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Spatial-Spectral Reweighted Collaborative Sparse Regression for Hyperspectral Unmixing

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Abstract. *Sparse unmixing of hyperspectral data plays an important role in hyperspectral image analysis. It aims at estimating the fractional abundances of pure endmembers based on the assumption that each mixed pixel in the hyperspectral image can be expressed in the form of linear combinations of a number of known and pure endmembers. The wide availability of large spectral libraries has fostered the role of sparse regression techniques in the task of characterizing mixed pixels in remotely sensed hyperspectral images. Introducing weighted factors to penalize the nonzero coefficients in the unmixing solution is a recent main trend. In this paper, based on collaborative sparse unmixing, considering the iterative reweighted algorithm and spatial-spectral information, a spatial-spectral reweighted collaborative sparse unmixing(*$\mathrm{S^2RCSU}$*) has been utilized. The weighted factor is introduced to promote the column sparsity of the fractional abundances in the sense and the row sparsity along the abundance vector corresponding to each endmember at the same time. Experimental results on both simulated data sets and real hyperspectral data sets demonstrate that the proposed method is an accurate and effective sparse unmixing algorithm when compared with other advanced hyperspectral unmixing methods.*
Keywords: Hyperspectral unmixing, Reweighted collaborative sparse regression, Spatial-spectral,Spectral unmixing

1. **Introduction.** Hyperspectral imagery unmixing has become an essential procedure, which decomposes a mixed pixel into a collection of constituent materials (called endmembers) and their relative proportions (called abundances). It aims at decomposing pixels of a hyperspectral image into constituent spectra and their relative proportions. Linear spectral mixing model and nonlinear spectral mixing model are two basic models in hyperspectral unmixing which were used to analyze the mixed pixel problem[1]. The linear model assumes that the spectral response of a pixel is given by a combination of the endmembers present in the pixel. Due to its computational tractability and flexibility, the linear mixture model has been widely applied for many different applications. The linear model exhibits practical advantages such as easy to implement and flexible to apply. However this model is barely true for the relatively low spatial resolution of state-of-the-art imaging spectrometers.

In order to solve the problems related to the unavailability of pure pixels, sparse unmixing which has gained much interest was developed[2]. It amounts to finding the optimal subset of signatures from a very large spectral library that can best model each mixed pixel. Because of the fact that there are typically only a few endmembers inside a pixel compared with the large spectral library, the sparse unmixing algorithm via variable splitting and augmented Lagrangian (SUnSAL)[3] is the seminal work developed for this purpose. The l_1 norm with convenient and efficient solution method is used in the sparse unmixing model in SUnSAL algorithm. But l_1 norm is not sparser enough, Xu proposed the $l_{1/2}$ regularization [4, 5, 6] instead of the l_1 regularizer for finding an accurate and effective solution.

Without considering spatial structure information of hyperspectral data, the high correlation of spectral libraries imposes some limitations to the performance of SUnSAL. Aiming at exploiting spatial contextual information, under the sparse regression framework some methods[7, 8, 9] were introduced, especially SUnSAL-TV[10]. The experiments of SUnSAL-TV algorithm indicate the potential of including spatial information on sparse unmixing for improved characterization of mixed pixels in hyperspectral imagery. The collaborative sparse unmixing algorithm via variable splitting and augmented Lagrangian (CLSUnSAL)[11] was proposed under the global assumption that all pixels in a hyperspectral image share the same active set of endmembers. Comparing with the SUnSAL algorithm, CLSUnSAL algorithm under the global assumption can highly improve the sparse solution that the abundance fractions corresponding to the fake endmembers are forced to be zero. Nevertheless, a main limiation of CLSUnSAL is endermembers tend to appear localized in spatially homogeneous areas intead of distributed over the full image. For settling this problem, we proposed the local collaborative sparse unmixing (LCSU)[12] which assumes that neighboring pixels share the same active set of endmembers.

