MATHEMATICAL MODELS FOR AUTOMATIC LINE DETECTION

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ABSTRACT
The goal of the work here reported Is to Investigate algorithms which find the real straight edges In scenes consisting of prismatic solids, taking Into account smooth variations In Intensity over faces, blurring of edges, and noise. To this end we give In thts paper a model of the appearance to an optical Input device of such scenes; and describe a sub-optimal statistical decision procedure, based on the model, for the Identification of a line within a narrow band on the field of view, given an array of Intensities from within the band.

I. INTRODUCTION

In the following six sections a model-based line predicate Is developed analytically. The remaining two sections summarize the results In a computationally feasible form, and discuss empirical Investigations of Its behavior. The reader may wish to examine these two sections prior to undertaking a study of the analysts upon which the results are based.

II. THE BASIS OF THE DECISION PROCEDURE

Our procedure for determining the existence of a real edge within a narrow sub-rectangle on some field of view, given a set of Intensity values J from within the sub-region, Is to compute a probability function of these Intensities, P(CB/J), and to compare this value with a preasslgned threshold value. P(CB/J) Is the conditional probability, given that J was obtained over the sub-region, that a boundary line was actually present and centered In the region. The use of probabilistic techniques reflects the assumption that noise prevents us from having access to the set of actual "nolse-free" Intensity values, Ij over the domain. This procedure can be shown (1, 2) to be optimal In the sense of making the best decision about the existence of a line In the region over which the Intensity sample J is taken, given only this Intensity sample, and no spatial context Information about the possible existence of other lines In the scene, e.g. cotermlnal with, or parallel to the line In question. In particular the procedure of accepting the hypothesis of the existence of an edge when PCCB/J) is above some threshold has the following properties:

1) Suppose It Is desired to keep the probability of a false positive error (false claim that a line exists) at or below a certain value. Then there exists a threshold for PCCB/J) which guarantees this bound on the false positive error rate and at the same time minimizes the probability of missing an actual line.

2) Similarly there exists a threshold for P(CB/J) which guarantees a particular bound on the probablllty of missing lines and at the same time gives a minimum rate of false positive errors.

3) Further, there exists a threshold value for PCCB/J) which guarantees that the sum of the probabilities of missing a line and of making a false positive error Is minimized.

The general strategy for establishing a value for PCCB/J) is based on some simple Bayeslan probability analysis. First, we note that by definition:

$$P(CB/J) = \sum_{I_i \in CB} P(I_i/J) \qquad \text{C2.1)}$$

where CB Is the set of I_i 's over narrow rectangular domains containing the image of a centered boundary line. Formula C2.1) follows from the fact that PCCB/J) is the conditional probability that the actual Intensity pattern over the given sub-region is some member of CB, given that the pattern J was obtained over the region. It Is thus the sum over all elements of CB, of the conditional probability of this element being the Intensity pattern, given that the pattern J was obtained. From Bayes' rule we have:

PCI, /J) - PCJ,I,)/PCJ)

where PCJ) Is the a priori probability of occurrence of the obtained Intensity pattern J and PCJ,I_i) Is the Joint probability of obtaining that pattern along with I_i being the real pattern of Intensities. Using this relation, (2.1) becomes:

$$PCCB/J) - \frac{\sum_{I_i \in CB} P(J, I_i)}{P(J)} \qquad (2.2)$$

Clearly P(J), the a priori probability of existence of J, is the sum of J's

Joint probability of existence along with some member of CB, and Its Joint probability of existence with a member of CN (the complement of CB In the set CI = $\{I_i\}$), since this exhausts all possible cases. Hence:

$$P(J) = \sum_{I_i \in CB} P(I_i, J) + \sum_{I_i \in CN} P(I_i, J)$$

so that (2.2) becomes:

$$P(CB/J) = \frac{\sum_{I_i \in CB} P(J, I_i)}{\sum_{I_i \in CB} P(J, I_i) + \sum_{I_i \in CN} P(J, I_i)} \quad (2.3)$$

Again, from Bayes' rule we have:

$$P(J, I_i) = P(J/I_i)P(I_i).$$

Thus we can express P(CB/J) as follows:

$$P(CB/J) = \frac{\sum_{I_i \in CB} P(J/I_i)P(I_i)}{\sum_{I_i \in CB} P(J/I_i)P(I_i) + \sum_{I_i \in CN} P(J/I_i)P(I_i)} \quad (2.4)$$

The remainder of this paper will be devoted to modelling the set CI, so as to provide a basis for expressing the various probabilities In the above expression. The end result will be a closed form threshold function Q(J) giving an approximate value of P(CB/J) as a function of the set of Intensities contained In the set J.

III. DEFINITION AND CATEGORIZATION OF CI

Mathematically the elements of CI are real valued functions of two variables defined over a narrow rectangular region of the plane. A member I_i of CI Is to be thought of as giving the value of the light Intensity, as a function of position, over some narrow rectangular sub region of a two dimensional Image. It should be emphasized that the Intensity functions we have In mind In this section are those giving the real Intensities over the sub regions In question, In the absence of any noise and distortion which would be introduced by some physical Instrument designed to obtain Intensity values. We shall limit out attention to Intensity functions taken over domains of a single preasstgned size. It will be apparent that no loss of generality will result from so doing.

