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Estimation of Minimum Quantization Levels by Using Reconstructed Histogram

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Abstract

The OK-quantization theory determines the minimum gray level by using the reproducibility of an image histogram. In many cases, it is ascertained by the human sense of sight that the minimum gray level obtained from this theory is appropriate. However, in order to put the OK-quantization theory into practical use, it is necessary to perform a validity evaluation of this theorem using a computer algorithm. In this research, the gray level of each pixel of a quantized image is first interpolated using the sampling function to create a reconstructed image with the average gray level of each pixel being as a new gray level. The gray level is then evaluated for validity by comparing the histogram of this reconstructed image with that of the original image. The experiment results have confirmed that the proposed method makes it possible to replace an estimation of minimum quantization levels that relies on the human sense of sight, with a computational algorithm.

1. Introduction

The OK-quantization theory [1]-[3] is modeled, as shown below, by handling the theoretical background of quantization as a restoration problem of the probability density function that governs a histogram where the gray level probability density function of image $f(x)$ is $p(f)$ and its Fourier transform $P(v)$, respectively. If $P(v)$ is strictly band limited by v_c as in eq. (1), the probability density function $p(f)$ can be completely restored from the digitization data by eq. (3) only when the quantization interval is set less than or equal to Δf as given by eq. (2).

$$P(v) = 0 \qquad (v \geq v_c) \tag{1}$$

$$\Delta f \leq \frac{1}{2v_c} \tag{2}$$

$$p(f) = \sum_k p(k\Delta f)\,\mathrm{sinc}\{2\pi(f - k\Delta f)\} \tag{3}$$

This Δf is the quantization interval that can completely restore the probability density function $p(f)$ of this image $f(x)$. Here, the probability density function $p(f)$ is estimated as follows. The histogram $h(k) = h_k$ of the digital image $f(i) = f_i$ observed from the image $f(x)$ provides a clue for estimating the probability density function $p(f)$. The histogram h_k assumed to be distributed on the limited discrete space, with its minimum quantization interval as 1, the max. gray level as $f_{\max}$, and a rect function used as an analysis function that is the basis, is estimated as $p^*(f)$ from eq. (4)

$$p^*(f) = \frac{\sum_{n=0}^{f_{\max}} h(n)\mathrm{rect}(f - n)}{S} \tag{4}$$

where;

$$S = \sum_{n=1}^{f_{\max}} h(n). \tag{5}$$

Thus, h_k ($k = 0,1,\cdots, f_{\max}$) is expressed as a function $p^*(f)$ [f: -∞, +∞] on the real space. In addition, Fourier transform $P^*(v)$ of $p^*(f)$ can be determined analytically from eq. (6).

$$\begin{aligned} P^*(v) &= \mathrm{FT}[p^*(f)] \\ &= \frac{1}{S}\sum_{n=0}^{f_{\max}} h(n)e^{-2\pi jnv}\mathrm{sinc}\,v \end{aligned} \tag{6}$$

Further, assuming the discrete Fourier transform of h_k to be $\widetilde{H}(v)$, the following equation is obtained.

$$\widetilde{H}(v) = \sum_{n=0}^{f_{\max}} h_n e^{-2\pi jnv} \tag{7}$$

Therefore, eq. (6) can be expressed as follows:

$$P^*(v) = \frac{1}{S}\widetilde{H}(v)\mathrm{sinc}\,v. \tag{8}$$

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2. Estimation of the minimum quantization gray level

The OK-quantization theorem is intended to theoretically forecast the minimum gray level (obtained by dividing 256 by the minimum quantization interval) that guarantees reproduction of the gray level probability density function. In general, the image quantized with the forecasted minimum gray level shows no deterioration. For example, Tamiya et al. pointed out that the minimum gray level that is visually confirmed is the gray level in which no false contour is felt in the image [4].

The standard image was shown to a subject, and the gray level at which no false contour can be recognized when the entire image was looked at without staring on a

Table 1. Two standard images.

False contour	Text	Parrots
○○	255～30	255～85
○	25～15	80～60
×	10～0	55～0

part basis was determined as the minimum quantization gray level. Preprocessing was then performed on the histogram of the standard image, after which the minimum quantization gray level obtained by applying the OK-quantization theorem was confirmed to be in agreement with that visually obtained.

Fig. 1 shows a couple of typical images taken from 17 standard images, obtained in reference [4], based on the visual experience with respect to the minimum and maximum quantization gray levels. The former is a text image with the smaller quantization levels and the latter is a parrot image with the larger quantization levels.

