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THE PROTON AND NEUTRON MAGNETIC MOMENTS IN LATTICE QCD

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ABSTRACT

We have computed the proton and neutron magnetic moments on the lattice by a Monte Carlo simulation of QCD in the quenched approximation. The results are in remarkable agreement with the experimental values.

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We can measure the magnetic moment of a Dirac particle through a measurement of its mass. In fact, the mass of a spin $\frac{1}{2}$ fermion in the presence of a weak external magnetic field $\vec{H}$, to lowest order in H, has the form :

$$m_n = m_o + \frac{e|\vec{H}|}{m_o}(n+1/2) + \vec{\mu}\cdot\vec{H} \quad (1)$$

where $\vec{\mu}$ is the magnetic moment, m_o is the mass of the particle in the absence of the magnetic field and e its electric charge. Different values of the integer n correspond to different Landau levels. The mass of the particle can be extracted from its propagator which is computed by Monte Carlo techniques [1)-6)].

On the lattice, a uniform, time-independent magnetic field $\vec{H}$ can be introduced by adding to the usual gluon field $U_\mu(\tilde{x})$ an external Abelian electromagnetic field $U_\mu^{ext}(\tilde{x})$. Taking $\vec{H} = (0,0,H)$ directed along the z axis we write :

$$\begin{aligned} &U_\mu(\tilde{x}) \rightarrow U_\mu(\tilde{x})\cdot U_\mu^{ext}(\tilde{x}) \\ &U_\mu^{ext}(\tilde{x}) = 1 \quad (\mu = t, z) \\ &U_x^{ext}(\tilde{x}) = \exp[-i\beta y] \quad ; \quad U_y^{ext}(\tilde{x}) = \exp[i\alpha x] \end{aligned} \quad (2)$$

$\tilde{x}$ is the generic point in four dimensions on the lattice. In this way we have a uniform flux of the magnetic field through an elementary plaquette in the x-y plane (see Fig. 1) :

$$\exp\left\{ie_f \oint_{\partial P} A_\mu dx^\mu\right\} = \exp\left\{ie_f \int_P \vec{H}\cdot d\vec{\sigma}\right\} = \exp\left[ie_f H a^2\right]$$

$$= U_x^{ext}(\tilde{x})\, U_y^{ext}(\tilde{x}+\hat{\mu}_x)\, U_x^{+ext}(\tilde{x}+\hat{\mu}_y)\, U_y^{+ext}(\tilde{x}) = \exp\left[i(\alpha+\beta)\right] \quad (3)$$

where a is the lattice opacing, $\hat{\mu}_{x,y}$ the unit vectors along the x, y directions and e_f the electric charge of the quark of flavour f coupled to the electromagnetic field. From Eq. (3) we find :

$$H = \frac{1}{e_f}\frac{(\alpha+\beta)}{a^2} \quad (4)$$

Working on a finite lattice with periodic boundary conditions $[F(\tilde{x}+N_{x,y}\,\hat{\mu}_{x,y}) = F(\tilde{x})]$ the minimum possible value of the magnetic field is :

$$H = \frac{1}{e_g}\,\frac{2\pi}{a^2\,[N_x+1]^2} \tag{5}$$

In practical cases this is always a very intense magnetic field. For example, for $a^{-1} \sim 1.4$ GeV $[\beta = 6/g_0^2 = 6$; g_0^2 = lattice coupling constant [5),6)] and $N_x = 5$ the minimum possible magnetic field would correspond to a shift of the proton mass :

$$\delta m = \mu H \sim 0.6\ \text{GeV} \tag{6}$$

Such a strong magnetic field would lead us out of the linear region where Eq. (1) is valid. In order to have a continuously variable magnetic field, we break the periodicity of the lattice in the following way :

$$\begin{aligned} &U_x^{ext}(\tilde{x}^* \equiv (t, N_x, y, z)) = 0 \quad ; \quad U_x^{ext}(\tilde{x} \neq \tilde{x}^*) = 1 \\ &U_{z,t}^{ext}(\tilde{x}) = 1 \quad ; \quad U_y^{ext}(\tilde{x}) = \exp\left[i\,(eH)\,x\,a^2\right] \end{aligned} \tag{7}$$

In practice, this means that we separate the copies of the periodic lattice along the x direction by an infinitely high barrier. The minimum nucleon energy is no longer zero but :

$$\Delta E \simeq \frac{1}{2m}\left[\frac{\pi}{(N_x+1)a}\right]^2 \tag{8}$$

m is the nucleon mass. This corresponds (for $a^{-1} = 1.42$ GeV, $N_x = 5$, $0.145 \leq K \leq 0.150$; K is the Wilson hopping parameter) to a relative increase of the estimated mass $\delta m/m \lesssim 5\%$. This small systematic effect should be partially compensated for in the computation of dimensionless quantities (for example the giromagnetic factor).