We will define a function to be a member of CI If It gives the Intensity over some rectangular sub region of a two dimensional Image of some scene having the following properties:

1) The scene consists of prismatic (plane-faced) solids.
2) The Intensity over the faces of the objects, and In the background, Is a smooth continuous function of position.
3) At the Images of the real edges In the scene, the Intensity Is either simply discontinuous In value (hereafter this will be referred to as an "edge"); or has an anaomalous value with equal Intensities Just to either side, as produced by a crack between blocks or by a highlight. (The latter will heafter be referred to as "lines").

Further, by assuming that the Intensity values are actually the logarithms of the Intensities In the scene, we have the additional properties:

4) The difference In Intensity across an "edge" will be approximately constant along Its length, although the absolute Intensity along the edge may smoothly vary,
5) The difference between the anomalous Intensity along a "line" and the Intensity of the Immediately surrounding area will be approximately constant along the length of the line.

Note that the possible existence of shadows need pose no problem, since real edges partially In shadow may be considered for the purposes of the model to be two distinct "lines" or "edges". Further, the edges of the shadows themselves have properties Identical to those of "edges", and may be so modeled.

We can be more specific about the previously Indicated division of CI Into two subsets CB and CN:

Let CB be the subset of CI consisting of all Images of "boundaries", I.e. "lines" or "edges", exactly centered In, and traversing the entire length of, the rectangular domain of definition. An example:

$$I(x,y) = \begin{cases} .1y + 2.5 & x < .5 \\ .1y + 3.5 & x \geq .5 \end{cases}$$

over the rectangle bounded by the points (0,0), (1,0), (0,10) and (1,10).

Let CN be the remainder of CI.

The class CB may be further subdivided In accordance with the distinction made earlier between an "edge" and a "line":

>Let CE be the subset of CB consisting of those functions representing simple discontinuities of Intensity. The function given as an example under the definition of CB above Is of this type.

>Let CL be the remainder of CB, I.e. those Intensity functions representing a

line of anomalous Intensity, such as from a highlight or crack between two blocks. An example:

$$I(x,y) = \begin{cases} -.4y + 3 & x = .5 \\ -.4y + 6 & x \neq .5 \end{cases}$$

over the rectangle bounded by the points (0,0), (1,0), (0,10) and (1,10).

Finally we may subdivide the class CN Into two sub-classes:

>Let CH be the subset of CN consisting of Intensity functions over rectangular sub-regions of the plane which are not crossed by Images of any 1 Ines of the scene. An example:

$$I(x,y) = .3x + .2y + 10.$$

over the region bounded by the points (0,0), (1,0), (0,10) and (1,10).

> Let CS be the remainder of CN. Intensity functions In this class are those over rectangular sub regions of the plane containing the Image of part of a line, or possibly a closely spaced pair.

The heirarchy of sub categories Just described is summarized In the following diagram:

- **CI**
  - **CB Lines centered In the rectangle**
    - **CL Cracks and highlights**
    - **CE Simple edges**
  - **CN The rest of CI**
    - **CH Homogeneous regions**
    - **CS Parts of lines etc.**

Figure 1.

IV. NUMERICAL CHARACTERIZATION OF THE MEMBERS OF CI AND A PROBABILITY DISTRIBUTION ON THEM

As a further step In modeling the set CI, we would like to be able to characterize an element of any of the above subsets by some simple numerical description. The functional descriptions given In the above examples are too extensive to lend themselves to the sort of analysis we wish to perform. By way of slmpllflcaton, we Intend to omit any reference to absolute Intensity, treating, for example, the following two functlons:

$$f(x,y) = \begin{cases} .2y + 3 & x < .5 \\ .2y + 5 & x \geq .5 \end{cases}$$

and

$$g(x,y) = \begin{cases} .2y + 4 & x < .5 \\ .2y + 6 & x \geq .5 \end{cases}$$

as equivalent, as they differ only by a constant over their domains. Members of CE and CH will thus be characterized only by their "relative amplitudes" or amplitude of discontinuity along their lengths. The above examples are misleadingly simple in that this relative amplitude has been a constant over the whole length of the domain. We actually Intend that there be a small "ripple" in the relative amplitude along the lines, as was Implied by the expression "approximately constant" In assumptions 4) and 5) above.

Although we may thus characterize a member of CL or CE by a nearly constant function of one variable, it is more practical for the following analysis to assume the domain of definition to be divided into n approximately square sub components and give a description In terms of the n average "relative amplitudes" over these regions. We propose the notation $I(a_1, \ldots, a_n)$ for a member of CL, and $I(b_1, \ldots, b_n)$ for a member of CE. The function f(x,y), a member of CE, Just referred to would be thus described by the 10-tuple (2, 2, .. , 2), and would be denoted by 1(2,

Members of CH are similarly treated, with average gradient within a square sub-region In place of average relative Intensity. For these elments, we will use the notation $I(c_1, \ldots, c_n)$.