Table 1 shows the reproduction experiment results obtained by determining the minimum quantization gray level while lowering the gray level of these two standard images by five gray levels each from the 255 gray levels. Mark ○○ indicates the gray level at which no image deterioration was felt, mark ○ the minimum quantization gray level (minimum gray level at which no false contour was recognized in the image), and mark × the gray level at which image deterioration was recognized, respectively. The reason for the minimum quantization gray level range being large is that there is a dispersion depending on subjects. The value of the minimum gray level was nearly in agreement with that stated in literature [4]. Then, how should this visually based minimum quantization gray level be verified for validity using a computer?

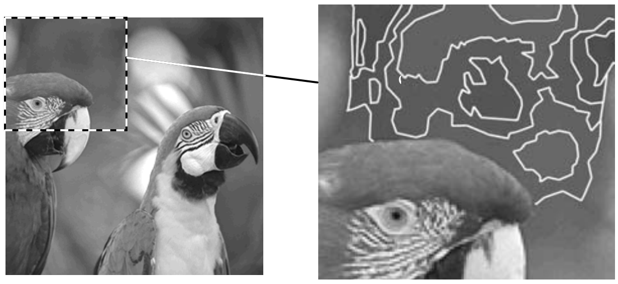
2.1. Direct comparison of images

One possible method is a comparison of a quantized image with its original image. For this purpose, using the normalized cross-correlation value R would be appropriate. The relationship between the quantization gray level obtained by reference [4] and the normalized cross-correlation value R with respect to the typical images "Text" and "Parrots" is shown below: Since the minimum quantization gray level of the text image is 15 to 25, the corresponding range of the normalized cross-correlation value R resulted in 99.52% to 99.82%.

On the other hand, since the minimum quantization

(a) Text. (b) Parrots.

Figure 1. Two standard images.



(a) 40 levels image. (b) Partial image.

Figure 2. False contour of the parrot image.

gray level of the parrot image was between 60 and 80, the corresponding range of the normalized cross-correlation value R resulted in 99.90% to 99.93%. From these results, it can be understood that the range of normalized cross-correlation value corresponding to a different minimum quantization gray level totally differs. For this reason, the quantization gray level cannot be judged for appropriateness from the normalized cross-correlation value R between the quantized image and the original image. In addition, in this method the variation in normalized cross-correlation value in response to the variation in quantization gray level is small, making it extremely difficult to handle the normalized cross-correlation value.

Then, what about comparing only the partial image deteriorated due to occurrence of the false contour instead of using the entire image? Figure 2 is a partial image that includes the false contour caused by quantization of the parrot image by 40 gray levels. While the normalized cross-correlation value R between the entire quantized image and the entire original image was 99.83%, that of the partial image was 99.70%, making no big difference. This indicates that even if a partial image is used, the problem does not improve.

2.2. Comparison using a reconstructed histogram

The reason why the false contour of a quantized image with a lower gray level cannot be visually recognized by

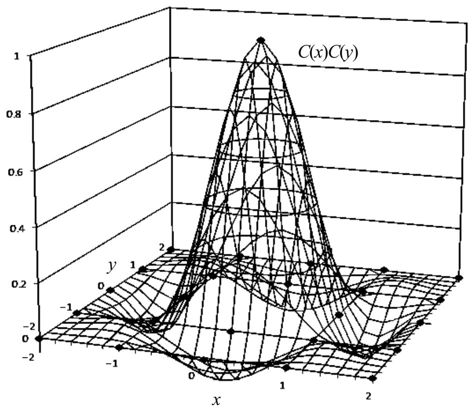


Figure 3. Interpolation function.

