

**Sai *et al.* Reply:** In Ref. [1], we demonstrated that there exists a nontrivial correction, arising from the viscosity of the electron liquid [2], to the conductance of nanoscale junctions calculated within the adiabatic local-density approximation to time-dependent density-functional theory (DFT). This dynamical correction cannot be captured by any static DFT functional, even the exact one. To provide an estimate of these effects, we derived Eq. (14) for the dynamical viscous resistance and evaluated that expression by employing the viscosity of bulk electrodes. In the preceding Comment [3], Jung, Bokes, and Godby (JBG) have used our Eq. (14) to calculate the dynamical resistance of two infinite jellium electrodes separated by a vacuum gap. By using the local-density-dependent formula for the viscosity coefficient as reported in Ref. [4], they conclude that the dynamical resistance is negligibly small in the specific cases they consider. Here we argue that their calculations do not preclude the possibility that the viscosity contribution to the conductance be large in realistic nanoscale structures, the systems of interest in Ref. [1].

Two main points support this statement. (i) Our Eq. (14) is an approximate formula, derived under certain physical assumptions (see below) in order to provide a qualitative understanding of the viscous effects in nanostructures. As noted in Ref. [1], Eq. (14) was derived assuming homogeneous density in the transverse direction *and* homogeneous current density in both the transverse and the longitudinal directions. The only contribution to the correction we included in Eq. (14) comes from density variations along the longitudinal direction. It is on the grounds of these assumptions that we adopted the viscosity of the bulk electrodes in Ref. [1]. We did not claim any quantitative accuracy of our estimates. In realistic nanoscale structures, the current density may vary rapidly in the transverse direction due to a decrease of velocity from the center of the channel to the sides of the conductors. The transverse density and current-density gradients can thus contribute significantly to the dissipation effects. The contribution can be further enhanced by the presence of turbulent eddies near the contacts [5,6]. To capture these gradient contributions, one needs to evaluate the dissipation power  $dE/dt = -\int e\vec{j} \cdot \vec{E}_{xc} d\vec{r}$  directly [and the associated resistance as  $R^{\text{dyn}} = (dE/dt)/I^2$ , with  $I$  the total current], because the nonconservative nature of the dynamical  $xc$  field makes it, in general, ambiguous to evaluate a line integral as in Ref. [1]. If the current density and viscosity are again assumed constant, the correction to the resistance evaluated from the power dissipated is given by  $R^{\text{dyn}} = (\eta/e^2 A^2) \int [\frac{4}{3}(\partial_z n^{-1})^2 + (\partial_\perp n^{-1})^2] d\vec{r}$ , where  $\perp$  represents the transverse direction,  $\eta$  is the (constant) viscosity, and  $A$  is the cross section of the nanostructure. This expression contains a positive transverse density gradient term which increases the dynamical effects evaluated in Eq. (14) of Ref. [1]. For a general current-carrying nanoscale system, a quantitative evaluation

of these corrections requires knowledge of their microscopic current and density distributions [5] and of the spatial variation of the viscosity, and the dissipation power (and associated resistance) must be evaluated numerically. (ii) To calculate the viscosity coefficient, JBG have applied the Conti-Vignale formula, Eq. (4.10) of Ref. [4]. This formula is more accurate than the high-density formula we have used in Ref. [1]. However, unlike what JBG suggest [3], the Conti-Vignale formula does *not* interpolate between the high-density and low-density limits of the homogeneous electron liquid. The simple reason for this is that the *exact* low-density limit of the viscosity of the electron gas is *unknown*. The formula instead comes from a fit to numerical results of Nifosi *et al.*, which are based on mode-mode coupling theory [7]. This theory is certainly not exact in the low-density limit [4]. Indeed, due to the strong correlation effects in the low-density electron gas, where the electrons are on the verge of crystallization, it is reasonable to suspect that the relative viscosity  $\eta/n$  might increase well above Eq. (4.10) of Ref. [4] with decreasing density. We conclude that the Comment [3], while interesting, should not be taken as an indication that the viscosity corrections to the conductance of real nanoscale structures are generally small. A more accurate treatment of the density and current-density distribution and of electronic correlations may yield much larger corrections in realistic systems.

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