The reader may note that no reference has been made to the remaining class composing CI, namely CS. Suffice It to say here that in the analysis which follows, such a characterization is unnecessary, provided one Is willing to trade computational feasibility for a small amount of mathematical exactness. The reader might further be puzzled that no reference is made In a description of a member of CL or CE to the underlying crosswise gradients to either side of the discontinuity, while members of CH are described entirely In terms of a gradlert. The answer Is that In the case of CH, the gradients are the principal effect, whereas in the case of members of CL and CE they are not. In the analysis which follows the gradients in the members of CE and CL are Indeed Ignored, since no provision is made for them in the n-tuple descriptions. However in (1) it is shown how these can be taken into acccount to a first order of approximation.

We are now In a position to put a probability distribution on the members of the sets CL, CE and CH based on the above chatracterlzatlon. Specifically, we shall express the a priori probability of existence of a member, e.g. of CL, as a relatively simple function of (a_{i1} , ... , a_{in}), the n relative amplitudes which are used to describe It.

To begin with, assume that each highlight or crack In scenes under consideration, I.e. each member of CL, has an "Idealized amplitude" a. This should be thought of as the relative amplitude which would occur along the whole length of the highlight or crack in the absence of minor Irregularities in lighting or defects In the objects themselves. Let us further assume that the probability distribution P(a) on this parameter Is known. Finally let us assume that for all highlights or cracks with Idealized amplitude a, the various actual relative amplitude values a, are statlstleally Independent; and that they have a normal distribution with mean a and standard deviation σ Independent of a. From this It follows that the probability PC 1 (a_d , .. , $a_{,n}$)/a)), I.e. the probability of existence of a crack or highlight with actual relative amplitudes a_{i1} , ... a_{in} along Its length, given that the Idealized amplitude was a, is given by:

$$P(a) \prod_{s=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{a_{is} - a}{\sigma}\right]^2} \quad da$$

To obtain P(I(a_{i1} , ... , a_{in})), the a priori probability of existence of the highlight or crack with relative amplitudes a_{ij} , ... , a_{in} , we need only sum the above expression over all values of a. Hence for I, a member of CL we have:

$$P(I_i) = P(I(a_{i1}, \ldots, a_{in})) = \sum_a P(a) \prod_{s=1}^{n} F(a, a_{is}) \qquad (4.1)$$

where:

$$F(a, a_{is}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{a_{is} - a}{\sigma}\right]^2} da_{is} \qquad (4.2)$$

Similar formulas hold for members of CE and CH, with b and c In place of a, P' and P" in place of P, and a' and a" In place of σ .

V. THE PROBABILISTIC RELATIONSHIP BETWEEN AN OBTAINED SET OF INTENSITIES AND A NOISE FREE INTENSITY FUNCTION

In this section we shall specify a plausible relationship between obtained "nolsy" (tensity samples taken over long rectangular regions and actual possible intensity functions over such regions, i.e. the members of CL, CE and CH. The object will be to give analytic closed form expressions for the values of the conditional probabilities $P(J/I_i)$as apear In the decision predicate (2.4). The relationship between an obtained sample and possible actual Intensity fuctlons is determined by two effects. The first of these Is the blurring Inherent in any optical system, which turns theoretically sharp Intensity discontinuities Into smooth slopes, and turns highlights, hypothesized as lines of discontinuity of virtually no width, Into smooth ridges. The second effect Is random time noise.

It will be assumed In what follows that the degree of blurring of edges that Is Inherent In the Intensity samples J obtained by an actual optical Input device from scenes of the sort described Is homogeneous and Isotropic over all scenes under consideration. This assumption might be violated If the scenes had suflclent depth that some parts would be significantly out of focus. However In using the device mentioned In (1) and In the last part of this paper It Is possible to have the optical focus blurring dominated by other blurring effects, which are spatially unlform.

It Is Instructive In considering the relationship between an obtained Intensity sample J and members of CL CE or CH to consider the result of applying this blurring to members of the latter sets. These blurred Intensity functions will be denoted by the symbol I_i'. It Is easy to show that If blurring Is uniform and Isotropic over a field of view, It acts on an Intensity function I(x,y) to produce a blurred function I'(x,y) as follows:

$$I'(x,y) = \iint B(x-t, y-u)\, I(t,u)\, dt\, du$$

where B(x,y) Is the Intensity function resulting from applying the blurring to an Intensity function having the value zero except at the origin of the Image space. Now If I_i belongs to the class CL then the corresponding blurred version I has the property that Its value along any normal to the major axis of the rectangular domain is given by ag(x) + b, where a Is the relative amplitude of the discontinuity at the point where the normal Intersects the major axis, and g(x) is related In a simple way to the function B(x,y) Just mentioned. It should be emphasized that g(x) Is not dependent on any property of I, other than that It Is a members of CL. Similar results hold for members of CE with a function h(x) In place of g(x); the latter being the Integral of the former. Typically g(x) will resemble a "bell curve"; and h(x) a simple step function with rounded corners. The case of members of CH Is similar, but wlih the simple linear function l(x)=mx In place of g(x) or h(x). These properties will be useful later In this section.