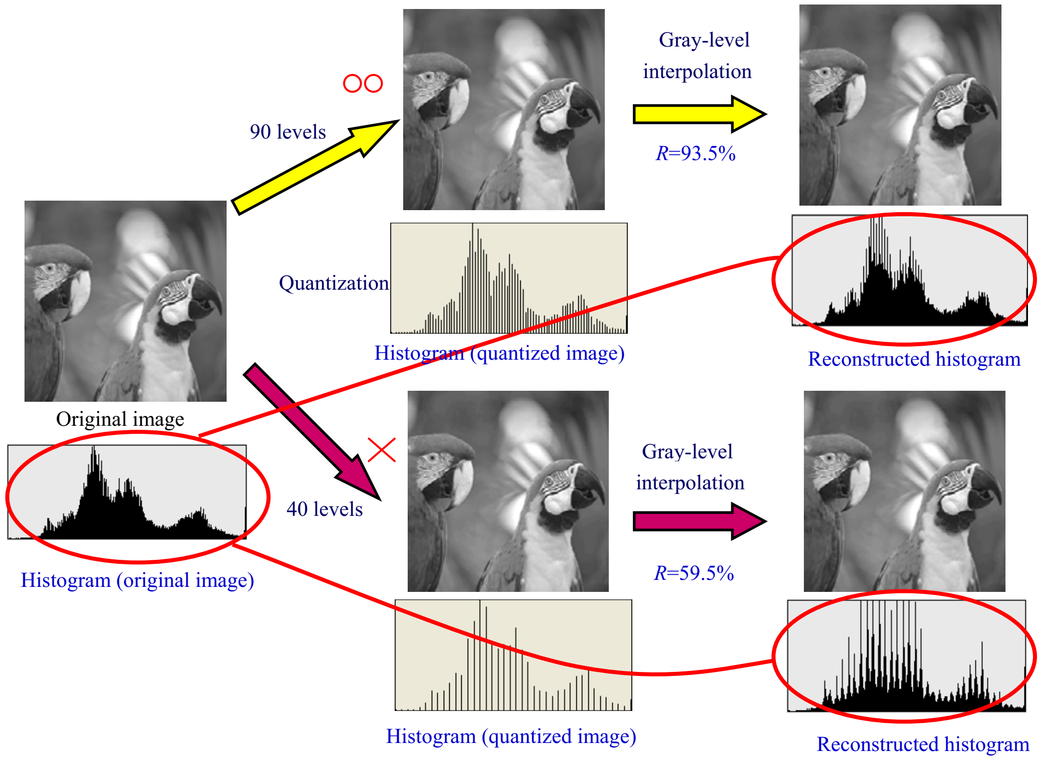


Figure 4. Reconstructed histogram.

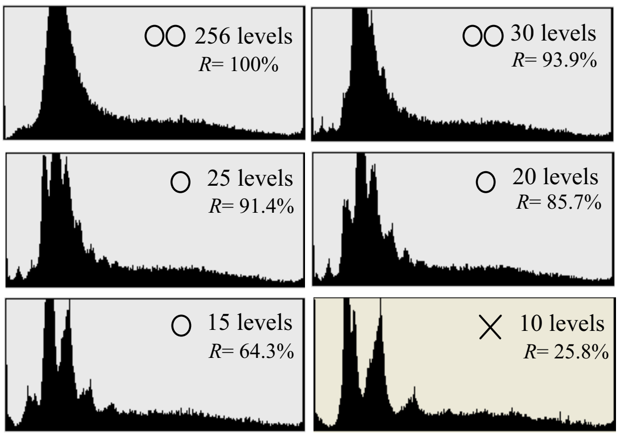


Figure 5. Reconstructed histogram of the text image.

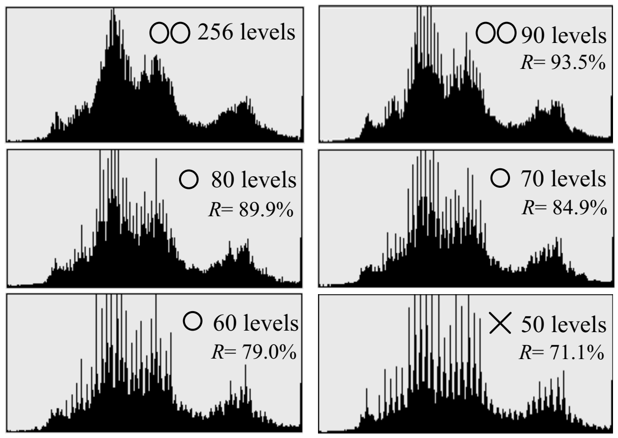


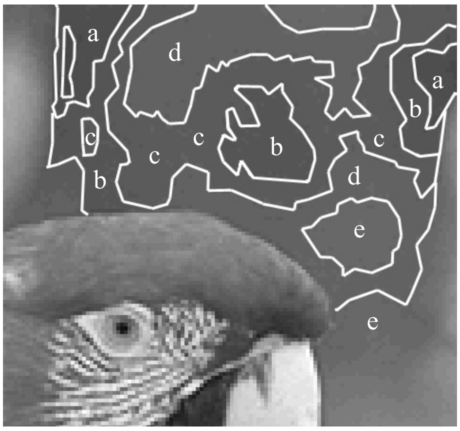
Figure 6. Reconstructed histogram of the parrot image.

looking at the image can be presumed to be that a human recognizes a type of analog image in which the gray level between respective pixels is interpolated and the space coordinates and the gray level are both the continuous values.

