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The Logical Difference For Fuzzy $\mathcal{EL}^+$ Ontologies

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Abstract. Ontologies undergo changes for reasons such as changes in knowledge, meeting varying application requirements. Thus, for different versions of a considered ontology, it is important to clarify the difference between them. The difference above refers to the logical difference, not syntactic one. Examples of the logical difference include the difference in taxonomy, concept subsumption difference and query difference. These has been well investigated for the lightweight description logic $\mathcal{EL}$ because of its tractability and successful application in bio-medical ontologies.

Fuzzy $\mathcal{EL}^+$ has been put forward and applied in view-based searching in Semantic portals. Thus comes the problem of comparing fuzzy ontologies of different versions and clarifying the difference. In this paper, we define the logical difference of two fuzzy $\mathcal{EL}^+$ ontologies. For fuzzy $\mathcal{EL}^+$ ontologies of different versions we investigate how to compute the difference in taxonomy and concept subsumption difference. We also explore how to compute approximation of the logical difference of two $\mathcal{EL}$ terminologies. Our work can be applied in the scenario of $\mathcal{EL}^+$ ontologies with access control if the set of all access rights is a linear order.

1 Introduction

Ontologies undergo changes for reasons such as changes in knowledge, meeting varying application requirements [1]. Thus, for different versions of a considered ontology, it is important to clarify the difference between them. The difference hereinabove refers to the logical difference of two ontologies, that is, the set of entailments implied by one ontology, but not by the other [2, 3]. For example, if a general concept inclusion (GCI) $C \sqsubseteq D$ is implied by one ontology, but not the other, then it is in the set of the logical difference, which is the difference in concept subsumption. In the above case, if C and D are simple concept names, then this kind of difference is called the difference in taxonomy. Query answering difference refers to the difference in query answering by two ontologies [3]. These has been well investigated for the lightweight description logic $\mathcal{EL}$ because of its tractability and successful application in bio-medical ontologies [2, 4, 5].

Fuzzy $\mathcal{EL}^+$ [6] has been put forward and applied in ontology alignment [7] and view-based searching in Semantic portals [8]. The fuzzy description logic allows fuzzy subsumption, and has scalable classification algorithm. Thus comes the problem of comparing fuzzy ontologies of different versions, as done in classic DL ontologies.

In this paper, we investigate how to clarify the difference between two fuzzy $\mathcal{EL}^+$ ontologies. First, we define the logical difference in fuzzy $\mathcal{EL}^+$. Then we investigate how to compute the difference in taxonomy and concept subsumption difference, and discuss how to compute approximation of the logical difference. Some of our investigation is about the following case. The second fuzzy ontology is obtained by modifying the degrees of truth of some axioms in the first fuzzy ontology. Thus, for some given entailments of interest, we want to compute the change of their degrees of truth.

As shown in [9, 10], there are some shared principles between reasoning in fuzzy $\mathcal{EL}^+$ and ontologies with access control. Thus, our results can be applied in the scenarios of $\mathcal{EL}^+$ ontologies with access control. More concretely, if the set of all access rights is a linear order, we can treat the access right of an axiom (or entailment) as its degree of truth. Thus, after modifications on some axioms' access right, the problem of computing the access right of the entailment is the same as the logical difference problem described in last paragraph.

This paper is structured as follows. In the next section, we introduce the preliminary about fuzzy $\mathcal{EL}^+$ and its reasoning methods. In section 3, we define the logical difference of two fuzzy ontologies. Then, in section 4, we investigate how to compute the difference in taxonomy, the concept subsumption difference, and approximation of the difference. At last, in section 5, we conclude the paper and discuss future work related to the logical difference.