The geometry over the Image plane of the points at which Intensities are to be obtained has thus far been left unspecified. We will stipulate that this geometry be the same for all of the square sub-regions Into which the rectangular domain of the line predicate Is divided. In particular we stipulate that It consist of a set of m points lying along a line traversing the square sub-region through Its center, normal to the major axis of the main rectangular band.

The discussion of the "blurred" Intensity functions 1_1' can now be carried one step farther by considering the resut of restricting these functions to the finite set of points whose geometry has just been described. The resulting Intensity function, which we shall denote by the symbol I,", are clearly formally similar to the sets J of obtained Intensity values. The nature of the relationship between these two classes, and Its Implications In developing closed form expressions for the conditional probabilities $P(J/I_i)$ will be discussed presently. Meanwhtle, various properties of the functions I," will be noted. First, each consists of n m-tuples of values, each m-tuple being the value of the corresponding function I,' over a set of m points lying along one of a set of equally spaced normals to a m,aJor axis of the domain of definition of I, . Secondly, If I_1 Is in the class CE, CL or CH, then any m-tuple from a corresponding I" Is determined entirely from the values of two parameters, and from whether Ii came from CL, CE or CH. This follows from the properties of the functions I,' mentioned In the paragraph before last, and from the uniformity of the geometry of the points In the domain of the function 1_1". This may be formally restated as follows: There exists an m-tuple (V_1 , ... , V_m) such that If I(a,I, ... , $a_{,n}$) Is a member of CL, then the corresponding I_i" is given by:

$$I_i'' = [(a_{i1} V_1, \ldots, a_{i1} V_m) \quad \ldots\ldots\ldots\ldots \quad (a_{in} V_1, \ldots, a_{in} V_m)] \tag{5.1}$$

For I, a member of CE there exists a corresponding m-tuple (V_1' , •• , V_m'); and for I, a member of CH an m-tuple (V" , ... , V_m"). These m-tuples are simply the values of the functions g(x), h(x) and I(x) mentioned In the discussion of the functions 1_i', suitably scaled.

We now address ourselves to an expression for the value of P(J/I,). This, It may be recalled, Is the conditional probability of obtaining the Intensities comprising J, over the region In question, given that the actual Intensity over the region Is given by the function I, . If the latter Is a member of CL, CE or CH, then the corresponding I_i" as given by (5.1) gives what the values In the set ought to be In the absence of noise. This noise Is assumed to be uniform over the field of view of the Intensity measuring Instrument; and values x of Intensity taken repeatedly at a particular point are assumed to have a distribution around a mean of x given by:

$$\frac{1}{r\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{x - \bar{x}}{r}\right]^2} dx$$

Thus If the sample J Is expressed In the form of n m-tuples, as was I_i" In (5.1), by:

$$J = [(u_{11}, \ldots, u_{1m}) \quad \ldots\ldots\ldots\ldots \quad (u_{n1}, \ldots, u_{nm})] \tag{5.2}$$

then P(J/I,) Is given by:

$$P(J/I_i) = P(J/I(a_{i1}, \ldots, a_{in})) = \prod_{s=1}^{n}\prod_{t=1}^{m} \frac{1}{r\sqrt{2\pi}} e^{-\frac{1}{2}(u_{st} - a_{is} V_t)^2} du_{st} .$$

This may be expressed In a form similar to (4.1):

$$P(J/I_i) = P(J/I(a_{i1}, \ldots, a_{in})) = \prod_{s=1}^{n} G(a_{is}, u_{s1}, \ldots, u_{sm}) \tag{5.3}$$

where:

$$G(a_{is}, u_{s1}, \ldots, u_{sm}) = \frac{1}{[r\sqrt{2\pi}]^m} e^{-\frac{1}{2}\sum_t (u_{st} - a_{is} V_t)^2} du_{s1} \ldots du_{sm} . \tag{5.4}$$

VI. TOWARD A CLOSED FORM FOR PCCB/J)

In this section we shall show how to

express certain sums constituting the numerator and denominator of our line decision function (2.4) as closed form functions of the Intensity values from a rectangular region. First, let:

$$CL(J) = \sum_{I_i \in CL} P(J/I_i)P(I_i) \qquad (6.1)$$

$$CE(J) = \sum_{I_i \in CE} P(J/I_i)P(I_i) \qquad (6.2)$$

$$CH(J) = \sum_{I_i \in CH} P(J/I_i)P(I_i) \qquad (6.3)$$

$$CS(J) = \sum_{I_i \in CS} P(J/I_i)P(I_i) \qquad (6.4)$$

and note that P(CB/J) In (2.4) may be expressed In terms of the foregoing as:

$$P(CB/J) = \frac{CL(J) + CE(J)}{CL(J) + CE(J) + CH(J) + CS(J)} \qquad (6.5)$$

It Is the goal of this section to give closed form expressions for the sums CL(J), CE(J), and CH(J) making use of the closed form expressions (4.1), (4.2), (5.3) and (5.4). The matter of the sum CS(J) will be deferred to the next section.

