8. Appendix

8.1. Proof of Lemma 1

Proof. When using $\bar{K}$ defined in (3), the matrix Q in (1) becomes $\bar{Q}$ as given below:

$$\bar{Q}_{i,j} = \begin{cases} y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j) & \text{if } \pi(\boldsymbol{x}_i) = \pi(\boldsymbol{x}_j) \\ 0 & \text{if } \pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j). \end{cases} \quad (7)$$

Therefore, the quadratic term in (1) can be decomposed into

$$\boldsymbol{\alpha}^T \bar{Q} \boldsymbol{\alpha} = \sum_{c=1}^{k} \boldsymbol{\alpha}_{(c)}^T Q_{(c,c)} \boldsymbol{\alpha}_{(c)}.$$

The constraints and linear term in (1) are also decomposable, so the subproblems are independent, and concatenation of their optimal solutions, $\bar{\boldsymbol{\alpha}}$, is the optimal solution for (1) when K is replaced by $\bar{K}$. □

8.2. Proof of Theorem 1

Proof. We use $\bar{f}(\boldsymbol{\alpha})$ to denote the objective function of (1) with kernel $\bar{K}$. By Lemma 1, $\bar{\boldsymbol{\alpha}}$ is the minimizer of (1) with K replaced by $\bar{K}$, thus $\bar{f}(\bar{\boldsymbol{\alpha}}) \leq \bar{f}(\boldsymbol{\alpha}^*)$. By the definition of $\bar{f}(\boldsymbol{\alpha}^*)$ we can easily show

$$\bar{f}(\boldsymbol{\alpha}^*) = f(\boldsymbol{\alpha}^*) - \frac{1}{2} \sum_{i,j:\pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j)} \alpha_i^* \alpha_j^* y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j) \quad (8)$$

Similarly, we have

$$\bar{f}(\bar{\boldsymbol{\alpha}}) = f(\bar{\boldsymbol{\alpha}}) - \frac{1}{2} \sum_{i,j:\pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j)} \bar{\alpha}_i \bar{\alpha}_j y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j). \quad (9)$$

Combining with $\bar{f}(\bar{\boldsymbol{\alpha}}) \leq \bar{f}(\boldsymbol{\alpha}^*)$ we have

$$\begin{aligned} f(\bar{\boldsymbol{\alpha}}) &\leq \bar{f}(\boldsymbol{\alpha}^*) + \frac{1}{2} \sum_{i,j:\pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j)} \bar{\alpha}_i \bar{\alpha}_j y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j), \\ &= f(\boldsymbol{\alpha}^*) + \frac{1}{2} \sum_{i,j:\pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j)} (\bar{\alpha}_i \bar{\alpha}_j - \alpha_i^* \alpha_j^*) y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j) \quad (10) \\ &\leq f(\boldsymbol{\alpha}^*) + \frac{1}{2} C^2 D(\pi), \text{ since } 0 \leq \bar{\alpha}_i, \alpha_i^* \leq C \text{ for all } i. \end{aligned}$$

Also, since $\boldsymbol{\alpha}^*$ is the optimal solution of (1) and $\bar{\boldsymbol{\alpha}}$ is a feasible solution, $f(\boldsymbol{\alpha}^*) < f(\bar{\boldsymbol{\alpha}})$, thus proving the first part of the theorem.

Let σ_n be the smallest singular value of the positive definite kernel matrix K. Since $Q = \text{diag}(\boldsymbol{y}) K \text{diag}(\boldsymbol{y})$ and $y_i \in \{1, -1\}$ for all i, Q and K have identical singular values. Suppose we write $\bar{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^* + \Delta\boldsymbol{\alpha}$,

$$f(\bar{\boldsymbol{\alpha}}) = f(\boldsymbol{\alpha}^*) + (\boldsymbol{\alpha}^*)^T Q \Delta\boldsymbol{\alpha} + \frac{1}{2} (\Delta\boldsymbol{\alpha})^T Q \Delta\boldsymbol{\alpha} - \boldsymbol{e}^T \Delta\boldsymbol{\alpha}. \quad (11)$$

The optimality condition for (1) is

$$\nabla_i f(\boldsymbol{\alpha}^*) \begin{cases} = 0 & \text{if } 0 < \alpha_i^* < C, \\ \geq 0 & \text{if } \alpha_i^* = 0, \\ \leq 0 & \text{if } \alpha_i^* = C, \end{cases} \quad (12)$$

where $\nabla f(\boldsymbol{\alpha}^*) = Q\boldsymbol{\alpha}^* - \boldsymbol{e}$. Since $\bar{\boldsymbol{\alpha}}$ is a feasible solution, it is easy to see that $(\Delta\boldsymbol{\alpha})_i \geq 0$ if $\alpha_i^* = 0$, and $(\Delta\boldsymbol{\alpha})_i \leq 0$ if $\alpha_i^* = C$. Thus,

$$(\Delta\boldsymbol{\alpha})^T (Q\boldsymbol{\alpha}^* - \boldsymbol{e}) = \sum_{i=1}^{n} (\Delta\boldsymbol{\alpha})_i ((Q\boldsymbol{\alpha}^*)_i - 1) \geq 0.$$

