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Protection on Complex Networks with Geometric and Scale-Free Properties

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Abstract. *The mitigation of epidemic spreading through complex networks has been attracting more and more interest recently. The fundamental issue is how to effectively allocate immunization resources due to their high cost. Previous works have considered a lot of immunization strategies such as immunizing nodes in order of degree or betweenness centrality. In this paper, we propose a new immunization strategy based on the combination of closeness and betweenness centralities to find a subset of nodes whose immunization efficiently reduces the network vulnerability. Experiments show that for networks with geometric and scale-free properties, our strategy can get higher efficiency compared with the targeted immunization strategies based on single node centrality.*
Keywords: Immunization, Complex networks, Vulnerability, Epidemic spreading.

1. **Introduction.** Many complex interacting systems such as social interactions and the Internet can be modeled as networks where the components of the system are represented by the nodes of the network and the interactions between the components are represented by the links connecting nodes in the network [1]. Epidemic spreading processes such as infectious diseases and computer viruses are resulting in severer damage in larger networks of modern society. Due to limited budgets for network immunization, a well-established strategy should identify a small subset of nodes whose immunization results in the minimal network vulnerability.

Recently, several immunization strategies have been proposed. The most common one is called the targeted immunization algorithm which first ranks nodes based on an importance factor and then immunizes a fraction of nodes with the highest priority [2, 3, 4]. The importance factor of a node is also called node centrality. The most commonly used node centralities are degree centrality, betweenness centrality, eigenvalue centrality and closeness centrality. By far the largest amount of work has focused on the effect of removing nodes uniformly at random or in decreasing order of their degree. [14, 15, 16] study this question in considerable detail, and also discuss the related issue of percolation on networks. However, less is known about how the structure of networks change when nodes are removed according to non-local strategies of their importance. In [17] the effect of removing nodes both in decreasing order of degree and of betweenness centrality is considered. Related work considering the effect of removing nodes based on betweenness is also described in [18]. In this paper, we focus on how to find a limited subset of nodes whose immunization minimizes the network vulnerability using adaptive strategies of node centrality.

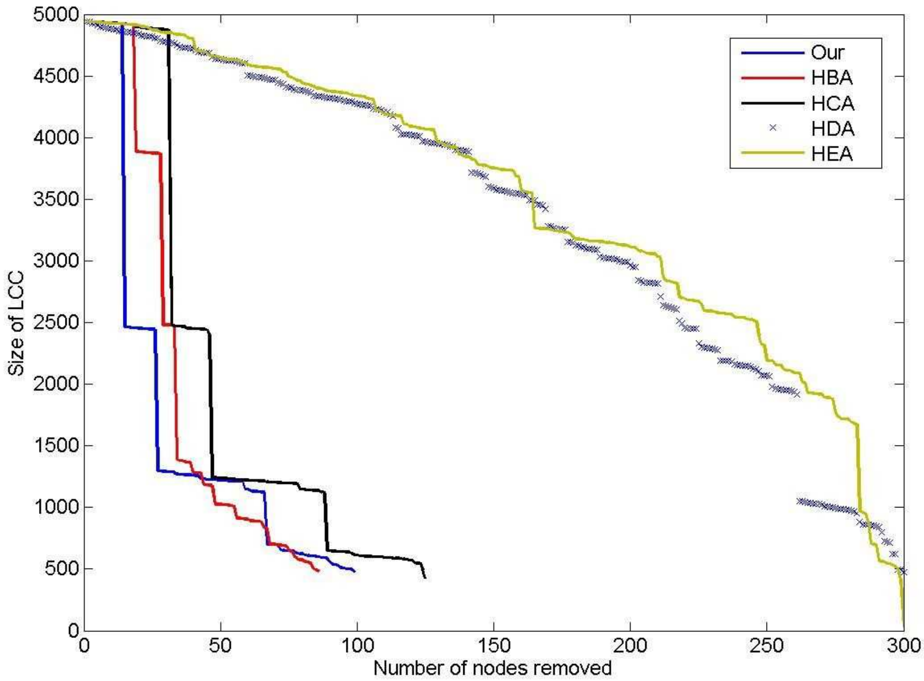


FIGURE 1. The size of LCC, versus the number of immunized nodes for HDA, HBA, HCA, HEA and our strategy for the power grid network with 4940 nodes.