Now (6.1) may be rewritten using (4.1) and (5.3) as:

$$CL(J) = \sum_{I_i \in CL}\left[\prod_{s=1}^{n} G(a_{is}, u_{s1}, \ldots u_{sm})\right]\left[\sum_a P(a)\prod_{s=1}^{n} F(a, a_{is})\right]$$

Because summation of I_i over elements of CL Is simply summation over all possible n-tuples of $(a_{i1}, \ldots, a_{in})$, this formula becomes, with a bit of rearranging:

$$CL(J) = \sum_a P(a)\sum_{a_{i1}}\cdots\sum_{a_{in}}\prod_{s=1}^{n} F(a, a_{is})G(a_{is}, u_{s1}, \ldots u_{sm})$$

with, again, F and G given by (4.2) and (5.4) respectively. Also, It ts easy to show that the product over s and the sum over the a_s 's may be Interchanged, so that:

$$CL(J) = \sum_a P(a)\prod_{s=1}^{n}\sum_{a_{is}} F(a, a_{is})G(a_{is}, u_{s1}, \ldots u_{sm}) \qquad (6.6)$$

As a consequence of Ignoring absolute Intensity we may assume that the obtained Intensities have been normalized by sub regions so that:

$$\sum_t V_t = 0$$

$$\sum_t u_{st} = 0$$

and using the definitions:

$$a_s^* = \sum_t u_{st} V_t \Big/ \sum_t V_t^2 \qquad (6.7)$$

$$b = \sum_t V_t^2 \qquad (6.8)$$

$$R_s = \sum_t (u_{st} - a_s^* V_t)^2 \qquad (6.9)$$

It Is not difficult to rewrite (5.4) as:

$$G(a_{is}, u_{s1}, \ldots, u_{sm}) = \frac{1}{(r\sqrt{2\pi})^m} e^{-\frac{1}{2}\left[\frac{b(a_{is} - a_s^*)^2 + R_s}{r^2}\right]} \qquad (6.10)$$

Combining (6.6) with (4.2) and (6.10):

$$CL(J) = \sum_a P(a)\prod_{s=1}^{n}\sum_{a_{is}} \frac{1}{\sigma\sqrt{2\pi}\,(r\sqrt{2\pi})^m} \times e^{-\frac{1}{2}\left[\frac{b(a_{is} - a_s^*)^2 + R_s}{r^2} + \frac{(a - a_{is})^2}{\sigma^2}\right]} \qquad (6.11)$$

with b, a_s^*, and R_s given by (6.7) (6.8) and (6.9). The Inner sum may be put In the form:

$$A\sum e^{-Ba + Ca + D} \; da$$

which admits of the approximation:

$$A\sum e^{Bx^2 + Cx + D} \approx$$

$$A\int e^{-Bx^2 + Cx + D} = \frac{A\sqrt{\pi}}{\sqrt{B}} e^{C^2/4B + D}$$

Applying this approximation to the Inner sum In (6.11) we have, after considerable algebra:

$$CL(J) \approx \sum_a P(a)\prod_{s=1}^{n}\frac{r}{\sqrt{r^2 + b\sigma^2}\,(r\sqrt{2\pi})^m} \times e^{-\frac{1}{2}\left[\frac{R_s}{r^2} + \frac{b(a_s^* - a)^2}{r^2 + b\sigma^2}\right]}$$

where the differentials have been omitted for clarity. This may alternatively be expressed:

$$CL(J) \approx \sum_{a} P(a)\left[\frac{r}{(r\sqrt{2\pi})^{m}\sqrt{r^2+b\sigma^2}}\right]^{n} e^{-\frac{1}{2}\left[\frac{\sum_s R_s}{r^2} + \frac{b\sum_s (a^*_s - a)^2}{r^2+b\sigma^2}\right]}$$

Assuming the Idealized relative amplitudes have a normal distribution with mean 0 and standard deviation p, then:

$$P(a) = \frac{P(CL)}{\rho\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\frac{a}{\rho}\right]^2}$$

where P(CL) Is the a priori probability of a "line". This latter expression for CL(J) may be put Into the form:

$$A\sum_{a} e^{-Ba^2 + Ca + D}$$

and approximated as before to yield:

$$CL(J) \approx P(CL)\left[\frac{r}{(r\sqrt{2\pi})^{m}\sqrt{r^2+b\sigma^2}}\right]^{n}\left[\frac{\sqrt{r^2+b\sigma^2}}{\sqrt{b\sigma^2+nb\rho^2+r^2}}\right] \times \qquad (6.12)$$

$$e^{-\frac{1}{2}\left[\frac{\sum_s R_s}{r^2} + \frac{b\sum_s a^{*2}_s}{b\sigma^2+nb\rho^2+r^2} + \frac{nb^2\rho^2\sum_s (a^*_s - \bar{a}^*)^2}{(b\sigma^2+r^2)(b\sigma^2+nb\rho^2+r^2)}\right]}$$

Similar formulas exist for CE(J) and CH(J).

