IAPR Workshop on CV - Special Hardware and Industrial Applications OCT.12-14, 1988, Tokyo

ON A FAST PIECE-WISE LINEAR HOUGH TRANSFORM PLHT AND ITS APPLICATION

Hiroyasu KOSHIMIZU+ **Munetoshi NUMADA++**

+ : Chukyo University 101 Tokodate, Kaizu-cho Toyota, 470-03 JAPAN
++: Lossev Corp. 1331-1 Futuka, Fukuno, Toyama

Abstract Replacing the Hough calculation of the trigonomeric functions, sinθ and cosθ, by the piece-wise linear Hough function(PLH), the basic cost for the sinθ , cosθ and the multiplications is removed. The PLH function is directly introduced from the usual Hough function. The PLH function inherits the basic properties of the usual Hough function from the view point to extract the line patterns from the pattern space.

The computing cost of the PLH transform was reduced to about 1/6 of that of the usual Hough transform. It was also investigated that an additional property of the PLH transform contributes to reduce the memory cost to about 70 % of the usual Hough transform.

1.Introduction

Hough transform is one of the important methods to extract line patterns from the noisy and unclustered points of the image. As the edge or line patterns are the essential features in several industrial vision systems, it is practically required to make the Hough transform efficient from the view point of the computing and memory costs. A new fast Hough transform algorithm is introduced and its application is shortly presented in this paper.

From this point of view, it is important to reduce the computing cost to utilize the Hough transform in several application. One of the factors in order to realize this cost reduction is to reduce the number of Hough calculations with respect to the number of edge points and and resolution numbers of the parameter space. It is still expectative to decrease the computing cost for the core Hough calculation defined by eq.(1).

$$p = x.\cos\theta + y.\sin\theta \qquad (1)$$

θ : perpendicular angle from x-axis

p : the length of the perpendicular line

In this paper, replacing Hough calculation of the trigonometric functions by the piece-wise linear Hough(PLH) function which is composed of m pieces of line segments, the basic cost for the sinθ , cosθ and multiplications can be removed. It is shown in section 3 that the PLH function inherits the basic properties of the usual Hough function from the view point of to extract line patterns from the pattern space, and a few modifications of the pattern behavior in pattern space are also presented. In section 4, an new algorithm of piece-wise linear Hough transform(PLHT) is introduced, and some experimental results of PLHT are presented to demonstrate the reduction of the computing cost using an image of industrial engine parts.

2. Piece-wise Linear Hough Function

2.1 Introduction of the PLH Function

A new transform function, piece-wise linear Hough(PLH) function is introduced by eq.(2) based on the usual Hough transform function defined by eq.(1), where m is the number of the divided blocks of θ-axis in θ-p parameter space. In this equation, subscript k represents the division number, k=0,1,2,..,m, on the θ-axis.

$$p - p_{k-1} = \{(p_k - p_{k-1})/(\theta_k - \theta_{k-1})\}(\theta - \theta_{k-1}) \;;\; \theta_{k-1} \leqslant \theta \leqslant \theta_k \;;\; k=0,1,2,..,m \qquad (2)$$

As shown in Fig.1, PLH function has m+1 common points on the Hough sinusoidal curve at the interval $0 \leqslant \theta \leqslant \pi$ and the m piece-wise linear segments are defined among these m+1 common points. As the PLH function is coincident with the Hough curve at the common points at $\theta_0, \theta_1, .., \theta_k, .., \theta_m$, eq.(2) can be modified as eq.(3) by using eq.(1).

$$p-(x.\cos\theta_{k-1} + y.\cos\theta_{k-1}) = [\{x(\cos\theta_k - \cos\theta_{k-1}) + y(\sin\theta_k - \sin\theta_{k-1})\}/(\theta_k - \theta_{k-1})].(\theta - \theta_{k-1}) \qquad (3)$$

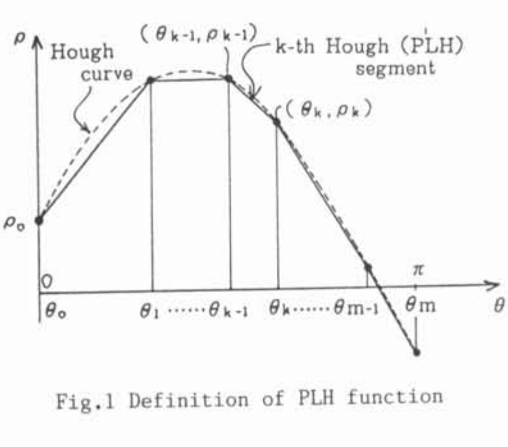


Fig.1 Definition of PLH function

$$p=x[\cos\theta_{k-1}+\{(\theta-\theta_{k-1})/(\theta_k-\theta_{k-1})\}(\cos\theta_k-\cos\theta_{k-1})]$$
$$+y[\sin\theta_{k-1}+\{(\theta-\theta_{k-1})/(\theta_k-\theta_{k-1})\}(\sin\theta_k-\sin\theta_{k-1})] \quad (4)$$

Therefore, the new expression of the PLH function can be given by eq.(4) for the substitution of the the usual Hough function.