Combining with (11) we have $f(\bar{\boldsymbol{\alpha}}) \geq f(\boldsymbol{\alpha}^*) + \frac{1}{2} \Delta\boldsymbol{\alpha}^T Q \Delta\boldsymbol{\alpha} \geq f(\boldsymbol{\alpha}^*) + \frac{1}{2} \sigma_n \|\Delta\boldsymbol{\alpha}\|_2^2$. Since we already know that $f(\bar{\boldsymbol{\alpha}}) \leq f(\boldsymbol{\alpha}^*) + \frac{1}{2} C^2 D(\pi)$, this implies $\|\boldsymbol{\alpha}^* - \bar{\boldsymbol{\alpha}}\|_2^2 \leq C^2 D(\pi) / \sigma_n$. □

8.3. Proof of Theorem 2

Proof. Let $\Delta Q = Q - \bar{Q}$ and $\Delta\boldsymbol{\alpha} = \boldsymbol{\alpha}^* - \bar{\boldsymbol{\alpha}}$. From the optimality condition for (1) (see (12)), we know that $\alpha_i^* = 0$ if $(Q\boldsymbol{\alpha}^*)_i > 1$. Since $Q\boldsymbol{\alpha}^* = (\bar{Q} + \Delta Q)(\bar{\boldsymbol{\alpha}} + \Delta\boldsymbol{\alpha})$, we see that

$$\begin{aligned} &(Q\boldsymbol{\alpha}^*)_i \\ &= (\bar{Q}\bar{\boldsymbol{\alpha}})_i + (\Delta Q \bar{\boldsymbol{\alpha}})_i + (Q\Delta\boldsymbol{\alpha})_i. \\ &= (\bar{Q}\bar{\boldsymbol{\alpha}})_i + \sum_{j:\pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j)} y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j) \bar{\alpha}_j \\ &\quad + \sum_j y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j) (\Delta\boldsymbol{\alpha})_j \\ &\geq (\bar{Q}\bar{\boldsymbol{\alpha}})_i - CD(\pi) - K_{max} \|\Delta\boldsymbol{\alpha}\|_1 \\ &\geq (\bar{Q}\bar{\boldsymbol{\alpha}})_i - CD(\pi) \\ &\quad - \sqrt{n} K_{max} C \sqrt{D(\pi)} / \sqrt{\sigma_n} \text{ (by Theorem 1)} \\ &= (\bar{Q}\bar{\boldsymbol{\alpha}})_i - CD(\pi) \left(1 + \frac{\sqrt{n} K_{max}}{\sqrt{\sigma_n D(\pi)}} \right). \end{aligned}$$

The condition stated in the theorem implies $(\bar{Q}\bar{\boldsymbol{\alpha}})_i > 1 + CD(\pi)(1 + \frac{\sqrt{n} K_{max}}{\sqrt{\sigma_n D(\pi)}})$, which implies $(Q\boldsymbol{\alpha}^*)_i - 1 > 0$, so from the optimality condition (12), $\alpha_i^* = 0$. □

8.4. Proof of Theorem 3

Proof. Similar to the proof in Theorem 1, we use $\bar{f}(\boldsymbol{\alpha})$ to denote the objective function of (1) with kernel $\bar{K}$. Combine (10) with the fact that $\alpha_i^* = 0 \ \forall i \notin S^*$ and $\bar{\alpha}_i = 0 \ \forall i \notin \bar{S}$, we have

$$\begin{aligned} \bar{f}(\boldsymbol{\alpha}^*) &\leq f(\boldsymbol{\alpha}^*) - \frac{1}{2} \sum_{i,j:\pi(\boldsymbol{x}_i) \neq \pi(\boldsymbol{x}_j) \text{ and } i,j \in S^*} (\bar{\alpha}_i \bar{\alpha}_j - \alpha_i^* \alpha_j^*) y_i y_j K(\boldsymbol{x}_i, \boldsymbol{x}_j) \\ &\leq f(\boldsymbol{\alpha}^*) + \frac{1}{2} C^2 D(\{\boldsymbol{x}_i\}_{i \in S^* \cup \bar{S}}, \pi). \end{aligned}$$

The second part of the proof is exactly the same as the second part of Theorem 1. □

8.5. Clustering time vs Training time

Our DC-SVM algorithm is composed of two important parts: clustering and SVM training. In Table 5 we list the time taken by each part; we can see that the clustering time is almost constant at each level, while the rest of the training time keeps increasing.

