3(x, y', z') \supset y = y' \wedge z = z')$$
$$\forall x, y.\, \neg\mathbf{A_1}\bot_1 \wedge \mathbf{K_1}(\exists z.\, \mathsf{Person}_1(x, z, y)) \wedge \neg\mathbf{A_3}\neg(\exists z.\, \mathsf{Person}_3(x, z, y)) \supset \mathbf{K_3}(\exists z.\, \mathsf{Person}_3(x, z, y))$$
$$\forall x, y, z.\, \neg\mathbf{A_4}\bot_4 \wedge \mathbf{K_4}(\mathsf{Citizen}_4(x, y, z)) \wedge \neg\mathbf{A_3}\neg\mathsf{Person}_3(x, y, z) \supset \mathbf{K_3}\mathsf{Person}_3(x, y, z)$$

theory $\mathcal{T}_K(P_3)$

$$\mathbf{K_4}(\forall x, y, y', z, z'.\, \mathsf{Citizen}_4(x, y, z) \wedge \mathsf{Citizen}_4(x, y', z') \supset y = y' \wedge z = z')$$
$$\mathbf{K_4}(\forall x, y, z.\, \mathsf{S}_4(x, y, z) \supset \mathsf{Citizen}_4(x, y, z))$$

theory $\mathcal{T}_K(P_4)$

Figure 2: Theories $\mathcal{T}_K(P_1), \ldots, \mathcal{T}_K(P_4)$ modeling the P2P system of Figure 1 in $\mathsf{K45}_n^A$

mantics to P2PDISs that, besides other things, exactly captures the two notions of inconsistency tolerance above defined. Indeed, from the above formalization it follows that:

- when inconsistency arises between local data and non-local data in a peer, i.e., when data coming from the peer sources through the local mapping contradicts the data retrieved by a peer through a P2P mapping, then the peer always prefers the local data. Formally, in this case there is one $\mathsf{K45}_n^A$-model for the P2PDIS, which represents the situation in which non-local data is discarded;
- when inconsistency arises between two different pieces of non-local data, i.e., when a piece of data retrieved by a peer through a P2P mapping contradicts another piece of data retrieved through the P2P mappings, then no preference is made between these two pieces of information, in the sense that in this case there are two $\mathsf{K45}_n^A$-models for the P2PDIS, each of which represents the situation in which one of the two pieces of data is discarded.

Conclusions

In this paper we have proposed a first attempt to define a *multi-modal*, *first-order*, *nonmonotonic* family of logics. In particular, the logics $\mathcal{S}_n^A$ presented in this paper generalize recent approaches in epistemic logic and nonmonotonic modal logics in many respects.

We have also illustrated the need for multi-modal nonmonotonic logics in the field of distributed systems. Interestingly, the possibility of modeling knowledge in distributed systems was also the initial motivation behind one of the first nonmonotonic modal logics, i.e., Halpern and Moses' logic of minimal knowledge (Halpern & Moses 1985).

An interesting extension of the present work is towards reasoning in the logics $\mathcal{S}_n^A$. The first results in this direction appear in (Calvanese *et al.* 2005), in which an algorithm is presented for reasoning in the restricted fragment of the logic $\mathsf{K45}_n$ which is able to logically modal information in P2P systems.

Finally, it would be very interesting to investigate whether the logics $\mathcal{S}_n^A$ can be characterized by fix-point semantics, in a way analogous to other families of nonmonotonic modal logics (Tiomkin & Kaminski 1990; Marek & Truszczyński 1993; Schwarz 1992b).

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