Interregional Transportation Modeling for the Far East of Russia Macro-region

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Abstract. The paper describes two mathematical models of trade flows among the territories of a region or country. First model uses the approach of modeling the complex communication system to determine the most probable values of trade flows in a case of incomplete information about the system. Second one is based on multi-commodity network flow equilibrium approach. Transport costs between the territories are modeled within the framework of gravity model. The payment for transportation depends on the distance between the regions, this distance is estimated as the shortest way length in a given transport network or geographical distance. The mathematical formulation of the problems belongs to the class of convex mathematical programming problems and assumes the numerical solution of nonlinear optimization problem with linear constraints. The paper demonstrates the simulation of interregional freight traffic of the Russian Far East region.

Keywords: spatial, interregional, transportation, multi-commodity, multimodal, gravity, entropy, network, flow, equilibrium, model

1 Introduction

Simulation of interregional flows was proposed by Wassily Leontief [1]. The researchers of international trade and regional economy develop gravity modeling approach [2] to explain the exports and imports flows of goods and services within a multiproduct equilibrium flows on the transport network.

A.G. Wilson and others developed a more general entropy modeling approach [3] to take into account the incompleteness of information in the application to an equilibrium modeling for complex communication systems which is applied to simulate interregional multi-product flows. In 1970s it was shown that gravity modeling approach and the principle of entropy maximization [4] are interrelated in many ways.

Boyce and others [5]–[7] further applied Wilson's approach for practical applications of the transport system of the USA regarding the configuration of the transport network and multimodal flows concerning various modes of transport. The entropy approach in

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Soviet transportation science has been widely used for planning spatial development of cities and industries. At present these approaches are now widely used for passenger and freight traffic modeling in the transportation systems in Russia and abroad [8]–[10]. Input data can be incomplete and possible modification assuming interval input data can be done as discussed in [11].

2 Gravity model of trade flows

In this section the mathematical model, program implementation and visualization of trade flows for the Far East of Russia Macro-region is given.

2.1 Mathematics of trade flows model

Consider the model of the economy of k regions in each of which there is an n products. Let z_{ij}^{rm} is an unknown number of a product of r-th type, $r = 1, \ldots, n$ delivered from the i-th region in the j-th, $i, j = 1, \ldots, k$, the mode of transport is defined as $m = 1, 2, \ldots, M$ where M is the number of different types of transport used for transportation. Here and below the superscripts correspond to the product type and mode of transport, subscripts correspond regions. Note that the flow z_{ii}^{rm} is not necessarily assumed to be zero. The case it is strictly positive corresponds the part of the region production that is consumed in the same region.

The total flow from region r to the j-th region ("total consumption" of the product r to the region j) by mode of transport m equals $\sum\limits_{m=1}^{M} \sum\limits_{i=1}^{k} z_{ij}^{rm}$. The last sum also equals to V_j^r that is a known cumulative import of product r to the region j given by official statistics. Total export of product r from the region i in other regions ("total production" of product r to the region i) transported by all modes of transport is $\sum\limits_{m=1}^{M} \sum\limits_{j=1}^{k} z_{ij}^{rm}$ is also known from official statistics and is defined as W_i^r.

The production and consumption of each product $r = 1, \ldots, n$ defined in the above way is subject to obvious balance equations

$$\sum_{m=1}^{M} \sum_{i=1}^{k} \sum_{j=1}^{k} z_{ij}^{rm} = \sum_j V_j^r = \sum_i W_i^r, \tag{1}$$

which imposes an additional restriction on the given values of V_j^r and W_i^r.

However, since the real system of regions may be not closed and we can observe the trade of the assumed regions with others, the aforementioned balance (1) formed on the statistical data would not be observed. This means that there is a flow of products between these k regions and other unknown "external" regions in relation to the considered system of regions. The problem is complicated by the fact that neither the total import or export of such "external" regions are known. Obviously in this case the model requires modification.

To solve this problem let's aggregate the "external" regions to $(k+1)$-th region and let's consider additional flows $z_{i\,k+1}^{rm}$ and $z_{k+1\,j}^{rm}$ which are unknown and moreover are unidentified.