Due to the appreciable performance by considering spatial structure information in hyperspectral data[13], we propose a new hyperspectral unmixing strategy of spatial-spectral reweighted collaborative sparse unmixing which is called S^{2}RCSU. The proposed method aims at incorporating spatial-spectral information and seeking the efficient sparser regularizer by introducing a weighted factor into collaborative sparse unmixing which promotes the column sparsity of the fractional abundances in the sense and the row sparsity along the abundance vector corresponding to each endmember at the same time. Comparing to the SUnSAL and CLSUnSAL algorithms, the experimental results demonstrate that the proposed S^{2}RCSU algorithm can achieve a better hyperspectral unmixing accuracy. The rest of this letter is organized as follows. Section 2 is the introduction of classical collaborative sparse unmixing algorithm. In Section 3 the proposed S^{2}RCSU algorithm is described in detail. Section 4 presents the experimental results. Conclusions are drawn in Section 5.

2. **Collaborative Sparse Unmixing Algorithm.**

As assuming the spectral endmembers as a known large spectral library , the sparse unmixing model can be formulated as follows:

$$Y = AX + N \tag{1}$$

where Y is an $L \times 1$ column vector of a mixed pixel, and mixed pixel always contains r pure spectral endmembers, X is the fractional abundance vector which related to the library $A \in R^{L \times p}$. Based on the fact that $p \gg r$, the X vector will have many values of zero, which we call it as sparseness. Then, we can formulate the unmixing problem as a $l_q(0 \leq q \leq 1)$ norm optimization problem.

As we all known, l_0 regularization sparse unmixing can be written as:

$$\min_X \frac{1}{2} \left\| AX - Y \right\|_F^2 + \lambda \left\| X \right\|_0 \qquad s.t. X \geq 0, 1^T X = 1 \tag{2}$$

where λ is the regularization parameter, $\|X\|_0$ represents the l_0 norm of the vector of X , which $X \geq 0$ counts the nonzero components in the matrix X , and $1^T X = 1$ denotes the sum of the abundance in a mixed pixel is 1.

The problem in (2) is nonconvex and very difficult to solve. However based on a certain condition of the restricted isometric property, the l_1 norm can approximately replace the l_0 norm, so the objective function can be written as follows:

$$\min_X \frac{1}{2} \|AX - Y\|_F^2 + \lambda \|X\|_1 \qquad s.t. X \geq 0, 1^T X = 1 \tag{3}$$

where $\|X\|_1$ denotes the l_1 norm. The optimization problem in (3) is convex and efficient, though the l_1 regularization always introduces extra bias in sparse unmixing. Then it cannot seek the fractional abundance with the much sparser solutions.

Based on the sparsity property of the regularizer and its influence on the unmixing performance has not been thoroughly investigated, Bioucas-Dias and Iordache proposed the $l_{2,1}$ mixed norm, called collaborative regularization, which globally impose sparsity among the endmembers collaboratively for all pixels. Let $X = [x_1, x_2, \cdots, x_n]$ be the abundance fractions which correpsonds to the sparse solution, where n is the number of pixels in the image, the collaborative sparse regression can be given as:

$$\min_X \frac{1}{2} \|AX - Y\|_F^2 + \lambda \sum_{k=1}^{m} \left\|X^k\right\|_2 \qquad s.t. X \geq 0, 1^T X = 1 \tag{4}$$

where X^k denotes the k-th line of the matrix X , the $\sum_{k=1}^{m} \|X\|_2$ is the so-called $l_{2,1}$ mixed norm which promotes sparsity among the lines of X, m is the number of lines in X and also the number of the endmembers in spectral library.

3. **Proposed Method.** Based on the iterative reweighted l_1 minimization algorithm[14, 15, 16] which consists of solving a sequence of weighted l_1 minimization problems where the weights used for the next iteration are computed from the value of the current solution, and considering the spatial-spectral information, a spatial-spectral reweighted collaborative sparse unmixing (named S^{2}RCSU) has been utilized into the classical collaborative sparse regression formulation. Then the formulation (4) can be written as:

$$\min_X \frac{1}{2} \|AX - Y\|_F^2 + \lambda \sum_{u=1}^{m} \sum_{v=1}^{n} \|W \odot X^{u,v}\|_{2,1/2} \qquad s.t. X \geq 0, 1^T X = 1 \tag{5}$$