VII. RATIONALE FOR IGNORING THE CLASS CS

A an analysis similar to the foregoing/ with the goal of obtaining a closed form expression for CS(J), may be avoided. It has been shown In (1) that Q(J) given by:

$$Q(J) = \frac{CL(J) + CE(J)}{CL(J) + CE(J) + CH(J)} \qquad (7.1)$$

has the following properties:

1) Q(J) and PCCB/J) are nearly equal when J consists of Intensity values from a narrow region which contains no part of a "boundary", I.e. an edge, hlghllght or crack.

2) Q(J) and P(CB/J) are nearly equal when J consists of Intensities from within a narrow rectangular region containing a centered boundary line.

3) Values of Q(J) for J's from various narrow rectangular regions approximately centered on a boundary line have a maximum for the set J from a region In which the boundary Is exactly centered.

If care Is exercised, Q(J) thus serves as a substitute for PCCB/J). Problems do arise, as the foregoing would predict, when Q(J) is applied to Intensity sets from a narrow rectangular band crossed at an angle of, e.g., 30 degrees by an Intense boundary line. It was necessary In practise to Include a heuristic procedure for eliminating the spurious line evidence produced by these oblique crossings of a region by an Intense boundary.

VIM. SUMMARY: A LINE DETECTION PROCEDURE

The foregotng analysis yields a four step procedure for deciding on the existence of a line between some pair of points on the Image of a scene consisting of plane faced prismatic solids.

The first step Is to make n equally spaced scans across the proposed edge. This yields n m-tuples of Intensity values. The second step Is to compute six values, a*, Ri , a*', R,', a*" and R" from each of these m-tuples. Tne third step is to compute from these 6n values the value of:

$$Q'(J) = \left(\sum_i R''_i + K''_1 \sum_i a^{*''2}_i + K''_2 \sum_i (a^{*''}_i - \bar{a}^{*''})^2\right) - \qquad (8.1)$$

$$\mathrm{Min}\left(\left(\sum_i R_i + K_1 \sum_i a^{*2}_i + K_2 \sum_i (a^*_i - \bar{a}^*)^2\right),\ \left(\sum_i R'_i + K'_1 \sum_i a^{*'2}_i + K'_2 \sum_i (a^{*'}_i - \bar{a}^{*'})^2\right)\right)$$

where the summations are taken from i=1 to 1=n, and the six (positive) constants K1, .. K2" are derivable from the constants In the exponents of (6.12). Finally this value Is compared with a threshold, and a line Is claimed to exist If the value exceeds It.

The calculation required to obtain the six values a*, ... , R," from an m-tuple of Intensity' values Is quite simple. Consider the Intensities obtained from the I-th scan to be an m-vector $\bar{U}_i$. Then there exist m-vectors $\bar{V}$, $\bar{V}'$ and $\bar{V}''$ such that:

$$a^*_i = C_1 \bar{V}\bullet\bar{U}_i$$

$$a^{*'}_i = C'_1 \bar{V}\bullet\bar{U}_i \qquad (8.2)$$

$$a^{*''}_i = C''_1 \bar{V}\bullet\bar{U}_i$$

where • Is the standard dot-product operator. The vectors V, V' and V" are constants, the same for each value of I. Each "R" value Is a linear combination of a corresponding "a*" value with the dot product of U, with Itself, I.e., for some C_2 $C_2^{\prime}$ ... $C_3^{\prime\prime}$ independent of i:

$$R_i = C_2 a_i^* + C_3 \bar{U}_i \bullet \bar{U}_i$$

$$R_i^{\prime} = C_2^{\prime} a_i^{*\prime} + C_3^{\prime} \bar{U}_i \bullet \bar{U}_i \qquad (8.3)$$

$$R_i^{\prime\prime} = C_2^{\prime\prime} a_i^{*\prime\prime} + C_3^{\prime\prime} \bar{U}_i \bullet \bar{U}_i$$

The values a*,R1,, ... R", derived from the m successive 'intensity values from some short scan across a scene, have a simple Intuitive significance. Recall that they are obtained from (8.2) and (8.3) using three m-tuples of constants symbolized as the vectors $\bar{V}$, $\bar{V}^1$ and V". The m-tuple of values comprising, e.g., V, are simply the m Intensity values which would be obtained from scanning across a perfectly clean paradigm highlight by an Intensity measuring device whose blurring characteristics are identical with the one used In obtaining U,, but which has a negligible noise level. Of course the separation of the m points Involved In this "paradigm highlight profile" must be the same as the actual separation of the pojnts at which the Intensities comprising U, are obtained. Now suppose that the "paradigm profile" V Is least squares fitted to the set of Intensities U, . The factor by which the former must be multiplied In order to achieve this best fit Is simply some constant multiple of ai*. We shall thus refer to a value of a* as the "highlight distinctness" of the set of Intensities U, from which It was obtained; and refer to the value of R, as the "highlight similarity" of Ui . It may be shown that a* has the property of being near zero when the "shape" of the profile Ui from which It was computerd Is not at all hlghlight-1lke. We may similarly refer to a_i^{*1} as the "edge distinctness" of Ui ;' and to $R_i^{\prime}$ as the "edge similarity" Finally V" 'is simply a linearly Increasing sequence of values; so that a*" is a measuer of local gradient and R" a measure of the aparent homogenlty of the surface across which the corresponding U_i was obtained.