Table 5: Run time (in seconds) for DC-SVM on different levels (covtype dataset). We can see the clustering time is only a small portion compared with the total training time.

Level	4	3	2	1	0
Clustering	43.2s	42.5s	40.8s	38.1s	36.5s
Training	159.4s	439.7s	1422.8s	3135.5s	7614.0s

8.6. Comparison with Bagging Approach

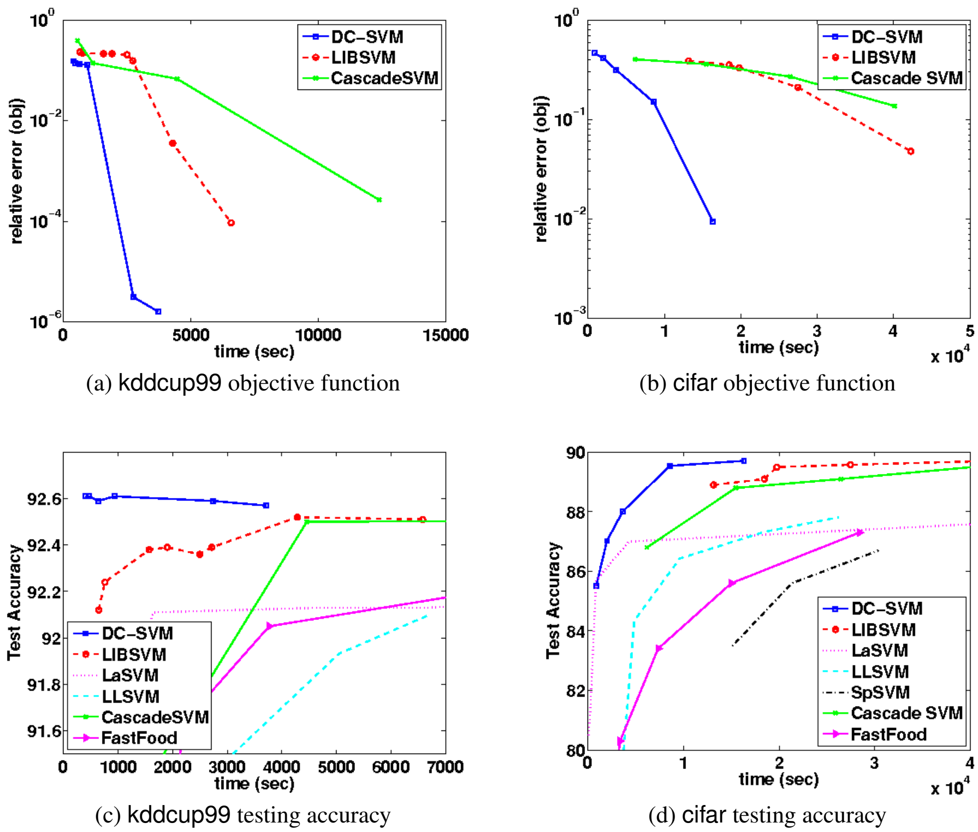
Boostrap aggregating (bagging) is a machine learning approach designed to improve the stability of machine learning algorithms. Given a training set with n samples, bagging generates k training sets, each by sampling $\bar{n}$ data points uniformly from the whole dataset. Considering the case that $\bar{n} = n/k$, then the bagging algorithms is similar to our DCSVM (early) approach, but with the following two differences:

- Data partition: bagging uses random sampling while DCSVM (early) uses clustering.
- Prediction: bagging uses voting for classification task, while DCSVM (early) using the nearest model for prediction.

Under the same k, both DCSVM (early) and bagging trains the k subsets independently, so the training times are identical for both algorithms. We compare the classification performance under various values of k in Table 6 on ijcnn1, covtype, and webspam datasets. The results show that DCSVM (early) is significantly better than bagging in terms of prediction accuracy.

Table 6: Prediction accuracy of DC-SVM (early) and bagging under various values of k. We can see that DCSVM (early) is significantly better than bagging.

k	ijcnn1		covtype		webspam	
	DCSVM (early)	Bagging	DCSVM (early)	Bagging	DCSVM (early)	Bagging
256	98.16%	91.81%	96.12%	83.41%	99.04%	95.20%
64	98.35%	95.44%	96.15%	88.54%	99.23%	97.13%
16	98.46%	98.24%	96.16%	91.81%	99.29%	98.28%



(a) kddcup99 objective function

(b) cifar objective function

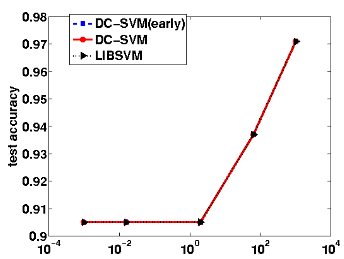
(c) kddcup99 testing accuracy

(d) cifar testing accuracy

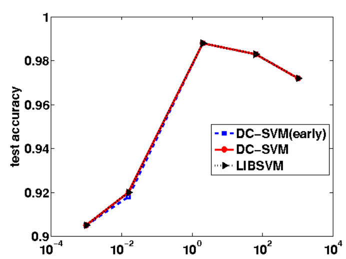
Figure 5: Additional comparison of algorithms using RBF kernel on the kddcup99 and cifar datasets.