where the operator $\odot$ denotes the element-wise multiplication of two variables, $X^{u,v}$ is the u-th line of the matrix X and v-th column of the matrix X , and λ is a regularization parameter which controls the degree of sparseness. The $\sum_{u=1}^{m} \sum_{v=1}^{n} \|X^{u,v}\|_{2,1/2}$ is the so-called $l_{2,1/2}$ mixed norm which promotes sparsity among the lines and columns of X. The W is a weighted factor which is introduced to promote the column sparsity of the fractional abundances in the sense and the row sparsity along the abundance vector corresponding to each endmember at the same time. On account of iterative reweighted minimization algorithm which can enhance the sparsity of endmembers in spectral library and improve the sparsity of fractional abundances, the weighted vector is inserted for the next iteration which is computed from the value of the current solution. In the interest of achieving much sparser solution of the optimization, we let

$$W_j^{k+1} = \frac{1}{\left|x_j^k\right| + \sigma} \tag{6}$$

where σ is a small value constant for controlling the value of weighted vector and preventing the error of the algorithm when x_j^k approaches to zero. As to ensure reliable performance for the algorithm, the parameter σ would automatically be adapted to the dynamic range and the sparsity of the object under study. Comparing to the paper [16], the introduced weighted vector is quite simple and easy to calculate but stably to catch the sparser solution. Based on the reweighted algorithm, when there is a sparse solution in optimal problems it can find the desirable answer with convergence may occur in just very few steps.

Given the objective function (6), we write the following constrained formulation:

$$\begin{aligned} &\min_{\boldsymbol{U},\boldsymbol{V_1},\boldsymbol{V_2},\boldsymbol{V_3},\boldsymbol{V_4}} \frac{1}{2}\left\|\boldsymbol{V_1}-\boldsymbol{Y}\right\|_F^2 + \lambda\left\|\boldsymbol{W}\odot\boldsymbol{V_2}\right\|_{2,1/2} + \boldsymbol{l}_{R^+}(\boldsymbol{V_3}) + \boldsymbol{l}_{\{1\}}(\boldsymbol{1}^T\boldsymbol{V_4}) \\ &\textit{subject to} \qquad \boldsymbol{V}_1 = \boldsymbol{AU}, \boldsymbol{V_2} = \boldsymbol{U}, \boldsymbol{V_3} = \boldsymbol{U}, \boldsymbol{V_4} = \boldsymbol{U} \end{aligned} \tag{7}$$

where $\boldsymbol{l}_{R^+}(\boldsymbol{V_3})$ represents abundance non-negativity constrains (ANC), $\boldsymbol{l}_{\{1\}}(\boldsymbol{1}^T\boldsymbol{V_4})$ denotes the sum of the abundance in a mixed pixel is 1 (ASC). Then based on the ADMM algorithm [17], the augmented Lagrangian of problem (7) can be written in a compact form as follows:

$$\min_{\boldsymbol{U},\boldsymbol{V}} \boldsymbol{g(V)} \quad \textit{subject to} \quad \boldsymbol{GU}+\boldsymbol{BU}=\boldsymbol{0} \tag{8}$$

where

$$\boldsymbol{g(V)} \equiv \frac{1}{2}\left\|\boldsymbol{V_1}-\boldsymbol{Y}\right\|_F^2 + \lambda\left\|\boldsymbol{W}\odot\boldsymbol{V_2}\right\|_{2,1/2} + \boldsymbol{l}_{R^+}(\boldsymbol{V_3}) + \boldsymbol{l}_{\{1\}}(\boldsymbol{1}^T\boldsymbol{V_4}) \tag{9}$$

With these definition in place, we can implement the alternative direction method of multiplier to solve the optimization problem involved in S^2RCSU, as shown in Algorithm 1.

4. **Experimental Results.** In this section, we illustrate the unmixing performance of the proposed spatial-spectral reweighted collaborative sparse unmixing using simulated data sets and real hyperspectral data sets in comparison to SUnSAL, CLSUnSAL. Regarding the performance discriminators adopted in our experiments, the quality of the results is measured using the signal-to-reconstruction error as follows:

$$SRE(dB) = 10\log_2\left(E(|X|_2^2)/E(|X-\widehat{X}|_2^2)\right) \tag{10}$$

where X represents the true abundances,$\widehat{X}$ represents the estimated abundances, and $E(\cdot)$ represents the expectation function. The larger the SRE(dB) is, the more accurate the unmixing is.