Trade flows are carried out by economic agents under the influence of the transportation costs which principally depends on the geographical distance between regions. Consider the gravity model for transportation costs which can be represented by $v_{ij}^{rm} = \exp(-d^{rm}T_{ij})$, where v_{ij}^{rm} is a priori defined the flow of products from the i-th region in the j-th, T_{ij} is an assessment of the geographical distance between regions i and j, and d^{rm} are the parameters that are responsible for the flow sensitivity to distance for the product r and the mode of transport m used to transport it. Parameters d^{rm} are non-negative which means that the higher the value of the distance between, the smaller an amount of flow between the regions i and j is. It is additionally assumed that $v_{ii}^{rm} = 0$ for $T_{ii} = 0$ and $v_{ij}^{rm} = v_{ji}^{rm}$ because of $T_{ij} = T_{ji}$.

Calibration of non-negative parameters d^{rm} with actual official statistics is a separate problem of applied statistics. This assessment is carried out by methods such as least squares (LS) applied to the regression model which is represented by a linear by parameters model $\ln v_{ij}^{rm} = \alpha - d^{rm}T_{ij} + \delta^{rm}$ for all l$i, j = 1, \ldots, k$ and $i > j$ where δ^{rm} is a normally distributed residuals of the regression for all r and m.

In papers [3], [4], [10] it is considered an approach of modeling flows in a communication networks corresponding to the principle of the most likely values of the distribution of flows in conditions of incomplete information when only some balance equations for these flows are given. Adaptation of this approach for the model of interregional trade flows makes it necessary to minimize the non-linear functions of the form $\sum\limits_{r=1}^{n} \sum\limits_{m=1}^{M} \sum\limits_{i,j=1, i \neq j}^{k+1} z_{ij}^{rm} \ln(z_{ij}^{rm}/\nu_{ij}^{rm})$ on the set of unknown flows z_{ij}^{rm}.

The presence of such features makes it necessary to specify strictly positive trade flows z_{ij}^{rm} which is modeled by specifying lower restrictions on flows by preassigned small parameter $\varepsilon > 0$.

Then we solve a nonlinear optimization problem with already considered balance equations as linear constraints and the objective function that is motivated by the most probable flows approach in a case of incomplete information about the communication system [3, 10, 11]:

$$\sum_{r=1}^{n} \sum_{m=1}^{M} \sum_{i,j=1, i \neq j}^{k+1} z_{ij}^{rm} \ln(z_{ij}^{rm}/\hat{v}_{ij}^{rm}) \to \min_{\{z_{ij}^{rm}\}}, \tag{2}$$

where $\hat{v}_{ij}^{rm} = \exp(-\hat{d}^{rm}T_{ij})$, $\hat{d}^{rm}$ are known estimates of the parameters and constraints of the problem are given further:

$$\sum_{m=1}^{M} \sum_{i=1}^{k+1} z_{ij}^{rm} = V_j^r, \tag{3}$$

for all $j = 1, 2, \ldots, k$ and $r = 1, 2, \ldots, n$,

$$\sum_{m=1}^{M} \sum_{j=1}^{k+1} z_{ij}^{rm} = W_i^r \tag{4}$$

for all $i = 1, 2, \ldots, k$ and $r = 1, 2, \ldots, n$,

$$z_{ij}^{rm} \geq \varepsilon > 0 \tag{5}$$

for all $i, j = 1, 2, \ldots, k+1, r = 1, 2, \ldots, n, m = 1, 2, \ldots, M$.

2.2 The solution and visualization

Implemented computer software for interregional trade simulation is used to determine the equilibrium interregional freight traffic in the transport network of railway, road and sea transport of the Far Eastern regions of Russia. The data is used as input data of Rosstat official statistical handbooks of different years "The regions of Russia. Socio-economic indicators of the interregional trade". The main products (commodities) are foodstuffs, fuel, goods for technical purposes.

As the points of import and export products corresponding administrative centers of 9 Far East regions are considered: Primorsky Krai (Vladivostok), Khabarovsk (Khabarovsk), the Amur Region (Blagoveshchensk), the Jewish Autonomous Region (Birobidzhan), the Republic of Sakha - Yakutia (Yakutsk), Magadan region (Magadan), Sakhalin region (Yuzhno-Sakhalinsk), Kamchatka region (Petropavlovsk-Kamchatsky), Chukotka Autonomous Okrug (Anadyr).

Estimates of the distances between regions are shown in Table 1 which correspond to the shortest paths between the administrative centers of the regions in question in the transport network of railway, road and sea transport of the Far East of Russia.