2. **Problem Statement and Preliminaries.** Degree centrality defined as number of links connected to a node estimates the immediate impact of node infection. The method of immunizing nodes with highest degree centralities (HD) can efficiently reduce the growth rate of epidemic diseases [5].

Betweenness centrality is the proportion of a node lying on the shortest path between other nodes. It tests the level at which one given node connects to other nodes in the network and is defined as:

$$C_B(i) = \sum_{k,j} \frac{P_i(k,j)}{P(k,j)} \tag{1}$$

where k and j represent two nodes in the network, $P(k,j)$ is the total number of shortest paths between Node k and Node j in the network, and $P_i(k,j)$ is the number of those shortest paths that contain Node i. The method of immunizing nodes with highest betweenness centralities (HB) can eliminate lots of disease transmission routes.

Closeness centrality describes the level at which a given node can on average reach all other nodes in the network and is defined as:

$$C_C(i) = \sum_{j=1}^{N} \frac{1}{d(i,j)} \tag{2}$$

where i is a node in question and j is another node in the network, and $d(i, j)$ is the length of the shortest path between Node i and Node j. The method of immunizing nodes with highest closeness centralities (HC) can prevent disease transmission through the network center.

Eigenvector centrality considers the importance of neighbors of a node and is defined as:

$$\lambda\nu = \mathbf{A}\nu \tag{3}$$

where λ is the largest eigenvalue of the matrix $\mathbf{A}$ and ν is the corresponding eigenvector. $\mathbf{A}$ is called adjacency matrix which is a square matrix whose elements indicate whether pairs of nodes are adjacent or not in the network. For simple network, its element $\mathbf{A_{i,j}}$ is one when there is an link from node i to node j, and zero when there is no link. The diagonal elements of the matrix are all zero. The eigenvector centrality of Node i is the i-th entry in this eigenvector. The method of immunizing nodes with highest eigenvector centralities (HE) can prevent disease transmission among important nodes.

A simple improvement of the above strategies is to recalculate node centralities after immunizing a node. We call these adaptive strategies HDA, HBA, HCA and HEA corresponding to HD, HB, HC and HE respectively [6, 7].

3. **Proposed Scheme.** As we know, the network vulnerability can be defined as the size of the largest connected component (LCC) of a network. The reason is that in the worst case, if there is a single source of infection, the maximum number of infected individuals is equal to the size of LCC [8, 9]. A connected component is defined as a subset of nodes which all are reachable from each other. The immunized nodes can be removed from the network since they are neither infected nor infect others. Therefore, the problem of network immunization with limited budgets can be translated into the question of how to efficiently remove a certain number of nodes to reduce the size of LCC of a network. In other words, we always hope that the size of LCC of the obtained network is as small as possible after a fraction of nodes have been removed. Our question now becomes how to remove a given number of nodes which results in the least size of LCC. It is notable that the optimal node set to be removed is not necessarily composed of the k most optimal individuals when considered alone[10]. In other words, the targeted immunization which selects k most important nodes for immunization [6, 7] fails to solve the immunization problem of finding an optimal set of k nodes whose immunization minimizes the network vulnerability. This kind of problem has been proven to be NP-hard[11].

Among the adaptive strategies using node centrality, HBA is the most impressive one. It outperforms other strategies such as HDA, HCA and HEA in many kinds of network[12]. To our knowledge, there is no other strategy which is superior to HBA in terms of reducing the size of LCC. In this paper, we do not intend to propose a strategy exceeding HBA in any kind of network but to provide an alternative strategy when HBA do not perform optimally in some networks.

Our strategy is essentially a revised version of HBA strategy, which can be described as below:

Input: an undirected unweighted network G

Output: a set of immunized nodes

Repeat following steps until the size of LCC is smaller than $p * S$, where p is the threshold and S is the initial size of G.

Step 1. Calculate the highest betweenness centrality score of G and denote it by β.

Step 2. Choose nodes whose betweenness centrality score is larger than $\beta * w$, where w is the threshold with $0 < w < 1$.