To find the values of H where the linear approximation is valid, we studied the behaviour of the nucleon propagator as a function of H for a particular gauge configuration at $K = 0.145$. As in Refs. 1) to 6), we define :

$$G_{\alpha\beta}(t) = \sum_{\vec{x}} G_{\alpha\beta}(\tilde{x}) \tag{9}$$

$G_{\alpha\beta}(\tilde{x})$ is the nucleon propagator, $\vec{x}$ are the spatial components and $G_{\alpha\beta}(t)$ is the propagator for a nucleon at rest. We expect that :

$$R(t) \equiv \left[\frac{G_\uparrow - G_\downarrow}{G_\uparrow + G_\downarrow}\right]_t - \left[\frac{G_\uparrow - G_\downarrow}{G_\uparrow + G_\downarrow}\right]_{(t-1)} \simeq \mu H \tag{10}$$

independent of t for small values of H. $G_{\uparrow,\downarrow}$ is the spin up, spin down propagator at distance t.

In Fig. 2, we give $R(t=5)$ as a function of H. The value of H chosen for the actual measurement is $H = 0.075$ which is well inside the linear region. It should be noticed that with too small a value for the magnetic field it would be impossible to extract the value of the magnetic moment with a reasonable error because of the limited precision ($\lesssim 1\%$) reached in the evaluation of the quark propagator. (greater precision would considerably increase the computer time needed). We have checked on a single gauge configuration at $K = 0.150$ that by changing the precision from 10^{-2} to 10^{-4} in the computation of the quark propagator the final result changes at the level of 2%.

Notice that in Fig. 2, $R(t=5)$ is different from zero for $H = 0$ as well because for one (or a few) gauge field configuration only, the symmetry for a reflection of the z axis is not exactly satisfied. In the following we will always use :

$$\tilde{R}(t) \equiv R(t)\Big|_{H=0.075} - R(t)\Big|_{H=0} \tag{11}$$

to correct this statistical effect.

A possible source of systematic errors is due to the presence of several different states propagating simultaneously with the nucleon in the same channel [as discussed in Refs. 5) and 6)]. Using a lattice of $5^3 \times 10$ it is, in practice, impossible, in the evaluation of the magnetic moment, to separate completely the low-lying states from higher excitations. To give a rough idea of the size of this systematic error we will give below the value of the magnetic moment extracted from $\tilde{R}(t=4)$ and $\tilde{R}(t=5)$.

To estimate the statistical error we used the same method as in Ref. 5), dividing the full set of gauge field configurations into two clusters and computing the average value and the dispersion on these clusters. We used eight practically uncorrelated gauge field configurations (they are separated by 400 Monte Carlo sweeps) chosen from those computed in Ref. 5).

In the Table, we give a complete list of the parameters used in this Monte Carlo experiment together with the results for the proton and neutron giromagnetic factor $g_{P,N}(4,5)$ extracted from $\tilde{R}(4)$ and $\tilde{R}(5)$ for $K = 0.145$, 0.1475 and 0.150, respectively.

$$g_{P,N} = \frac{2\, m_{P,N}\, \mu_{P,N}}{e} \tag{12}$$

$\mu_{P,N}$ and $m_{P,N}$ are the proton/neutron measured magnetic moments and masses respectively ; e is the unit electric charge.

Because of the rather large statistical fluctuations at large K, it is clear that it is not possible to extract any K dependence. It should be noticed that the drift of the values of $g_{P,N}$ with the mass of the quarks is very small compared with the variation of the quark mass $m_q = 1/2a(1/K - 1/K_C)$ (in the quark model one expects $g_P \sim m_P/m_q$) and it may be attributed to the mixing of the low-lying states with other excitations. The averages over K, weighted with the statistical errors, give *) :

$$g_P(t=4) = 2.99 \pm 0.17 \;;\quad g_P(t=5) = 2.96 \pm 0.58 \quad (\exp \sim 2.79)$$

$$g_N(t=4) = -1.91 \pm 0.08 \;;\quad g_N(t=5) = -1.93 \pm 0.45 \quad (\exp \sim -1.90)$$

The g_P/g_N ratio averaged over the eight gauge configurations is

$$g_P/g_N = -1.60 \pm 0.15 \qquad (\exp \sim -1.47)$$

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*) Note that the results for $t = 4$, with smaller statistical errors, are affected by larger systematical errors.

In spite of the presence of various systematic effects, the results appear in remarkable agreement with the experimental values. It would be very interesting, with a larger lattice, to compute $g_{P,N}$ much nearer to the critical K. The technique we used to compute the anomalous magnetic moment can easily be extended to the computation of the hadronic form factors and matrix elements. This experiment took about 9 h CPU time on a CDC 7600.

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K	$g_P(4)$	$g_P(5)$	$g_N(4)$	$g_N(5)$
0.145	2.76±0.18	2.89±0.29	-1.76±0.09	-1.87±0.24
0.1475	2.96±0.17	3.09±0.53	-1.89±0.08	-2.02±0.42
0.150	3.18±0.15	3.33±0.91	-2.02±0.07	-2.21±0.68

TABLE - The value of $g_{P,N}$ (see text) for different values of the Wilson hopping parameter K. We used a $5^3 \times 10$ lattice, $\beta = 6$, 8 gauge field configurations and a relative precision in the inversion of the propagator $\lesssim 1\%$.