It may be shown that the threshold procedure Involving (8.1) Is approximately equivalent to the procedure Involving (7.1) and (6.12). The proof of approximate equivalence, which will not be given explicitly here, depends on two properties of the latter two expressions. First, If the three terms given by (6.12) are plugged Into (7.1), and the (omitted) mn differentials and mn factors of r, common to all terms, are cancelled; then the resulting coefficients of e are all less than 1. Second, It may be shown (1) that If the value of (7.1) Is, e.g., above .95, then the two terms of the numerator are necessarily quite unequal. It appears to be necessary to threshold at above this value If the predicate Is to be applied a large number of times to avoid a significant number of false positive errors.

IX. EMPIRICAL INVESTIGATIONS

Investigations of the present line detection procedure were based on Intensity Information obtained from a computer controlled Image dissector (Information International "VIdIssector"). This device makes available to the computer Intensity values from any point on a 40000, by 40000,grd. Intensity values are logarItmIcally scaled with about 80 gray levels for every factor of two In actual Intensity.

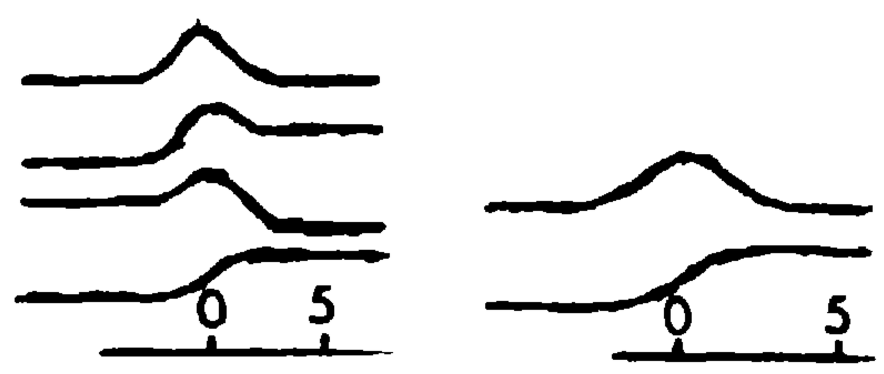
Several dozen scenes, mostly consisting of plane-faced white prismatic solids, were analysed, with the following conclus Ions:

>ERIFICATION OF UNDERLYING ASSUMPTIONS. It was first necessary to Investigate the validity of various assumptions up on which the foregoing analysis Is based, namely:

1) Intensity values supplied by the Image dissector have an appreciable random noise component which Is Gaussian, and whose level Is Independent both of Intensity and location.
2) Profiles of Intensity along scans normal to real edges In analysed senes are either cliff-shaped or peak-shaped.
3) Various profiles of one or the other of these two types differ only In amplitude, and not In width or skewness.
4) Intensity scans taken normal to a real edge, at regular Intervals along Its length show a good degree of consistency as to amplitude and type.

The first assumption Is theoretically guaranteed by the design of the Image dissector used. Various empirical Investigations were made which verified this.

Assumptions two and three were found to be largely true In that the shape of an Intensity profile across a particular edge appeared to be Independent of position on the field; and also the shapes of Intensity profiles taken across various highlights or edges between equally lighted uniform faces differed only by multiplicative constants. However, In the scenes analysed there appeared to be a larger variety of types of edge profiles than expected, particularly across rather subtle boundaries. A reasonable representative set of six profiles Is Illustrated In figure 2.



1 unit = .1% of field width

Figure 2.

Assumption 4 appeared to be true to a marked degree for Intense edges. However this was not always the case for very subtle edges. The Implications of this will be discussed later.

The existence of more than two edge types does not especially Invalidate the foregoing theory and analysis, which may clearly be generalized to six types Instead of two. This results In a decision function similar to (8.1) but with the minimum taken over six expressions Instead of two. These six expressions are calculated using (8.2) and (8.3) and a set of six m-vectors of constants. Each of these vectors Is obtained from one of the profiles of Figure 2, and consists of a set of successive values taken at equally spaced points. The number (m) and spacing will now be discussed.

>GEOMETRY OF THE SAMPLING DOMAIN. Two spatially local parameters govern the acquisition of Intensity Information to be given to the present line predicate. These are the length of each of the n sub-scans, and the number (m) of Intensities In each.

It seemed to be of prime Immportance that the length of the scan be Just enough to encompass the entire shape of the various profiles. Referring to figure 2, this would appear to be about 15 units or 1.5% of the width of the field. If this were much wider, a greater diversity of paradigm profiles would have to be used reflecting various different combinations of gradient values (slopes) of the "tails" of the profiles. Experiments Involving scan lengths of 1%, *2%* and 1.5% of the field width confirmed the relative superiority of the latter.

The most Important criterion for establishing point separation seemed to be that the spacing be suflclently fine that a scan across a highlight would Include at least one value suflclently close to the center of the peak as to have a value within, e.g., 90% of the maximum value at the peak. Referring to Figure 2, the reader may see that the corresponding separation Is one or two units, or about .1% of the field width of the present Instrument. Experiments Involving a spacing of .05% Indicated no particular advantage over .1%.