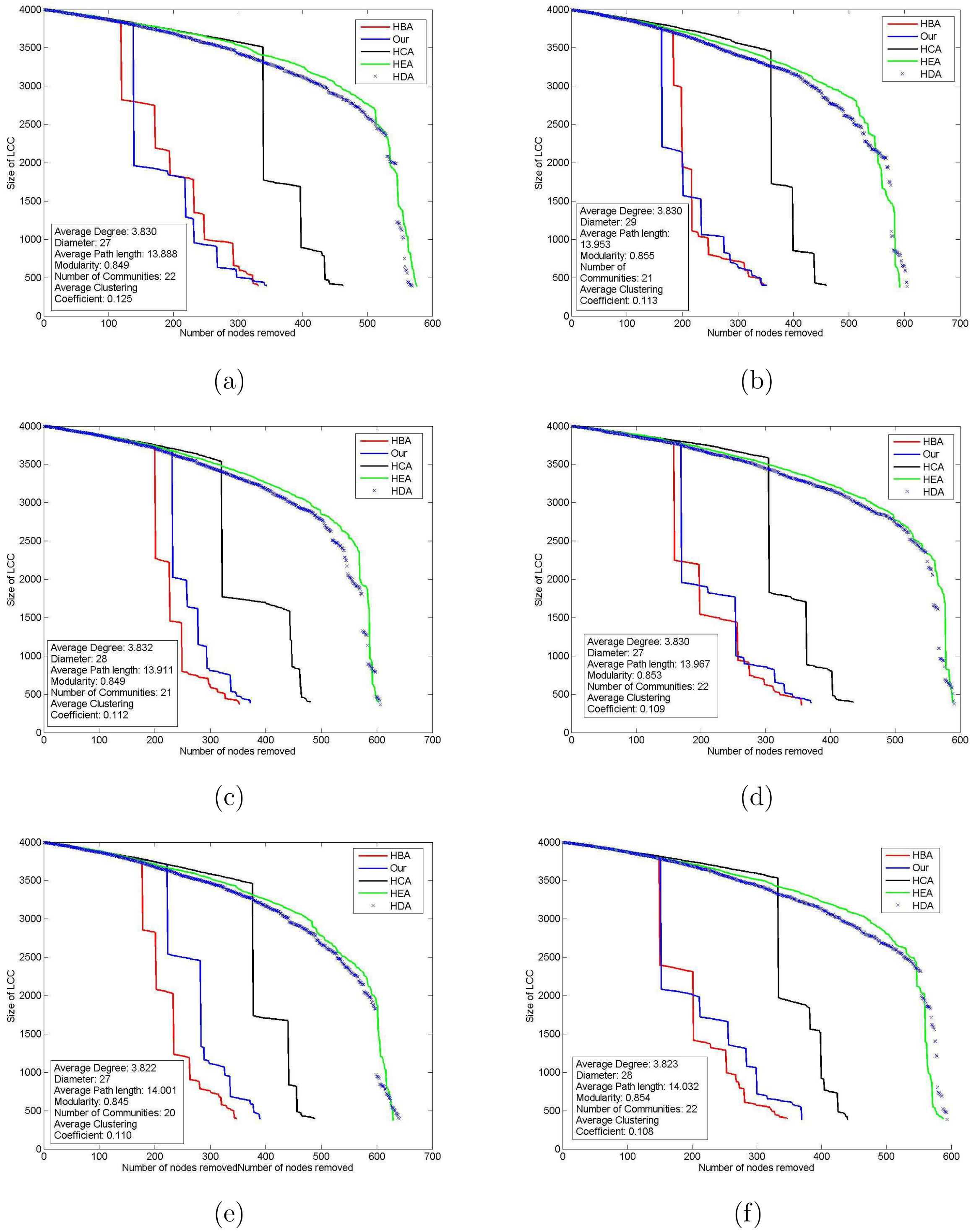
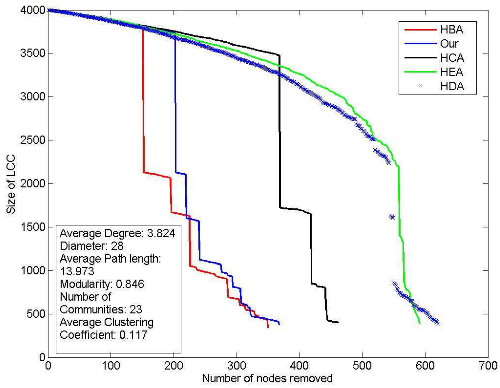


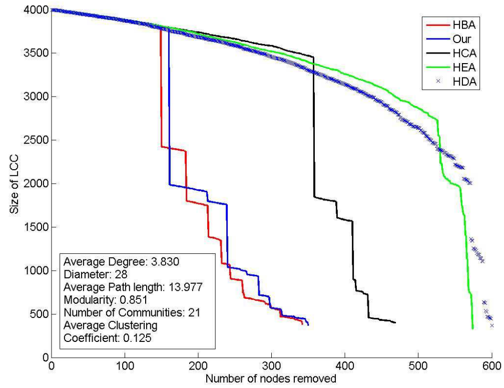
FIGURE 2. The size of LCC versus the number of immunized nodes under HDA, HBA, HCA, HEA and our strategies(denoted by BC) for geometric preferential attachment model networks of size 4000 with different parameters.