Table 1. Estimates of the distances between the regions (Russian Far East), km

Regions	1	2	3	4	5	6	7	8	9
1	0	762	1410	938	3314	2490	990	2490	4490
2	762	0	779	176	2552	2534	1034	2534	4534
3	1410	779	0	603	2035	3182	1813	3182	5182
4	938	176	603	0	2376	2710	1210	2710	4710
5	3314	2552	2035	2376	0	1736	3586	2736	4736
6	2490	2534	3182	2710	1736	0	1500	1000	3000
7	990	1034	1813	1210	3586	1500	0	1500	3500
8	2490	2534	3182	2710	2736	1000	1500	0	2000
9	4490	4534	5182	4710	4736	3000	3500	2000	0

Primorskiy kray - 1, Khabarovskiy kray - 2, Amurskaya oblast' - 3, Evreyskaya avtonomnaya oblast' - 4, Respublika Sakha (Yakutiya) - 5, Magadanskaya oblast' - 6, Sakhalinskaya oblast' - 7, Kamchatskiy kray - 8, Chukotskiy avtonomnyy okrug - 9.

Table 2. Total outflow (production) from the regions (Russian Far East)

Goods Regions	Fuel, thous. tons	Coal, thous. tons	Furniture, thous. cub. m.	Meat and poultry, tons	Cars, units
1	11.2	11.9	0.2	25.9	1175
2	507	263	47.7	431	0.01
3	0.01	495	2	2007	0.01
4	0.01	0.01	0.01	0.01	0.01
5	0.01	3514	10	0.01	0.01
6	0.2	81.1	0.01	0.01	0.01
7	0.01	61	0.01	0.01	0.01
8	0.01	0.01	0.01	0.01	0.01
9	0.01	129	0.01	0.01	0.01

Primorskiy kray - 1, Khabarovskiy kray - 2, Amurskaya oblast' - 3, Evreyskaya avtonomnaya oblast' - 4, Respublika Sakha (Yakutiya) - 5, Magadanskaya oblast' - 6, Sakhalinskaya oblast' - 7, Kamchatskiy kray - 8, Chukotskiy avtonomnyy okrug - 9.

Table 3. Total inflow (consumption) of different goods (Russian Far East)

Goods Region	Fuel, thous. tons	Coal, thous. tons	Furniture, thous. cub. m.	Meat and poultry, tons	Cars, units
1	459	2179	167	662	951
2	63.5	3386	6.6	2048	1424
3	138	693	0.01	154	921
4	45.2	377	0.01	2.4	2
5	42.4	81.4	5.4	485	661
6	0.8	323	0.01	5	33
7	21.6	0.01	0.01	5	564
8	37.2	202	0.01	9.5	27
9	0.2	77.8	0.1	0.01	0.01

Primorskiy kray - 1, Khabarovskiy kray - 2, Amurskaya oblast' - 3, Evreyskaya avtonomnaya oblast' - 4, Respublika Sakha (Yakutiya) - 5, Magadanskaya oblast' - 6, Sakhalinskaya oblast' - 7, Kamchatskiy kray - 8, Chukotskiy avtonomnyy okrug - 9.

Table 4 shows the result of the coal traffic simulation as solution of the problem to determine the most probable movement of goods in the system of the Far Eastern regions and trade with other regions. External to the system of the regions in question in the territory of the aggregated region that table shows the last row (column) under the number “10”.

Table 4. The simulation results. Transportation of coal, thous. tons

Regions	1	2	3	4	5	6	7	8	9	10
1	0	5.53	2.29	0.78	0.72	1.45	0	0.78	0.36	0
2	100.69	0	44.63	5.18	19.6	52.13	0	27.98	12.79	0
3	172.49	184.94	0	16.44	14.47	60.61	0	32.53	13.53	0
4	0	0	0	0	0	0	0	0	0	0
5	1133	1693.23	301.64	180.99	0	92.41	0.01	78.16	34.55	0.01
6	20.95	41.37	11.61	5.08	0.85	0	0	0.7	0.54	0
7	12.86	26.08	10.21	3.5	2.71	3.03	0	1.63	0.97	0
8	0	0.01	0	0	0	0	0	0	0	0
9	33.11	64.88	16.57	7.74	2.03	3.44	0	1.23	0	0
10	705.91	1369.96	306.05	157.29	41.03	109.91	0	58.99	15.06	0

Primorskiy kray - 1, Khabarovskiy kray - 2, Amurskaya oblast' - 3, Evreyskaya avtonomnaya oblast' - 4, Respublika Sakha (Yakutiya) - 5, Magadanskaya oblast' - 6, Sakhalinskaya oblast' - 7, Kamchatskiy kray - 8, Chukotskiy avtonomnyy okrug - 9, Other regions - 10.

The visual presentation of the data from Tab. 4 is shown in Fig. 1.