2.2 Generation Structure of the PLH function

When let K and $K^{(k)}$ be the numbers of the resolutions in the whole range of θ-axis and the k-th block $[\theta_{k-1}, \theta_k]$ respectively, as shown in Fig.2, the inclination $T^{(k)}$ of the line segment in the k-th klock can be calculated by eq.(5). Therefore, as the minute augment $\Delta p^{(k)}$ of the line segment can be defined by eq.(6), the line segment can be generated incrementally as given by eq.(7). Equation (7) gives the basis to generate the PLH function exclusively with the addition operation without multiplication. In addition, the other computing cost can be reduced to m+1 calculations of the trigonometric functions for the substitution of K trigonometric calculations of the usual Hough curve generation.

$$T^{(k)} = \{(p_k - p_{k-1})/(\theta_k - \theta_{k-1})\} \quad (5)$$
$$p^{(k)} = T^{(k)}(\theta_k - \theta_{k-1})/K^{(k)} = (p_k - p_{k-1})/K^{(k)} \quad (6)$$
$$\Delta p_{kk} = p_{kk-1} + \Delta p^{(k)} \quad (7)$$

,where initial condition: $kk=0$, $p_0 = p_{k-1}$

$kk=0,1,2,\ldots,K^{(k)}-1$

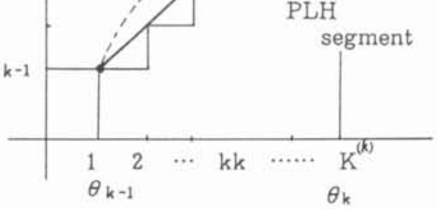


Fig.2 Incremental generation of PLH function

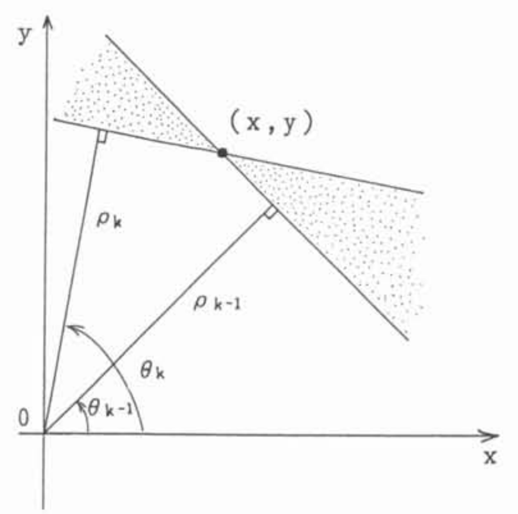


Fig.3 The behavior of the detected lines correspoinding to a PLH segment

3. Interpretation of the PLH Function

A pictorial interpretation of the behavior of the PLH function and the corresponding line patterns can be given in Fig.3. The locus of the parameter pair (θ, p) at the k-th klock where $\theta_{k-1} \leq \theta \leq \theta_k$ represent the all line patterns passing through a point (x,y) in the pattern space.

3.1 Relation between Hough and PLH functions

As the set of the line patterns are precisely described by eq.(4), the further interpretation of the PLH function is given as follows:

(a)the the case of $\theta=\theta_k$

A parameter pair (θ, p) represents a line pattern given by eq.(8) which is easily derived from eq.(4). As this equation is just the same expression of the usual Hough transform, it is clear that θ coincides just with the perpendicular angle θ of the Hough transform defined by eq.(1). This situation is valid for all θ_k, $k=0,1,2,..,m$.

$$p=x.\cos\theta_k + y.\sin\theta_k \quad (8)$$

(b)for the case of $\theta_{k-1} < \theta < \theta_k$

As a parameter pair (θ, p) at the block $[\theta_{k-1}, \theta_k]$ represents a line pattern given by eq.(4), it is easily known that the parameter θ does not coincide with the perpendicular angle given in eq.(1). However, if let the angle θ' be the equivalent perpendicular angle, where $p = x.\cos\theta' + y.\sin\theta'$, θ' can be exactly provided by eq.(9).

$$\theta' = \cos^{-1}[\cos\theta_{k-1} + \{(\theta-\theta_{k-1})/(\theta_k - \theta_{k-1})\}(\cos\theta_k - \cos\theta_{k-1})] \quad (9)$$

It is clear that the behaivior of θ' is characterized by the biased sinusoidal function of the angle θ as shown in Fig.4.

