

Relating paths in transition systems: the fall of the modal μ -calculus

Catalin Dima¹, Bastien Maubert², and Sophie Pinchinat³

¹ Université Paris Est, LACL, UPEC, Créteil, France.
dima@u-pec.fr

² Università degli Studi di Napoli Federico II
bastien.maubert@gmail.com

³ Université de Rennes 1, IRISA, Rennes, France.
sophie.pinchinat@irisa.fr

This is an extended abstract of a paper presented at MFCS 2015 [11].

Monadic second-order logic (MSO) is considered as a standard for comparing expressiveness of logics of programs. Ground-breaking results concerning expressiveness and decidability of MSO on infinite graphs were obtained first on “freely-generated” structures (words, trees, tree-like structures, etc.) [28,30], then on “non-free” structures like grids [18] or infinite graphs generated by regularity-preserving transformations [10,8]. In all the above settings, the syntax of MSO utilizes one or more binary relation symbols which are interpreted using the binary edge relations of the graph structure. Additionally, much attention has been brought to the study of enrichments of MSO with unary predicate symbols or with the “equal level” binary predicate (MSO^{eq}) [12,27].

For many of these settings, MSO has been compared with automata and with modal logics. Standard results on trees are Rabin’s expressiveness equivalence between MSO with two successors and automata on binary trees [23], and Janin and Walukiewicz’s result [16] showing that the bisimulation-invariant fragment of MSO interpreted over transition coincides with the μ -calculus. Notable exceptions to the classical trilogy between MSO, modal logics and automata are MSO on infinite partial orders – where only partial results are known [5,9,25] – and MSO^{eq} – where, similarly, only partial results are known [27].

More recently, there has been an increased interest in the expressiveness and decidability of logics defined on structures in which two “orthogonal” relations are considered: the so-called temporal epistemic (multi-agent) logics [13], which combine time-passage relations and epistemic relations on the histories of the system. Time-passage relations classically represent the evolution of the system, while each epistemic relation captures some agent’s partial observation of the system by relating indistinguishable histories. They allow to reason about what these agents know about the state of the system along its executions. We may incidentally identify now an important sub-domain in verification which

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is concerned with the expressivity, decidability and axiomatizability of logics of knowledge and time [15,6,29,13,14,17].

The natural question that arises regarding logics of agents that combine time and knowledge is whether a similar trilogy can be established or not. In particular, does there exist a natural extension of MSO, of the μ -calculus, and of tree automata for the temporal epistemic framework, and how would they compare? To the best of our knowledge, these questions remain open. Only partial results exist on relations between some extensions of MSO, μ -calculus, tree automata and other logics of knowledge and time [27,26,29,20].

The first observation is that appropriate extensions of MSO, of the μ -calculus and of tree automata would rely on two sorts of binary relations: those related to the behaviour of the system and those related to epistemic features. While the temporal part of these logics naturally refer to a tree-like structure, the epistemic part requires, in order to model *e.g.* powerful agents that remember the whole past, to consider binary relations defined on histories. The models of such an extension of MSO neither are tree-like structures, nor grid-like structures, nor graphs within the Caucal hierarchy. The proposals in this direction that we know about are [29,26,20] and [1]. [29] mentions an encoding of LTL with knowledge into Chain Logic with equal-level predicate, which is a fragment of MSO^{eq} . [26] introduces the epistemic μ -calculus and studies its model-checking problem. [1] studies an extension of the epistemic μ -calculus, and [20] proposes a generalization of tree automata, called jumping tree automata, which is suited to the study of temporal epistemic logics.

In this work, we develop a general setting in which models are transition systems, *i.e.* directed graphs with atomic propositions, or predicates, on vertices/states and labels on edges/transitions, together with a binary relation over their finite executions, also called paths or histories. Such relations are called *path relations*, and their definition is general enough to capture all indistinguishability relations considered in temporal epistemic logics, and more. We propose extensions of MSO and of the μ -calculus, respectively called the *monadic second order logic with path relation* and the *jumping μ -calculus*.

MSO with path relation is an extension of MSO interpreted over unfoldings of transition systems equipped with a path relation, so that a first-order variable x refers to a node in the tree-unfolding of a transition system, *i.e.* a finite execution (or path, or history). The syntax is that of MSO on graphs with an additional special binary relation symbol \rightsquigarrow . A formula of the form $x \rightsquigarrow y$ holds in a transition system if the path represented by x is related to the one represented by y , according to the binary relation over paths that equips the system.