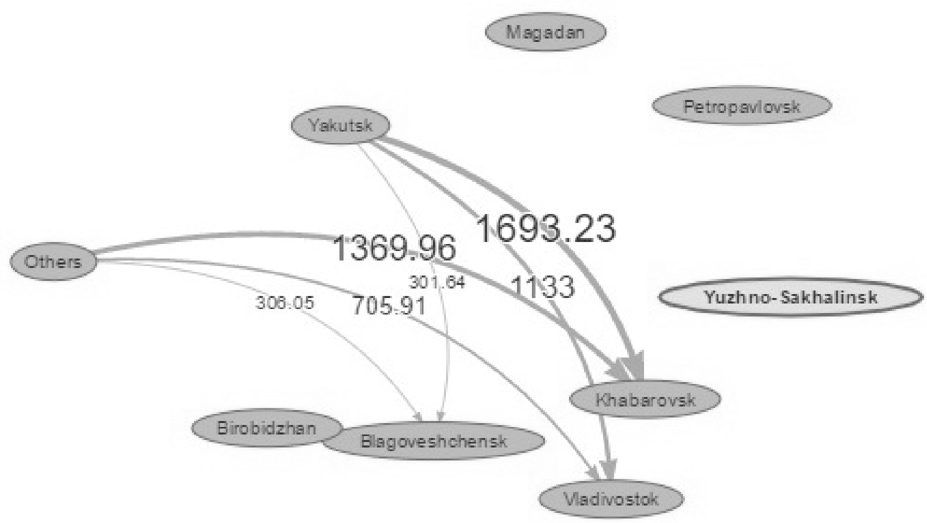

Fig. 1. The simulation results. Transportation of coal, thous. tons

3 Network flow model of interregional trade

In this section the mathematical model within the more general network flow equilibrium framework is discussed and the visualization of simulated trade flows for the Far East of Russia Macro-region is given.

3.1 Mathematical model

Total network equilibrium problem takes the form of the following nonlinear optimization problem

$$\sum_{m=1}^{M}\sum_{l=1}^{L_m}\int_0^{f_l^m} c_l^{(m)}(y)\,dy \to \min\,. \tag{6}$$

Here

$c_l^{(m)}(y)$ is the cost of flow y moving on arc l by m-th type of transportation, further called “mode”,

M is a number of modes of transport used for transportation,

L_m is a number of network arcs for m-th mode,

$f_l^m = \sum\limits_{p=1}^{P_m} d_{lp}^m h_p^m$ is a flow along the arc l of transport network graph for m-th type of transportation, where P_m is a number of all possible paths between any pair of different regions on the m-th mode. Numbers d_{lp}^m equals to one if for fixed m the arc l is a part of path p. For fixed m the elements $\{d_{lp}^m\}$ form a matrix of dimension $L_m \times P_m$.

Then let’s define unknown variables h_p^m that is a total flow along the path p for mode m and z_{ij}^m is an unknown trade flow from the region i to the region j by mode m.

Then we have such constraints:

$$\sum_{p=1}^{P_{ij}^m} h_p^m = z_{ij}^m, \tag{7}$$

where P_{ij}^m is a number of all possible paths between the region i and j for mode m which can be represented as

$$\sum_{p=1}^{P_m} a_{ij}^p h_p^m = z_{ij}^m, \tag{8}$$

where $a_{ij}^p = 1$, if the path p connects region i and j.

Balance constraints for transportation between the regions have the form

$$\sum_{m=1}^{M}\sum_{i=1}^{N} z_{ij}^m = V_j,\ \sum_{m=1}^{M}\sum_{j=1}^{N} z_{ij}^m = W_i. \tag{9}$$

Value V_j is a known cumulative import to the region j and total export from the region i in other regions is also known as W_i both given by official statistics.

Solving the problem (6) under constraints (8) and (9) equilibrium distribution of flows over the network expressed complementary slackness conditions for flows in a form

$$h_p^m(\sum_l^{L_m} c_l^m(f_l^m)d_{lp}^m - u_{ij}^m) = 0, \tag{10}$$

which reflects the principle of Wardrop for network equilibrium that, firstly, if the flow h_p^m along the path p is not equal to zero i.e. $h_p^m > 0$, then the total cost of flow moving $c_p^m = \sum_{l=1}^{L_m} c_l^m(f_l^m)d_{lp}^m$ on all the paths $p = \overline{1, P_{ij}^m}$ are equal to the equilibrium value costs u_{ij}^m which is independent of the path. Secondly, if for some way between the regions i and j total expenses is strictly greater than equilibrium value costs, i.e. $c_p^m > u_{ij}^m$, then all $h_p^m = 0$. All these mean that none of the unloaded paths do not have a lower cost for transportation than c_p^m.