3.2 PLH Function as a Line Detector

It is known that the PLH function inherits the basic properties of the usual Hough function as the line pattern detector.

(a)Property-1 : All line patterns passing through a point (x,y) in x-y space can be uniquely represented by a PLH function, as the PLH function is a one-valued function w.r.t. θ.

(b)Property-2 : In the same way, it is clear that a pair (θ, p) in the parameter space represents a line pattern in the pattern space.

(c)Property-3 : As shown in Fig.5, the topological relations between any two PLH line segments are just the same as the usual Hough transform. Therefore, any two PLH functions intersect just once at the full range of the parameter θ, $0 \leq \theta \leq \pi$.

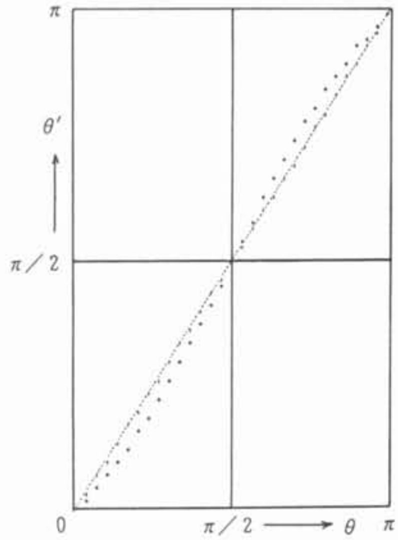
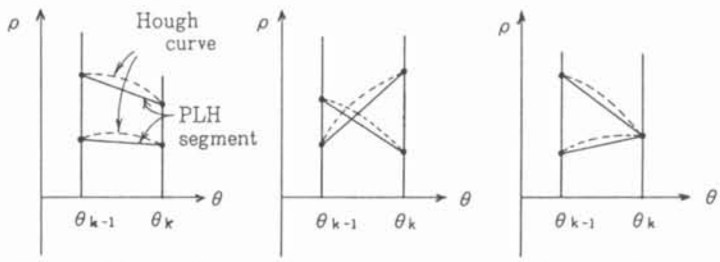


Fig.4 Equivalent perpendicular angle of the PLH function



(a)non-intersection (b)an intersection (c)a contact

Fig.5 Topological relations among PLH segments

From these discussion, it was clarified that the PLH function can be applicable to detect line patterns from the noisy and unclustered edge points in x-y space in the same way for the usual Hough transform.

4. Piece-Wise Linear Hough Transform Algorithm

This paper demonstrates not only to propose theoretically the new function for the line pattern detection, but also to make clear that the new PLH function provides a fast algorithm of the new Hough transform, piece-wise linear Hough transform(PLHT).

4.1 Algorithm:PLHT Algorithm

An algorithm to detect line patterns in x-y space using the PLH function can be introduced as follows. The notations used in the algorithm are defined below and are shown in Fig.6:

- N : the number of edge points
- b(i,j): 2-D array for the parameter space
- K,L : size of the array b, where K and L are the resolutions of the θ-ρ space
- K(k) : the number of the resolutions in the k-th block
- m : the number of the blocks, where K=K(1)+K(2)+...+K(m)
- u(1), v(1) : 1-D arrays to record a pair of array subscripts of b for θ- and ρ-value of the respective peaks detected from b(i,j), 1=1,2,..,p

PLHT Algorithm

①. Clear array b. block number k=0.
②. k=k+1 (If k>m then skip to ⑨.)
③. $C-=\cos\theta_{k-1}$ $C=\cos\theta_k$
$S-=\sin\theta_{k-1}$ $S=\sin\theta_k$
④. edge point number i=0
⑤. i=i+i, kk=0(If i>N then skip to ②.)
Calculate the inclination of the PLH segment at k-th block.

$p_{k-1}^i = x_i C- + y_i S-$
$p_k^i = x_i C + y_i S$
$\Delta p_i^{(k)} = (p_k^i - p_{k-1}^i)/K^{(k)}$
Set initial values: kk=o, $p_0^i = p_{k-1}^i$
⑥. kk=kk+1 (If kk>$K^{(k)}$ then skip to ⑤.)
Calculate
$p_{kk}^i = p_{kk-1}^i + \Delta p_i^{(k)}$
and generate PLH lines in array b.
⑦. Skip to ⑥.
⑧. Skip to ②.
⑨. Suppress the non maximum points in array b,
⑩. Detect the higher p peaks from the array b, and let the subscripts of tha array b be recorded to u(1),v(1),1=1,2,..,p
⑪. stop of PLHT

4.2 Evaluation of the Cost of PLHT algorithm

Figure 7 shows the loop scheme to execute the PLHT algorithm. Table 1 shows the results for the theoretical estimation of the computing costs. The multiplication operation of PLHT can be reduced to 2m/K of that of Hough transform. The trigonometric calculations of PLHT can be reduced to 2m/NK.