Step 3. For all the nodes chosen in Step 2, select the node with the highest closeness centrality score and remove it.

In our experiments, we let the w and p values be 0.92 and 0.1 respectively as default.



(g)



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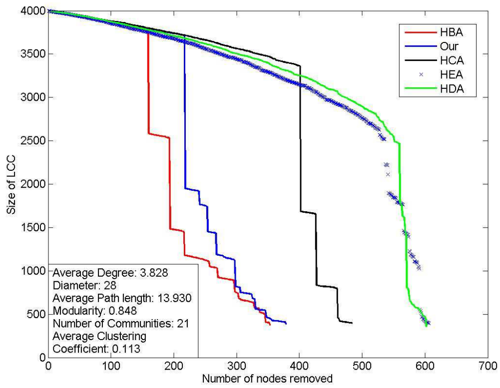

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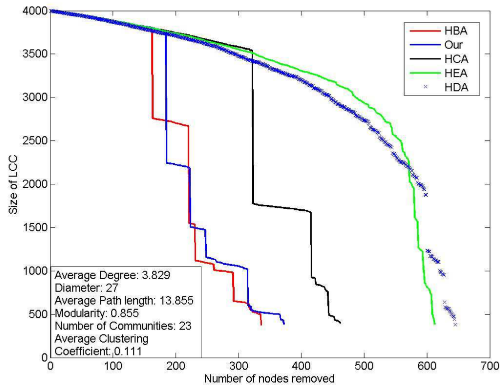

(j)

FIGURE 3. The size of LCC versus the number of immunized nodes under HDA, HBA, HCA, HEA and our strategies(denoted by BC) for geometric preferential attachment model networks of size 4000 with different parameters.

4. **Experimental Results.** In this section, we evaluate the effectiveness of our strategy by comparing it with HBA, HCA, HEA and HDA strategies in a so-called geometric preferential attachment network[13] and a real-world network (the power grid network of Western States of the United States). The geometric preferential attachment model combines geometric random graphs and preferential attachment graphs. Nodes are sequentially generated with random positions in space and therefore have a geometric distance between each other. Then nodes are connected within a fixed distance using the well-known preferential attachment method. We think this model is more precise to model some real-world networks than the commonly considered model networks such as ER and WS models. Here, for the geometric preferential attachment network, the size is 4,000 and the number of edges added to each node is 2 and the beta parameter which controls the reachable distance of nodes is 0.1.

We test the effectiveness of each strategy by plotting the size of LCC versus the number of nodes removed. We can see from Fig.1 that, in the power grid network, our strategy is better than the betweenness strategy when the size of LCC is between about 4900(99.2% of the initial size of LCC) and 1250(25.3% of the initial size of LCC). When the size of LCC is reduced to 50%, the nodes needed to be removed by our strategy is almost as half as it needed by HBA.

Fig.2 and 3 show the numeric results of applying different strategies to 10 geometric preferential attachment models with different parameters. In 4 cases (Figs.2(c), 2(e), 3(g) and 3(i)), our strategy is completely worse than HBA. In 2 cases(Figs.2(a) and 2(b)), our strategy has evident advantage over HBA and in the other 4 cases(Figs.2(d), 2(f), 3(h) and 3(j)), our strategy is superior to HBA in limited intervals.

From Figs. 1 and 2, we can see that the HCA curve approximately bisects the LCC size for each drop since a node with higher closeness centrality score is closer to the center of a network. The reason why our strategy beats HBA in some networks is that HBA always finds nodes with the highest betweenness but may fail to notice some center nodes with relatively low betweenness due to the structure of a network and it is those nodes that sharply reduce the LCC size. However, our strategy does not outperform the HBA strategy in networks without geometric distance and preferential-attachment properties such as ER and WS networks. One of the reasons is that there usually does not exist a fragile center part whose nodes are much less than other parts in this kind of networks due to the homogeneity.