The jumping μ -calculus is a generalization of the epistemic μ -calculus defined in [26]: it is also evaluated on tree-unfoldings of transition systems, and it features a *jumping modality* \boxtimes whose semantics relies on the path relation that equips the system. In case the path relation is seen as modelling histories' indistinguishability for some agent, this modality coincides with the classic knowledge operator K for this agent.

As in the classic setting of [16], definability in the jumping μ -calculus entails definability in MSO with path relation. It is the converse statement that we explore, that is the *expressive completeness* of jumping μ -calculus *w.r.t.* (the bisimilar invariant fragment of) MSO with path relation.

We first show that, just like alternating tree automata are equivalent to the μ -calculus, the *jumping tree automata* recently defined in [20] are equivalent to the jumping μ -calculus, and the two-way translation does not depend on the *a priori* fixed path relation. We then address, like in [16], the question whether for bisimulation-closed classes of models, definability in MSO with path relation implies definability in the jumping μ -calculus. A crucial parameter in this question is the complexity of the path relation one considers. We recall that, given a finite alphabet Σ , a binary relation over Σ^* is *regular* if there is a finite state automaton with two tapes on which it progresses synchronously (a *synchronous transducer*) that accepts a pair of words over Σ if, and only if, it is in the relation (see [2] for details). An example is the epistemic relation of an agent with synchronous perfect recall [3,4]. A relation over Σ^* is *recognizable* if there is a finite-state word automaton over $\Sigma \cup \{\#\}$, where $\#$ is a special separator symbol, that accepts precisely words of the form $w\#w'$ where (w, w') is in the relation (again refer to [2] for details). For example, epistemic relations of agents whose memory can be represented by finite state machines are recognizable relations (see [19]).

We establish the following results:

Theorem 1. *For any recognizable path relation, the jumping μ -calculus is expressive complete with respect to MSO with path relation.*

Theorem 2. *There are regular binary relations for which the jumping μ -calculus is not expressive complete with respect to MSO with path relation.*

Theorem 1 follows simply from the fact that, since recognizable relations are MSO definable, both our extensions of MSO and the μ -calculus collapse to the classic MSO and μ -calculus, respectively, when the path relation is recognizable.

Concerning transition systems with bounded branching degree, we obtain in addition that the jumping μ -calculus with recognizable path relation is at most exponentially more succinct than the μ -calculus, while its satisfiability problem is also EXPTIME-complete. These results rely on the effective translation of jumping tree automata equipped with recognizable path relations into alternating two-way tree automata [20].

To establish Theorem 2 we consider the case of the so-called *synchronous perfect recall* relation over paths [24,22], which is regular. We prove that the class of reachability games with imperfect information and perfect recall (with a fixed number of observations and actions) where the first player wins cannot be defined in the jumping μ -calculus, while being closed by bisimulation and definable in our extension of MSO. The proof heavily relies on the equivalence between the jumping μ -calculus and jumping automata, on which we exploit the “pigeon-hole principle”, as well as on the use of unobservable winning conditions. Indeed, we prove that if winning conditions are assumed to be observable, then the class of

imperfect-information (either reachability or parity) games where the first player wins is definable in the jumping μ -calculus.

Our expressivity incompleteness result has several impacts.

First, we obtain that the class of jumping tree automata is not closed under projection. Indeed, the (bisimulation-closed) second-order-quantification-free fragment of MSO with path relation can be embedded into jumping tree automata. Their closure under projection would therefore imply that they coincide with the full (bisimilar invariant) MSO with path relation, which contradicts our expressivity incompleteness result.

Regarding logics of programs, there has been some interest in comparing alternating-time temporal logics with fix-point logics. When agents have perfect information, the μ -calculus subsumes these logics (see for instance [21]). For imperfect information, our results show that the picture changes: because alternating-time temporal logics with imperfect information can express the existence of winning strategies in reachability games with imperfect information, Theorem 2 reveals that the powerful jumping μ -calculus does not subsume alternating-time temporal logics with imperfect information when we consider players with perfect recall.

We also believe that our incompleteness result impacts the axiomatizability of alternating-time temporal logics with imperfect information: the impossibility to express the existence of a winning strategy in reachability games with imperfect information and perfect recall in the jumping μ -calculus strongly suggests the absence of fix-point axioms for certain alternating temporal logics.

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