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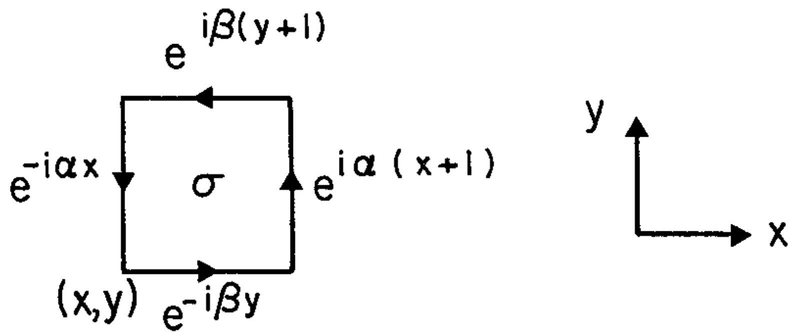
FIGURE CAPTIONS

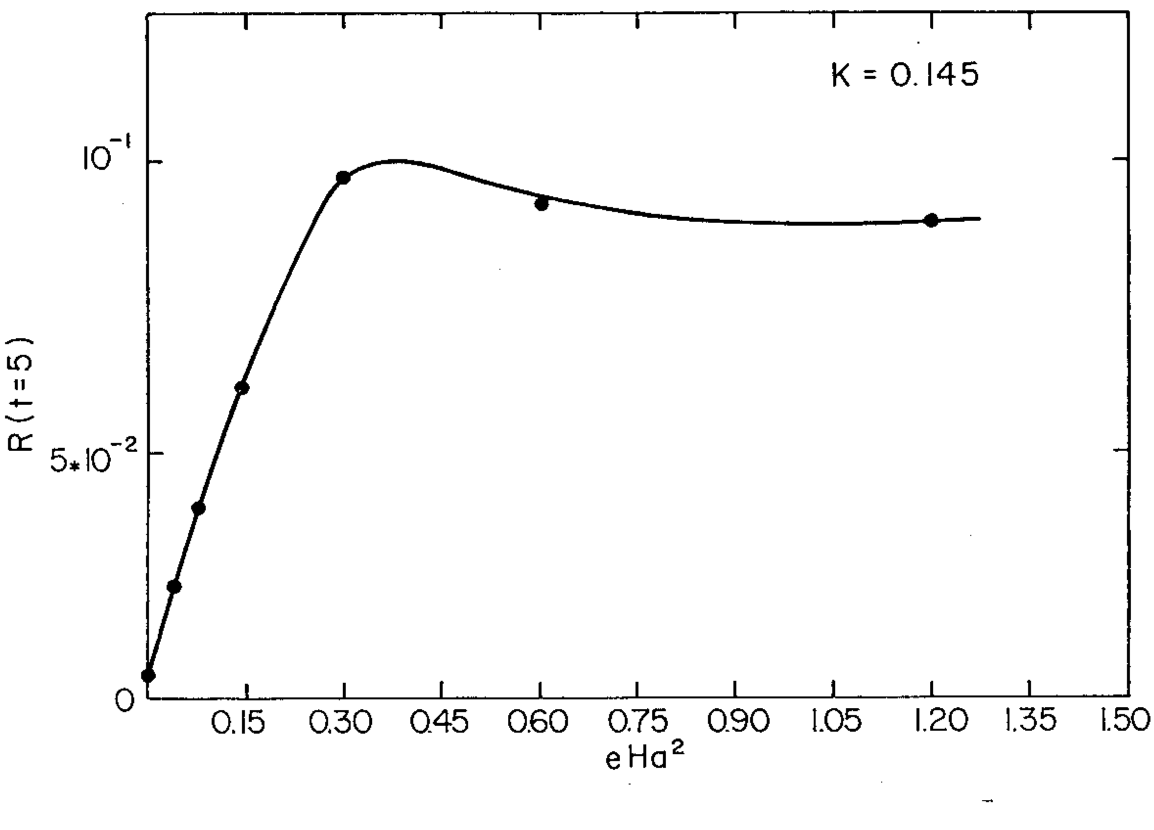
Fig. 1 The flux of the magnetic field $\vec{H}$ through an elementary plaquette σ in the x, y plane.

Fig. 2 R(t=5) defined in Eq. (10) as a function of H. Notice that its behaviour for large H is expected to go (approximately) as the difference of two hyperbolic tangents.



e^{iβ(y+1)}
e^{-iαx}
σ
e^{iα(x+1)}
(x,y)
e^{-iβy}
y
x

Fig. I



K = 0.145
10^{-1}
5*10^{-2}
0
R(t=5)
0.15
0.30
0.45
0.60
0.75
0.90
1.05
1.20
1.35
1.50
eHa^2

Fig. 2