A result of the simulation experiment is shown in Table 2. It was clarified that the PLHT algorithm can be executed about 6 times faster than the usual Hough transform.

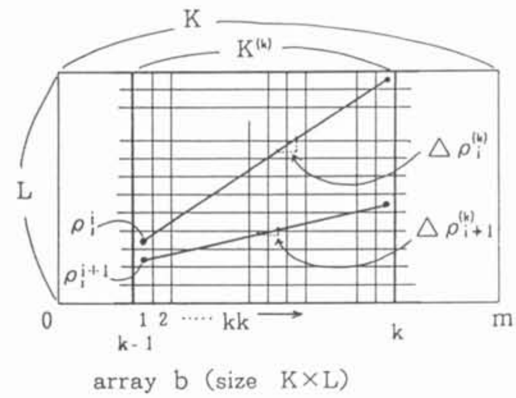


Fig.6 Notations for the PLHT algorithm

```
┌──────────k:block number
│            C=cosθ  C-=sinθ
│            S=sinθ  S-=sinθ
│   ┌────i:dot number
m   │       calc. Δρ_i^(k) using C,S,C- and S-
│   N ┌─kk:resolution number in k-th block
│   │ │     generation of PLH function
│   │ K        ρ_kk^i =  ρ_kk-1^i +   Δρ_i^(k)
│   │ └────────next kk
│   └──────────next i
└──────────────next k
        (a)   PLHT method

┌──────────k: resolution number in θ-axis
│ ┌────i:dot number
K │        ρ_k^i  =  x_i.cosθ  +  y_i.sinθ
│ N────────────── next i
└──────────────── next k
```

(b) usual Hough transform method

Fig.7 The loop structure of the transform

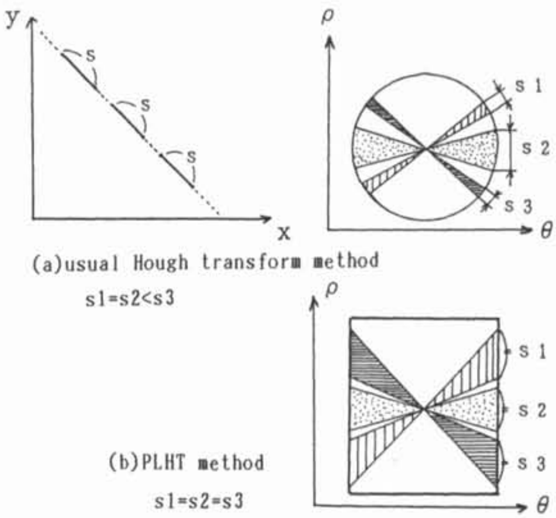


Fig.8 Geometric properties of the PLH function

5. Application of PLHT and Considerations

5.1 Ridge Line of the Engine Parts

From a camera image(256x256, 256 grey levels) of the engine parts shown in Photo.1, a major ridge line is extracted by the PLHT method. The distribution of the PLH functions in the parameter space is shown in Photo.2, where m=2 and K=L=256. In this case, Sobel operator was used and the number N of the candidate points was 3014.

It took about 4.1 seconds to execute PLHT algorithm(MC68000,12.5MHz). The detailed results of the computing cost are shown in Table 3.

5.2 Considerations

In addition to the reduction of the computing cost, the reduction of the memory cost can be provided by the PLHT method. If the number m of the equally devided blocks is 2, the maximum value ρ, ρmax, of the parameter space is evaluated by eq.(10), when a point (x_m, y_m) is at a longest away from the origin. Therefore, when $x_m = y_m$, about 30 % reduction of the memory cost can be provided by the PLHT method.

$$\rho_{max} = x_m = \mathrm{SQRT}(x_m^2 + x_m^2)/\mathrm{SQRT}(2) = \mathrm{SQRT}(x_m^2 + y_m^2)/\mathrm{SQRT}(2) = \rho_H/\mathrm{SQRT}(2) = 0.7.\rho \quad (10)$$

,where ρ_H is ρ value of the usual Hough transform

The collinear line segments in x-y space are transformed to the parameter space as shown in Fig.8. On the contrary to the usual Hough transform, the distribution of the PLH functions around the peak is radially homogenious in the sense that the area neighbouring the peak is constant.