Table 7: Comparison of DC-SVM, DC-SVM (early), and LIBSVM on ijcnn1 with various parameters C, γ. DC-SVM (early) is always 10 times faster than LIBSVM achieves similar testing accuracy. DC-SVM is faster than LIBSVM for almost every setting.

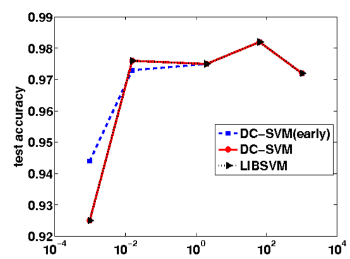
dataset	C	γ	DC-SVM (early)		DC-SVM		LIBSVM		LaSVM	
			acc(%)	time(s)	acc(%)	time(s)	acc(%)	time(s)	acc(%)	time(s)
ijcnn1	2^{-10}	2^{-10}	**90.5**	**12.8**	**90.5**	120.1	**90.5**	130.0	**90.5**	492
ijcnn1	2^{-10}	2^{-6}	**90.5**	**12.8**	**90.5**	203.1	**90.5**	492.5	**90.5**	526
ijcnn1	2^{-10}	2^{1}	**90.5**	**50.4**	**90.5**	524.2	**90.5**	1121.3	**90.5**	610
ijcnn1	2^{-10}	2^{6}	**93.7**	**44.0**	**93.7**	400.2	**93.7**	1706.5	92.4	1139
ijcnn1	2^{-10}	2^{10}	**97.1**	**39.1**	**97.1**	451.3	**97.1**	1214.7	95.7	1711
ijcnn1	2^{-6}	2^{-10}	**90.5**	**7.2**	**90.5**	84.7	**90.5**	252.7	**90.5**	531
ijcnn1	2^{-6}	2^{-6}	**90.5**	**7.6**	**90.5**	161.2	**90.5**	401.0	**90.5**	519
ijcnn1	2^{-6}	2^{1}	90.7	**10.8**	**90.8**	183.6	**90.8**	553.2	90.5	577
ijcnn1	2^{-6}	2^{6}	**93.9**	**49.2**	**93.9**	416.1	**93.9**	1645.3	91.3	1213
ijcnn1	2^{-6}	2^{10}	**97.1**	**40.6**	**97.1**	477.3	**97.1**	1100.7	95.5	1744
ijcnn1	2^{1}	2^{-10}	**90.5**	**14.0**	**90.5**	305.6	**90.5**	424.9	**90.5**	511
ijcnn1	2^{1}	2^{-6}	91.8	**12.6**	**92.0**	254.6	**92.0**	367.1	90.8	489
ijcnn1	2^{1}	2^{1}	**98.8**	**7.0**	**98.8**	43.5	**98.8**	111.6	95.4	227
ijcnn1	2^{1}	2^{6}	**98.3**	**34.6**	**98.3**	584.5	**98.3**	1776.5	97.8	1085
ijcnn1	2^{1}	2^{10}	**97.2**	**94.0**	**97.2**	523.1	**97.2**	1955.0	96.1	1691
ijcnn1	2^{6}	2^{-10}	**92.5**	**27.8**	91.9	276.3	91.9	331.8	90.5	442
ijcnn1	2^{6}	2^{-6}	94.8	**19.9**	**95.6**	313.7	**95.6**	219.5	92.3	435
ijcnn1	2^{6}	2^{1}	**98.3**	**6.4**	**98.3**	75.3	**98.3**	59.8	97.5	222
ijcnn1	2^{6}	2^{6}	**98.1**	**48.3**	**98.1**	384.5	**98.1**	987.7	97.1	1144
ijcnn1	2^{6}	2^{10}	**97.2**	**51.9**	**97.2**	530.7	**97.2**	1340.9	95.4	1022
ijcnn1	2^{10}	2^{-10}	**94.4**	**146.5**	92.5	606.1	92.5	1586.6	91.7	401
ijcnn1	2^{10}	2^{-6}	97.3	**124.3**	**97.6**	553.6	**97.6**	1152.2	96.5	1075
ijcnn1	2^{10}	2^{1}	97.5	**10.6**	**97.5**	50.8	**97.5**	139.3	97.1	605
ijcnn1	2^{10}	2^{6}	**98.2**	**42.5**	**98.2**	338.3	**98.2**	1629.3	97.1	890
ijcnn1	2^{10}	2^{10}	**97.2**	**66.4**	**97.2**	309.6	**97.2**	2398.3	95.4	909