5. **Conclusion.** In this paper, we have proposed a new strategy to find a subset of nodes whose removal minimizes the LCC size of a network. Our strategy provides an alternative to the HBA strategy when one deals with the immunization problem in networks with geometric and scale-free properties. Simulation results show that our strategy outperforms the HBA strategy under certain conditions. Although the statistics of the model networks are of little difference, the results of our strategy differ from each other when comparing with the HBA strategy. Which structure feature results in that is still unknown and thus finding the reason will be our future work.

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REFERENCES

[1] L. da F. Costa, O. N. Oliveira, G. Travieso, F. A. Rodrigues, P. R. Villas Boas, L. Antiqueira, M. P. Viana , and L. E. Correa Rocha, Analyzing and modeling real-world phenomena with complex networks: A survey of applications, *Advances in Physics*, vol. 60, no.3, pp. 329-412, 2011.

[2] C. Gao, J. Liu, and N. Zhong, Network Immunization with Distributed Autonomy-Oriented Entities, *IEEE Transactions on Parallel and Distributed Systems*, vol. 22, no. 7, pp. 1222-1229, 2011.

[3] C. M. Schneider, T. Mihaljev, S. Havlin, and H. J. Herrmann, Suppressing epidemics with a limited amount of immunization units, *Physical Review E*, vol. 84, no. 6, p. 061911, 2011.

[4] L. Hbert-Dufresne, A. Allard, J.G. Young, and L. J. Dub, Global efficiency of local immunization on complex networks, *Scientific Reports*, vol. 3, p. 6, 2013.

[5] J. Hadidjojo and S. A. Cheong, Equal graph partitioning on estimated infection network as an effective epidemic mitigation measure, *PloS One*, vol. 6, no. 7, p. e22124, 2011.

[6] Y. Chen, G. Paul, S. Havlin, F. Liljeros, and H. Stanley, Finding a Better Immunization Strategy, *Physical Review Letters*, vol. 101, no. 5, pp. 2-5, 2008.

[7] C. M. Schneider, T. Mihaljev, and H. J. Herrmann, Inverse targeting-An effective immunization strategy, *Europhysics Letters*, vol. 98, no. 4, p. 46002, 2012.

[8] L. Gallos, F. Liljeros, P. Argyrakis, A. Bunde, and S. Havlin, Improving immunization strategies, *Physical Review E*, vol.75, no. 4, pp. 1-4, 2007.

[9] T. Yoshida and Y. Yamada, Community structure based node scores for network immunization, PRICAI 2012: *Trends in Artificial Intelligence*, vol. 7458, no. 899902, pp. 1-6, 2012.

[10] S. P. Borgatti, Identifying sets of key players in a social network, *Computational and Mathematical Organization Theory*, vol. 12, no. 1, pp. 21-34, 2006.

[11] J. Aspnes, K. Chang, and A. Yampolskiy, Inoculation strategies for victims of viruses and the sum-of-squares partition problem, *Journal of Computer and System Science*s, vol. 72, no. 6, pp. 1077-1093, 2006.

[12] S. Iyer, T. Killingback, B. Sundaram, and Z. Wang, Attack robustness and centrality of complex networks, *PloS One*, vol. 8, no. 4, pp. 8-11, 2013.

[13] D. Abraham, M. Alan, and J. Vera, A geometric preferential attachment model of networks, *Internet Mathematics*, vol. 3, no. 2, pp.187-205, 2006.

[14] R. Albert, H. Jeong, A. Barabasi, Error and attack tolerance of complex networks, *Nature*, vol. 406 pp.378-382, 2000.

[15] D. Callaway, M. Newman, S. Strogatz, D. Watts, Network Robustness and Fragility: Percolation on Random Graphs, *Physical Review Letters*, vol. 85, no.25, pp.5468-5471, 2000.

[16] R. Cohen, S. Havlin, D. Ben-Avraham, Efficient immunization strategies for computer networks and populations, *Physical Review Letters*, vol. 91, no. 24, 247901, 2003.

[17] P. Holme, B. Kim, C. Yoon, S. Han, Attack vulnerability of complex networks, *Physical Review E*, vol. 65, no.5, 056109, 2002.

[18] P. Crucitti, V. Latora, M. Marchiori, A. Rapisarda, Efficiency of Scale-Free Networks: Error and Attack Tolerance, *Physica A: Statistical Mechanics and its Applications*, vol. 320, pp. 622-642, 2003.