4.1. **Simulated datasets experiments.** In this experiments, the spectral library A used in our experiments was generated by randomly extracting from the United State Geological Survey (USGS) library denoted splob06[18], which size was $A \in R^{224\times240}$ contains $m = 240$ members with $L = 224$ spectral bands.The reflectance values are measured for 224 spectral bands distributed uniformly in the interval $0.4 - 2.5\mu m$. The mutual coherences of the two libraries are very close to one.

Algorithm 1 $\mathrm{S^2RCSU}$ Algorithm

1: Intialization:

2: set $k = 0$, choose $\mu \geq 0$ $\quad \mathbf{U_{(0)}}, \mathbf{V_1^{(0)}}, \cdots, \mathbf{V_3^{(0)}}, \mathbf{V_4^{(0)}}, \mathbf{D_1^{(0)}}, \cdots, \mathbf{D_3^{(0)}}, \mathbf{D_4^{(0)}}$

3: Repeat:

4: $\mathbf{U}^{(k+1)} \leftarrow ((\mathbf{A^T A} + \mathbf{2I}))^{-\mathbf{1}}(\mathbf{A^T}(\mathbf{V_1^{(k)}} + \mathbf{D_1^{(k)}}) + (\mathbf{V_2^{(k)}} + \mathbf{D_2^{(k)}}) + (\mathbf{V_3^{(k)}} + \mathbf{D_3^{(k)}}) + (\mathbf{V_4^{(k)}} + \mathbf{D_4^{(k)}}))$

5: $\mathbf{V_1}^{(k+1)} \leftarrow \frac{1}{1+u}[\mathbf{Y} + \mu(\mathbf{AU}^{(k)} - \mathbf{D_1^{(k)}})]$

6: $\mathbf{V_2}^{(k+1)} \leftarrow \text{vector\_soft\_row\_1\_2}(\mathbf{U}^{(k+\mathbf{1})} - \mathbf{D_2^{(k)}}, \mathbf{W}\lambda/\mu)$

7: $\mathbf{V_3}^{(k+1)} \leftarrow \max\ (\mathbf{U}^{(k)} - \mathbf{D_3^{(k)}}, \mathbf{0})$

8: $\mathbf{V_4}^{(k+1)} \leftarrow (\mathbf{U}^{(k)} - \mathbf{D_3^{(k)}})$+repmat $((1\text{-sum}(\mathbf{U}^{(k)} - \mathbf{D_4^{(k)}}))/n, n, 1)$

9: Update Lagrange multipliers:

10: $\mathbf{D}^{(k+1)} \leftarrow \mathbf{D}^{(k)} - \mathbf{AU}^{(k+1)} + \mathbf{V}^{(k+1)}$

11: Update $\boldsymbol{k} \leftarrow \boldsymbol{k}+1$

12: Until the stopping criterion is satisfied

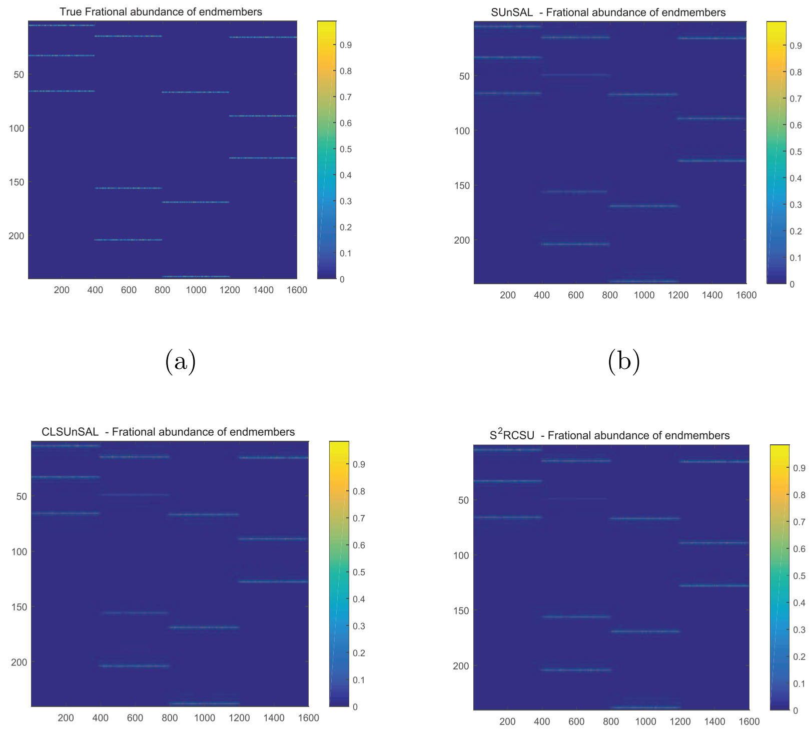
The simulated data cube (DC) used in our experiments have 4 different regions, a total of $n = 1600$ pixels, with three spectral signatures for each region selected from the library A. The fractional abundances of the endmembers follow a Dirichlet distribution. After generating the datacube, it was contaminated with i.i.d. Gaussian noise, for three levels of the signal-to-noise (SNR) ratio: 30, 40 and 50 dB.