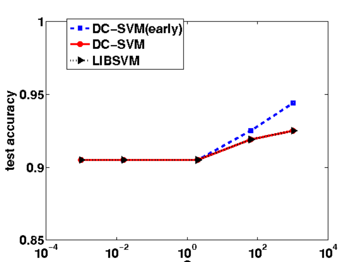
(a) ijcnn1 $C = 2^{-10}$



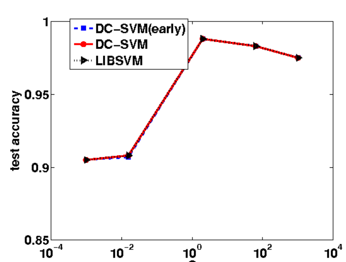
(b) ijcnn1 $C = 2^{1}$



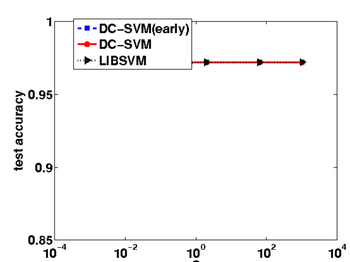
(c) ijcnn1 $C = 2^{10}$



(d) ijcnn1 $\gamma = 2^{-10}$



(e) ijcnn1 $\gamma = 2^{1}$

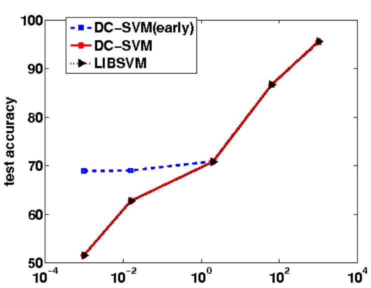


(f) ijcnn1 $\gamma = 2^{10}$

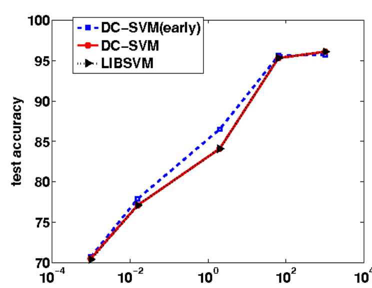
Figure 6: Robustness to the parameters C, γ on ijcnn1 dataset.

Table 8: Comparison of DC-SVM, DC-SVM (early) and LIBSVM on webspam with various parameters C, γ. DC-SVM (early) is always more than 30 times faster than LIBSVM and has comparable or better test accuracy; DC-SVM is faster than LIBSVM under all settings.

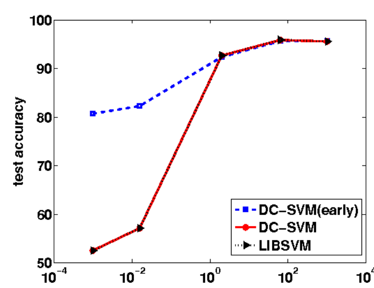
dataset	C	γ	DC-SVM (early)		DC-SVM		LIBSVM	
			acc(%)	time(s)	acc(%)	time(s)	acc(%)	time(s)
webspam	2^{-10}	2^{-10}	**86**	**806**	61	26324	61	45984
webspam	2^{-10}	2^{-6}	**83**	**935**	61	22569	61	53569
webspam	2^{-10}	2^{1}	87.1	**886**	**91.1**	10835	**91.1**	34226
webspam	2^{-10}	2^{6}	**93.7**	**1060**	92.6	6496	92.6	34558
webspam	2^{-10}	2^{10}	98.3	**1898**	**98.5**	7410	**98.5**	55574
webspam	2^{-6}	2^{-10}	**83**	**793**	68	24542	68	44153
webspam	2^{-6}	2^{-6}	**84**	**762**	69	33498	69	63891
webspam	2^{-6}	2^{1}	93.3	**599**	**93.5**	15098	93.1	34226
webspam	2^{-6}	2^{6}	**96.4**	**704**	**96.4**	7048	**96.4**	48571
webspam	2^{-6}	2^{10}	98.3	**1277**	**98.6**	6140	**98.6**	45122
webspam	2^{1}	2^{-10}	**87**	**688**	78	18741	78	48512
webspam	2^{1}	2^{-6}	**93**	**645**	81	10481	81	30106
webspam	2^{1}	2^{1}	98.4	**420**	**99.0**	9157	**99.0**	35151
webspam	2^{1}	2^{6}	**98.9**	**466**	**98.9**	5104	**98.9**	28415
webspam	2^{1}	2^{10}	98.3	**853**	**98.7**	4490	98.7	28891
webspam	2^{6}	2^{-10}	**93**	**759**	80	24849	80	64121
webspam	2^{6}	2^{-6}	**97**	**602**	83	21898	83	55414
webspam	2^{6}	2^{1}	98.8	**406**	**99.1**	8051	**99.1**	40510
webspam	2^{6}	2^{6}	**99.0**	**465**	98.9	6140	98.9	35510
webspam	2^{6}	2^{10}	98.3	**917**	**98.7**	4510	**98.7**	34121
webspam	2^{10}	2^{-10}	**97**	**1350**	82	31387	82	81592
webspam	2^{10}	2^{-6}	**98**	**1127**	86	34432	86	82581
webspam	2^{10}	2^{1}	**98.8**	**463**	**98.8**	10433	**98.8**	58512
webspam	2^{10}	2^{6}	**99.0**	**455**	**99.0**	15037	**99.0**	75121
webspam	2^{10}	2^{10}	98.3	**831**	**98.7**	7150	**98.7**	59126