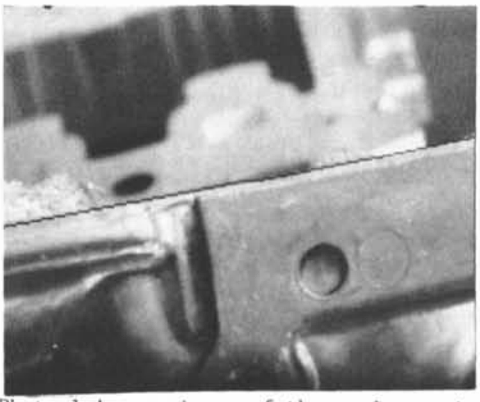


Photo.1 A grey image of the engine parts

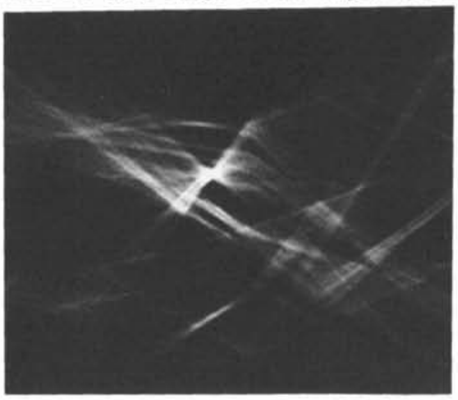


Photo.2 Distribution of the PLH functions

6. Conclusion

A new Hough transform method to extract a set of line patterns from x-y space, piece-wise linear Hough transform(PLHT), was proposed. The PLHT can be executed without the calculations of the trigonomeric and multiplication operations.

Furthermore, this method is suggestive enough to establish the more extended functions to provide a scheme to extract a set of line patterns in the same way of the Hough function.

The PLHT method is one of the fast Hough transforms. The computing cost of PLHT is realized by reducing the core multiplication and trigonometric calculations. It was known experimentally that about 1/6 reduction is provided by an industrial application.

As one of the coming problems, it is important to clarify the geometric interpretation of the behavior of the line patterns extracted by the PLHT method, and to establish the more extended functions applicable to the scheme of the Hough pattern extraction.

Acknowledgement

Authors wish to thank Mr. K.Murakami of Nagoya Municipal Industrial Research Institute for his helpful discussions.

References

1)Koshimizu,H. and Numada,M. :"On a fast Hough transform method based on piece-wise linear Hough function,"IEICE Technical Report, Vol.88, No.23, pp.1-8 (PRU88-1) (May 1988)

2)O'Gorman,F. and Clowes,M.B.:"Finding picture edges through collinearity of feature points," IEEE Trans.C, C-25, 4, pp.449-456(Apr.1976)

3)Onda,K. and Aoki,Y. :"A computational method for a fast Hough transform using the periodicity of the trigonometric function," Trans.IEICE, Vol.70-D, No.10, pp.2009-2011 (Oct.1987)

Table 1 Theoretical estimation of the computing cost

operation	usual Hough	PLHT method	efficiency(m=2)
C(cos)	NK	2m	4/NK
S(sin)	NK	2m	4/NK
*	2NK	4mK	4/K
+	NK	NK+2mN	1+4/K
-	0	mN	.
/	0	mN	.

N: the number of edge points
K: the total resolutions in θ-axis
m: the number of blocka in θ-axis

Table 2 Experimental estimation of the computing cost

	BASIC(PC9801VM)	ASSEMBY(MC68000)
usual Hough	128 sec	0.79
PLHT	49	0.13
efficiency	2.6 times	6

N=100, K=512, trigonometric calc.= using LUT

Table 3 Experimental estimation of computing cost

<table>
<tr><td colspan="4">detection of a line(p=1)</td></tr>
<tr><td rowspan="3">edge detection
(Sobel opr.)</td><td colspan="3">whole process of PLHT</td></tr>
<tr><td colspan="2">transform process</td><td>peak etection</td></tr>
<tr><td>address</td><td>disribution</td><td>(p=1)</td></tr>
<tr><td rowspan="3">1.6</td><td>2.0sec</td><td>2.1</td><td rowspan="2">0.2</td></tr>
<tr><td colspan="2">4.1</td></tr>
<tr><td colspan="3">4.3</td></tr>
<tr><td colspan="4">5.9 sec</td></tr>
</table>

N=3014, K,L=256; machine=LIP-10(MC68000,12.5MHz)
language=ASSEMBLY; image/parameter memory=static RAM