For a majority of subsequent experiments values of the (generalized) threshold function (8.1) and terms thereof, were computed (using (8.2) and (8.3)) from a specific set of 7 16-vectors of constants derived from the profiles In figure 2, matched to a spacing of .1% of the field width. Intensity values were read at this spacing In sets of 16. The reported results were based on exhaustive calculation of various "R" and "a*" values for scenes of prismatic solids. Typically 100 scans of 500 Intensity values each were made covering the entire scene. For each scan 485 values of each of the 7 "a*"'s and "R"'s were computed from each successive 16-tuple of Intensity values. These values were graphed or printed line by line for analysls.

>LOCAL SENSITIVITY. Restricting the function given by (8.1) (or rather the mentioned generalization of It) to a single scan ($n = 1$) results In a simplified formula, since the $(a^*-a^*)^2$ terms are all zero. Further, as will be later explained, the a^{*2} terms are of doubtful significance. There thus results a non-linear decision function based on the values of seven "R"'s.

This localized version of the present procedure was compared with two other local edge detection operators. The first operator Is due to Roberts (3) and uses the following function of four Intensity values v_1, v_2, v_3 and v_4 from the corners of a small square:

$$\sqrt{(v_1 - v_3)^2 + (v_2 - v_4)^2}$$

In (1) It was shown that the present edge predicate, even taking Into account Its use of more Intensity values, Is more than twice as sensitive as the Roberts operator. The present operator was also compared with a second difference operator discussed In *(4)* Involving four successive Intensity values V_1, v_2, v_3 and v_4, obtained at equal Intervals along a line normal to a proposed edge. The formula Is given by:

$$2(v_3 - v_2) - (v_2 - v_1) - (v_4 - v_3)$$

It may be shown by an argument similar to the one cited In connection with the Roberts operator that the present procedure Is slightly more sensitive In detecting edges than the second difference method provided the latter's point spacing Is wide relative to the blurring. Referlng to the edge profiles at the bottom of figure 2, the reader should note that If these four Intensity values are obtained at 1 unit Intervals around the center of the edge, the value of the operator Is considerably below that which would be obtained If the Intensity values were obtained at, e.g., four unit Intervals. Further If the edge were exactly centered on second or third point, the value of the operator would similarly be reduced. This four value operator thus becomes progersslvly less and less sensitive as the point spacing dereases. This Is true of the Roberts operator as well. On the other hand, In order to avoid missing highlights, the point spacing should be quite close.

These observations Indicate a marked superiority of the persent procedure In the local detection of edges, etc, provided the cost of obtaining Intensity values Is considerably greater than the cost of computation. In the present Image dissector- computer system this was Indeed the case. Intensity values required an average of one millisecond; whereas the entire time required to calculate a set of 7 "a*" and "R" values from a set of 16 Intensity values was approximately 500 microseconds. In order not to miss highlights, the successive Intensity values In a scan must be obtained at a relatively close spacing. The effective use of a simple second difference operator on such Intensity values would, as cited above, requtre that e.g., only every fourth Intensity value of the fine scan be used. Its sensitivity would thus be considerably below that of the present operator, which uses all the Intensity values.

>AVERAGE APPARENT DISTINCTNESS. As may be Inferred from the discussion earlier In this section, the terms In (8.1) Involving the sums over the values a* and a^{*1} give an average measure of how distinct an edge or highlight may be fitted to the obtained profiles. These terms seem to be of little value In predicting the existence of a real edge or line. Two reasons may explain this. First, large values actually provide negative evidence In (8.1) as to the existence of an edge or line, reflecting the fact that according to the model very distinct edges are relatively Infrequent. A second problem Is that a very distinct edge crossing the examined region at an oblique angle will produce a spurious high value In this sum.

>CONSISTENCY OF EDGE DISTINCTNESS. The seven sums over terms of the form $(a_i^*-\bar{a}^*)^2$, $(a^{*\prime}_i-\bar{a}^{*\prime})^2$, etc. In the generalized version of (8.1) have a simple Intuitive significance. Consider the sum corresponding to the uppermost profile of figure 2. This consists of n terms of the form $(a_i^*-\bar{a}^*)^2$, where a* Is the average of the n a*'s. Since each a* Is a measure of the "highlight ' distinctness" of the scan Intensities from which It was computed, the entire sum gives a measure of the variance of the highlight distinctness along the length of the proposed line. Thus each of the seven terms gives a measure of the consistency of the proposed line along Its length.

There was Indeed a striking consistency of this sort among the more distinct edges, hlghllghts and cracks, In the many scenes analysed. However, as was pointed out, the more subtle boundaries sometimes showed this consistency, and sometimes not. Since In effect a basic assumption of the model was partially violated, a certain doubt was cast on the validity of these terms. It might be fruitful to use a heuristic procedure which would Increase the likelihood of acceptance of a line If this consistency were found; but would have no effect If not. This requires further Invest-Igatlon.

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