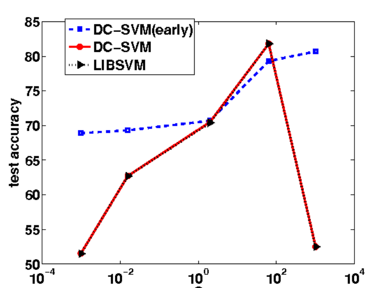
(a) covtype $C = 2^{-10}$



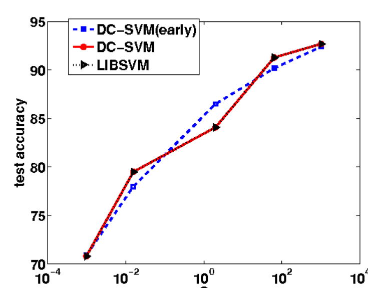
(b) covtype $C = 2^{1}$



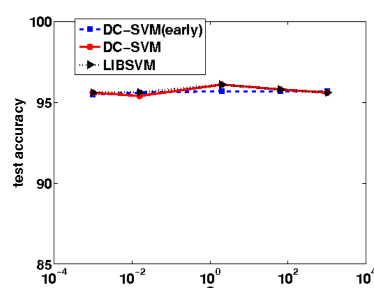
(c) covtype $C = 2^{10}$



(d) covtype $\gamma = 2^{-10}$



(e) covtype $\gamma = 2^{1}$

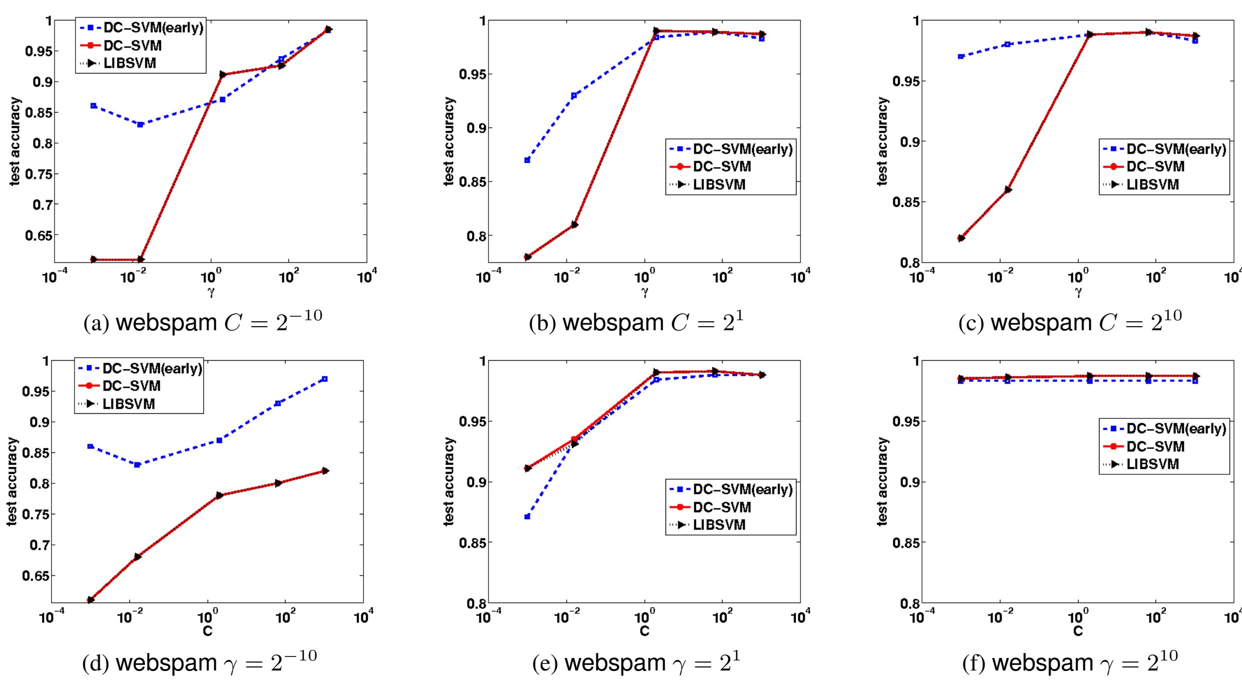


(f) covtype $\gamma = 2^{10}$

Figure 7: Robustness to the parameters C, γ on covtype dataset.