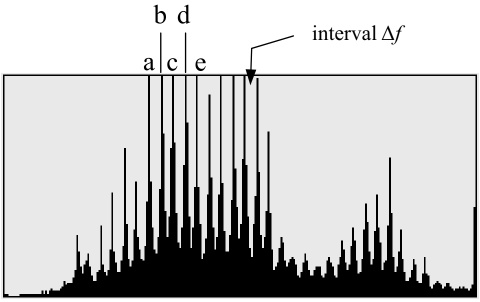
On the basis of this concept, replacing the evaluation method of the minimum quantization gray level based on human sense of vision with the computational algorithm will be as follows. First, the gray level of all pixels of the quantized image is interpolated using the sampling function to reconstruct an analog image (see Figure 3), from which a histogram is created, and here in this paper this will be called "a reconstructed histogram". The histogram of a quantized image is of a comb shape with gray levels only at each quantization interval. If no false contour occurs in the image, gray-level interpolation between pixels is appropriately performed by which the reconstructed histogram will be similar to the histogram of the original image. On the other hand, if a false contour occurs in the image, this means that gray-level interpolation between pixels will not be properly done, and therefore the reconstructed histogram will not be similar to the histogram of the original image, but will be close to the comb shape. Figure 4 shows the above procedure.

3. Experiments

This section describes the experiments done to check



(a) Partial image.



(b) Reconstructed histogram.

Figure 7. False contour of the parrot image.

the minimum quantization interval for validity using the proposed method.

Figure 5 is a reconstructed histogram obtained when the standard image “Text” was quantized at the gray levels before and after the minimum quantization gray level based on human sense of sight. Even if the gray level is lowered to 30, the histogram is almost the same as that for the 256 gray levels. The normalized cross-correlation value R at this time was high at R= 93.9%. The histogram shape began to collapse slightly at about the 25th gray level (R= 91.4%) and then collapsed greatly at the 10th gray level (R= 25.8%). From this, it can be thought that the gray level at which the reconstructed histogram shape begins to collapse (normalized cross-correlation value R being about R= 90%) and the upper limit of the minimum gray level based on human sense of sight agree with each other.

Figure 6 shows a reconstructed histogram obtained when the standard image “Parrots” was quantized around the minimum quantization gray level. Even if the gray level is lowered to 90, it was almost the same as the histogram for the 256 gray levels, and the normalized cross-correlation value at this time was high at R= 93.5%. The histogram shape started to collapse slightly at the 80th gray level (R= 89.9%) and collapsed greatly at 50 (R= 71.1%). Also with the parrot image, the gray level at which the reconstructed histogram shape began to collapse (R= 90%) and the upper limit of the minimum gray level based on the human sense of sight both agreed at 80.

Figure 7 (a) is the parrot image quantized at 40 gray levels, showing an enlarged view of the part where a false contour occurred. The reconstructed histogram in Figure 7 (b) shows that the pixels of the part where this false contour occurred correspond to the levels of the comb tooth sections. The comb tooth interval is Δf. The reason why the comb tooth shape was obtained unlike the histogram of the original image is that the quantization interval Δf was too large, allowing the level for the integral value within the pixels of the interpolation curve based on the sampling function, to be biased to gray levels at irregular quantization intervals. This very bias appears as a false contour on the image based on the human sense of sight.

4. Conclusions

This paper has proposed a method of realizing a validity evaluation of the minimum quantization gray level based on human sense of sight by a computational algorithm using the normalized cross-correlation value R between the reconstructed histogram and the histogram of an original image. For this purpose, the gray level of all the pixels of a quantized image was interpolated using the sampling function to create a reconstructed image, and a reconstructed histogram as this histogram was obtained.

In the normalized cross-correlation value between the quantized image and the original image, it was difficult to identify the minimum quantization gray level. However, in the experiment that verifies the proposed method where two typical images are used, the gray level at which the reconstructed histogram starts to collapse, and the upper limit of the minimum gray level based on human visual sense starts to agree with each other. The minimum quantization gray level can be determined as a gray level at which the normalized cross-correlation value R of the reconstructed histogram is 90%.

References

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