Especially, for all the algorithms, the input parameters have been carefully tuned for optimal performance, and all reported results are obtained from average of 20 algorithm executions. In Table 1, some parameter values are given, such as λ and σ, which are sensitive to different data and will influence the unmixing accuracy or have impact on the objective criterias optimization.

Table 1. SRE(DB) salues obtained by different unmixing algorithm in DC

Data	SNR	SUnSAL	CLSUnSAL	$\mathrm{S^2RCSU}\sigma = 1$
DC	30	4.67 ± 0.09 $\lambda = 2 \times 10^{-2}$	4.67 ± 0.08 $\lambda = 3 \times 10^{-3}$	5.95 ± 0.08 $\lambda = 1 \times 10^{-2}$
	40	8.49 ± 0.21 $\lambda = 2 \times 10^{-4}$	9.67 ± 0.22 $\lambda = 1 \times 10^{-3}$	10.71 ± 0.89 $\lambda = 2 \times 10^{-2}$
	50	14.84 ± 0.22 $\lambda = 1 \times 10^{-5}$	17.28 ± 0.19 $\lambda = 1 \times 10^{-4}$	25.03 ± 0.31 $\lambda = 6 \times 10^{-3}$

In Table 1, it shows a comparison between the three algorithms of simulated data cube for the stopping criterion is satisfied with $k = 700$. From Table 1, we can see that the proposed $\mathrm{S^2RCSU}$ algorithm obtain better SRE(dB) results than other algorithms in all cases. Furthermore, the SRE value is improved, especially the improvement of SRE value obtained with regard to the SUnSAL and CLSUnSAL algorithms is significant in the condition of low noise. This is based on $\mathrm{S^2RCSU}$ algorithm introduces a weighted factor which can promote the column sparsity of the fractional abundances in the sense and the row sparsity along the abundance vector corresponding to each endmember at the same



(a) (b) (c) (d)

Figure 1. Truth and estimated abundance of endmembers in DC with SNR of 50 dB. (a) truth abundance, (b) SUnSAL estimated abundance, (c) CLSUnSAL estimated abundance, and (d) S^2RCSU estimated abundance.

time. On the other hand, the reweighted algorithm also can help to find the desirable answer quickly with convergence may occur in just few steps.

Furthermore, Figure 1 shows the abundance maps estimated for endmembers in DC. They show a graphical comparison of the performances between S^2RCSU method and the other algorithms when compared with the truth abundance. Obviously, we can get that the abundance maps obtained by S^2RCSU are much more similar to the truth abundance maps from Figure 1. In one word, it can be seen that the proposed method combining collaborative sparse and iterative reweighted minimization algorithm can promote the spatial correlation and spectral sparseness on the solution and improve unmixing performance.

4.2. **Real hyperspectral data experiments.** In this section, In our real hyperspectral data experiments we resort to the well-known Airborne Visible Infrared Imaging Spectrometer (AVIRIS)[19] Cuprite data set for evaluation of the proposed approach, which is a common benchmark for validation of spectral unmixing algorithms. The portion used in experiments corresponds to a 250191-pixel subset, comprising 224 spectral bands between 400 and 2500 nm, with nominal spectral resolution of 10 nm. Prior to water absorption and low SNR, bands 1-2, 105-115, 150-170, and 223-224 are removed, only leaving 188 spectral bands.

The parameters in real spectral data experiments are set as $\lambda = 1 \times 10^{-4}$ in SUnSAL, $\lambda = 1 \times 10^{-3}$ in CLSUnSAL, $\lambda = 1 \times 10^{-3}$, $\sigma = 1$ in S^2RCSU. For the illustrative purposes, the Tricorder map was produced in 1995, but the publicly available AVIRIS Cuprite data were collected in 1997 which are adopt as a reference to make a qualitative analysis of the performances of the different sparse unmixing algorithms.