Table 9: Comparison of DC-SVM, DC-SVM (early) and LIBSVM on covtype with various parameters C, γ. DC-SVM (early) is always more than 50 times faster than LIBSVM with similar test accuracy; DC-SVM is faster than LIBSVM under all settings.

dataset	C	γ	DC-SVM (early)		DC-SVM		LIBSVM	
			acc(%)	time(s)	acc(%)	time(s)	acc(%)	time(s)
covtype	2^{-10}	2^{-10}	**68.9**	**736**	51.5	24791	51.5	48858
covtype	2^{-10}	2^{-6}	**69.0**	**507**	62.7	17189	62.7	62668
covtype	2^{-10}	2^{1}	**70.9**	**624**	70.8	12997	70.8	88160
covtype	2^{-10}	2^{6}	**86.7**	**1351**	**86.7**	13985	**86.7**	85111
covtype	2^{-10}	2^{10}	95.5	**1173**	**95.6**	9480	**95.6**	54282
covtype	2^{-6}	2^{-10}	**69.3**	**373**	62.7	10387	62.7	90774
covtype	2^{-6}	2^{-6}	**70.0**	**625**	68.6	14398	68.6	76508
covtype	2^{-6}	2^{1}	78.0	**346**	**79.5**	5312	**79.5**	77591
covtype	2^{-6}	2^{6}	87.9	**895**	**87.9**	8886	**87.9**	120512
covtype	2^{-6}	2^{10}	**95.6**	**1238**	95.4	7581	95.6	123396
covtype	2^{1}	2^{-10}	**70.7**	**433**	70.4	25120	70.4	88725
covtype	2^{1}	2^{-6}	**77.9**	**1000**	77.1	18452	77.1	69101
covtype	2^{1}	2^{1}	**86.5**	**421**	84.1	11411	84.1	50890
covtype	2^{1}	2^{6}	**95.6**	**299**	95.3	8714	95.3	117123
covtype	2^{1}	2^{10}	95.7	**882**	**96.1**	5349		$>$300000
covtype	2^{6}	2^{-10}	79.3	**1360**	**81.8**	34181	81.8	105855
covtype	2^{6}	2^{-6}	81.3	**2314**	**84.3**	24191	**84.3**	108552
covtype	2^{6}	2^{1}	90.2	**957**	**91.3**	14099	**91.3**	75596
covtype	2^{6}	2^{6}	**96.3**	**356**	96.2	9510	96.2	92951
covtype	2^{6}	2^{10}	95.7	**961**	**95.8**	7483	**95.8**	288567
covtype	2^{10}	2^{-10}	**80.7**	**5979**	52.5	50149	52.5	235183
covtype	2^{10}	2^{-6}	**82.3**	**8306**	57.1	43488		$>$ 300000
covtype	2^{10}	2^{1}	92.4	**4553**	**92.7**	19481	**92.7**	254130
covtype	2^{10}	2^{6}	95.7	**368**	**95.9**	12615	**95.9**	93231
covtype	2^{10}	2^{10}	**95.7**	**1094**	95.6	10432	95.6	169918