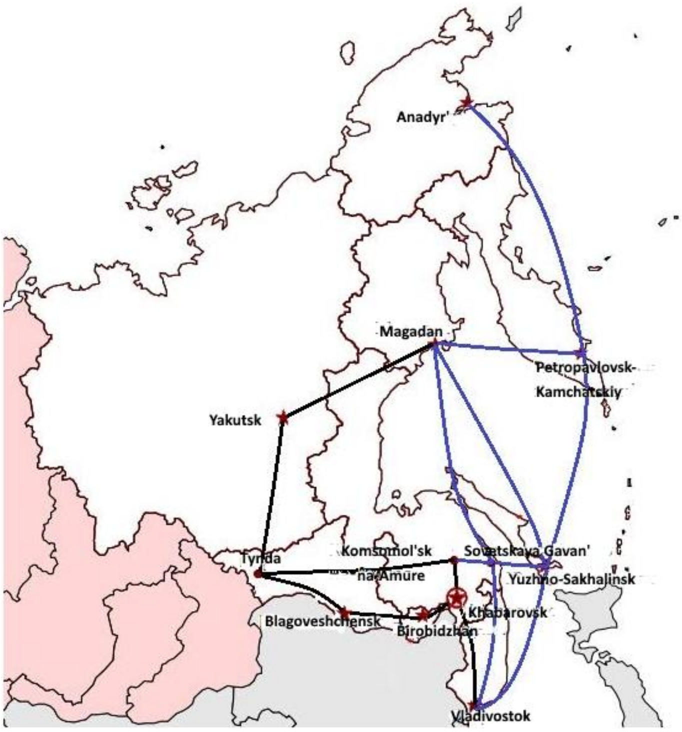

Fig. 2. Aggregated transportation network of Far East of Russia

3.2 The visualization of a solution

We consider 12 cities and administrative centers of corresponding regions of the Far East of Russia as a nodes of transport network: 1 - Vladivostok (Primorye), 2 - Khabarovsk (Khabarovsk Territory), 3 - Birobidzhan (Jewish autonomous region), 4 - Blagoveshchensk (Amur region), 5 - Yakutsk (Sakha-Yakutia), 6 - Magadan (Magadan region), 7 - Yuzhno-Sakhalinsk (Sakhalin region), 8 - Petropavlovsk (Kamchatka region), 9 - Anadyr (Chukotka Autonomous region), 10 - Komsomolsk-on-Amur (Khabarovsk Territory), 11 - Sovetskaya Gavan (Khabarovsk Territory), 12 - Tynda (Amur region).

Aggergated transportation network for all modes is represented on Fig. 2.

The visual presentation of the simulation for network flow model for aggregated is shown in Fig. 3.

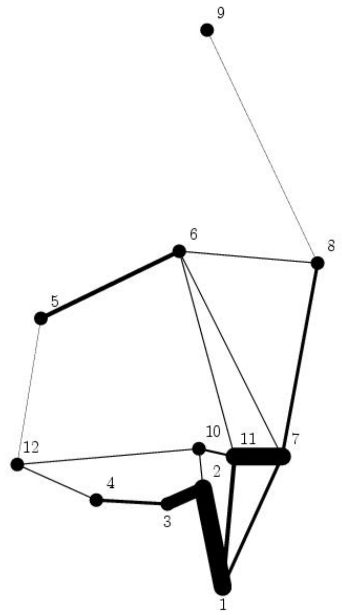

Fig. 3. The simulation result. Aggregated transportation flows

4 Conclusion

The paper sketches two mathematical models of trade flows among the territories of a region or country. They are based on a most probable values of flows in a case of incomplete information about the communication system and multi-commodity network flow equilibrium approach. The mathematics of the models assumes nonlinear convex optimization problem with linear constraints.

The paper demonstrates the simulation of interregional freight traffic of the Russian Far East region. The implemented software package is designed for professionals from various ministries and departments dealing with the problem of optimizing the planning and interregional flows industries products based on their multi-product in multimodal transport networks, and can be used for interregional trade simulation of other set of regions in Russia and the world.

Further research could be developed in a way of numerical algorithms design including parallel ones which could be efficient in a case of a huge number of constraints in the considered mathematical problems and therefore high-performance computations would be needed.

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