Table 2. Fractional abundance maps estimated by SUnSAL,CLSUnSAL and S^{2}RCSU, as compared with the classification maps produced by usgs tricorder software

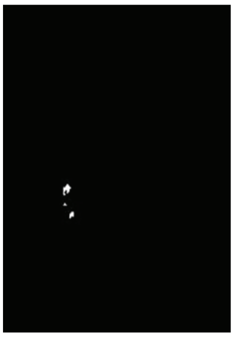
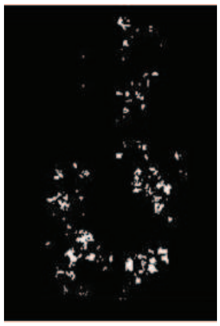
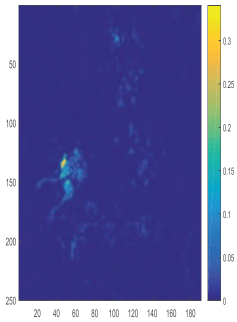
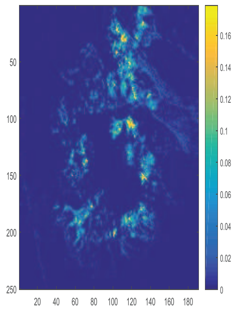
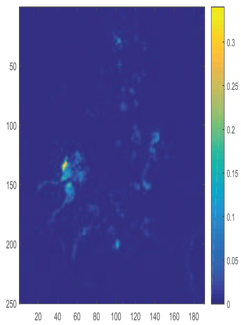
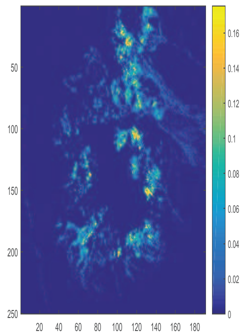
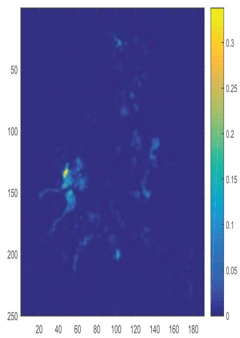
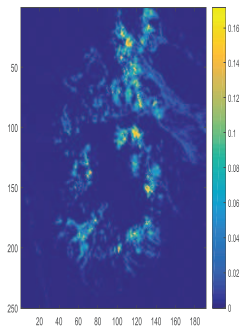
Algorithm/Materials	*buddingtonite*	*alunite*
Tricorder		
SUnSAL		
CLSUnSAL		
S^{2}RCSU		

Table 2 shows the fractional abundance maps estimated by proposed S2RCSU algorithm and the other algorithms with two different minerals of buddingtonite and alunite in a visual comparison. Thus, we can only make a qualitative analysis of the performances of different sparse unmixing algorithms by comparing their estimated abundances with the minerals map. From Table 2, due to comparison to the reference maps, we can see that the fractional abundance estimated by S^2RCSU algorithm is obviously much more accurate and comparable in the regions assigned to the respective materials. Thus, it can be concluded that S^2RCSU algorithm is a valid tool including spatial information and spectral information for sparse unmixing in real hyperspectral imagery data.

5. **Conclustions.** In this letter, based on collaborative sparse unmixing, considering the iterative reweighted algorithm and spatial-spectral information, a spatial-spectral reweighted collaborative sparse unmixing has been utilized. In the proposed method, the weighted factor is introduced to promote the column sparsity of the fractional abundances in the sense and the row sparsity along the abundance vector corresponding to each endmember at the same time. And we applied the $l_{2,1/2}$ norm instead of $l_{2,1}$ norm in collaborative sparse unmixing for achieving a sparser solution. Due to that, it aimed at incorporating spatial structure information and seeking the efficient sparser regularizer in just very few steps. Then the optimization problem was simply solved by the variable splitting and augmented Lagrangian algorithm with no variable introduced. The experimental results with simulated and real hyperspectral data sets demonstrated that the proposed S^2RCSU algorithm can achieve a better spectral unmixing accuracy.

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