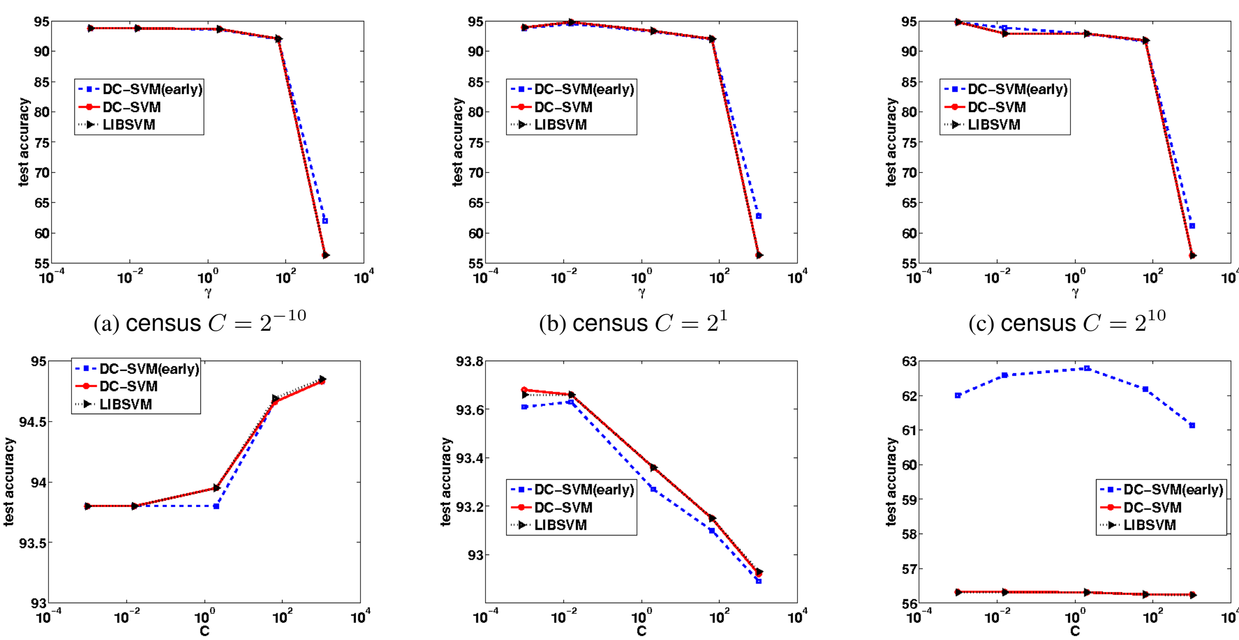


(a) webspam $C = 2^{-10}$
(b) webspam $C = 2^{1}$
(c) webspam $C = 2^{10}$
(d) webspam $\gamma = 2^{-10}$
(e) webspam $\gamma = 2^{1}$
(f) webspam $\gamma = 2^{10}$

Figure 8: Robustness to the parameters C, γ on webspam dataset.

Table 10: Comparison of DC-SVM, DC-SVM (early) and LIBSVM on census with various parameters C, γ. DC-SVM (early) is always more than 50 times faster than LIBSVM with similar test accuracy; DC-SVM is faster than LIBSVM under all settings.

dataset	C	γ	DC-SVM (early)		DC-SVM		LIBSVM	
			acc(%)	time(s)	acc(%)	time(s)	acc(%)	time(s)
census	2^{-10}	2^{-10}	**93.80**	**161**	**93.80**	2153	**93.80**	3061
census	2^{-10}	2^{-6}	**93.80**	**166**	**93.80**	3316	**93.80**	5357
census	2^{-10}	2^{1}	93.61	**202**	**93.68**	4215	93.66	11947
census	2^{-10}	2^{6}	91.96	**228**	**92.08**	5104	**92.08**	12693
census	2^{-10}	2^{10}	**62.00**	**195**	56.32	4951	56.31	13604
census	2^{-6}	2^{-10}	**93.80**	**145**	**93.80**	3912	**93.80**	6693
census	2^{-6}	2^{-6}	**93.80**	**149**	**93.80**	3951	**93.80**	6568
census	2^{-6}	2^{1}	93.63	**217**	**93.66**	4145	**93.66**	11945
census	2^{-6}	2^{6}	91.97	**230**	**92.10**	4080	**92.10**	9404
census	2^{-6}	2^{10}	**62.58**	**189**	56.32	3069	56.31	9078
census	2^{1}	2^{-10}	93.80	**148**	**93.95**	2057	**93.95**	1908
census	2^{1}	2^{-6}	94.55	**139**	**94.82**	2018	**94.82**	1998
census	2^{1}	2^{1}	93.27	**179**	**93.36**	4031	**93.36**	37023
census	2^{1}	2^{6}	91.96	**220**	**92.06**	6148	**92.06**	33058
census	2^{1}	2^{10}	**62.78**	**184**	56.31	6541	56.31	35031
census	2^{6}	2^{-10}	**94.66**	**193**	**94.66**	3712	**94.69**	3712
census	2^{6}	2^{-6}	94.76	**164**	**95.21**	2015	**95.21**	3725
census	2^{6}	2^{1}	93.10	**229**	**93.15**	6814	**93.15**	32993
census	2^{6}	2^{6}	91.77	**243**	**91.88**	9158	**91.88**	34035
census	2^{6}	2^{10}	**62.18**	**210**	56.25	9514	56.25	36910
census	2^{10}	2^{-10}	94.83	**538**	94.83	2751	**94.85**	8729
census	2^{10}	2^{-6}	**93.89**	**315**	92.94	3548	92.94	12735
census	2^{10}	2^{1}	92.89	**342**	92.92	9105	**92.93**	52441
census	2^{10}	2^{6}	91.64	**244**	**91.81**	7519	**91.81**	34350
census	2^{10}	2^{10}	**61.14**	**206**	56.25	5917	56.23	34906



(a) census $C = 2^{-10}$ (b) census $C = 2^{1}$ (c) census $C = 2^{10}$

(d) census $\gamma = 2^{-10}$ (e) census $\gamma = 2^{1}$ (f) census $\gamma = 2^{10}$

Figure 9: Robustness to the parameters C, γ on census dataset.