2 Preliminary

2.1 Fuzzy $\mathcal{EL}^+$

Fuzzy set theory has been well applied in representing knowledge with vagueness [11]. Let X be a set of elements. A fuzzy subset A of X, is defined by a *membership function* $\mu_A(x)$, or simply $A(x)$, of the form $\mu_A(x) : X \rightarrow [0, 1]$ [11]. This function assigns each $x \in X$ a value $n \in [0, 1]$ that represents the degree of x belongs to X. Then, under this assumption, the classical set operators and logical operators are performed by mathematical functions. For example, *fuzzy complement* is a unary function of the form $c : [0, 1] \rightarrow [0, 1]$, *fuzzy intersection and union* are two binary functions of the form $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$, and $u : [0, 1] \times [0, 1] \rightarrow [0, 1]$, called t-norm and t-conorm operations, respectively, and *fuzzy implication* also by a binary function $\mathcal{T} : [0, 1] \times [0, 1] \rightarrow [0, 1]$ [11]. Certainly, the definitions of these functions have to satisfy some properties to make sense. There are difference semantics according to the choices of these functins. We only introduce Gödel semantics, which is also the base of fuzzy $\mathcal{EL}^+$ introduced hereinbelow. As for other semantics, we refer to [11]. The t-norm is $t_G(a, b) = min(a, b)$, t-conorm $u_G(a, b) = max(a, b)$, and implication $\mathcal{T}_G(a, b) = b$ if $a > b$, $\mathcal{T}_G(a, b) = 1$ otherwise.

Then let us illustrate fuzzy $\mathcal{EL}^+$, denoted by f_G - $\mathcal{EL}^+$ [6]. Suppose N_C is a set of concept names, and N_R a set of role names. Then, $\top$ and elements in N_C are *concept descriptions*. If C and D are concept descriptions, then $C \sqcap D$ and $\exists r.C$ are also concept descriptions, where $r \in N_R$. The f_G - $\mathcal{EL}^+$ ontology consists of finite general concept inclusions $\langle C \sqsubseteq D, n \rangle$and role inclusions axioms (RIA) $r \subseteq s$, where $n \in (0, 1]$ and

$r, s \in N_R$. That is, in the f_G - $\mathcal{EL}^+$ ontology, fuzzy concept inclusions (f-GCI) are permitted, and $\langle C \sqsubseteq D, n \rangle$means that the degree of C is a subset of D is n. However, there is no fuzzy role inclusion permitted. The language is f_G - $\mathcal{EL}$ if there is no RIA in the ontology. In the later parts of the paper, we use *crisp DL ontology* to denote the ontology resulting from eliminating all the degrees of truth from the axioms.

The semantics of fuzzy DLs are defined through a *fuzzy interpretation*. A fuzzy interpretation consists of $(\Delta^{\mathcal{I}}, .^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty set of elements, and $.^{\mathcal{I}}$ is a fuzzy interpretation function, which maps,

- an individual name a to an element $a^{\mathcal{I}}$ in $\Delta^{\mathcal{I}}$,
- a concept name A to a membership function $A^{\mathcal{I}} : \Delta^{\mathcal{I}} \rightarrow [0, 1]$, and
- a role name r to a membership function $r^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \rightarrow [0, 1]$.

Then, the interpretation is extended as shown in Table 1.

Table 1. Syntax and Semantics of the fuzzy description logic $\mathcal{EL}^+$

Constructor	DL Syntax	Semantics
top	$\top$	$\top^{\mathcal{I}}(a) = 1$
conjunction	$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}}(a) = min(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))$
existential restriction	$\exists r.C$	$(\exists r.C)^{\mathcal{I}}(a) = sup_{b \in \Delta^{\mathcal{I}}}\{min(r^{\mathcal{I}}(a, b), C^{\mathcal{I}}(b))\}$
fuzzy GCIs	$\langle C \sqsubseteq D, n \rangle$	$inf_{a \in \Delta^{\mathcal{I}}}\{\mathcal{T}_G(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a))\} \geq n$
RIAs	$\mathrm{r}_1 \circ \ldots \circ r_k \sqsubseteq s$	$[\mathrm{r}_1^{\mathcal{I}} \circ^t \ldots \circ^t r_k^{\mathcal{I}}](a, b) \leq s^{\mathcal{I}}(a, b)$

2.2 Reasoning in Fuzzy $\mathcal{EL}^+$

The reasoning problem in fuzzy $\mathcal{EL}^+$ ontology is to decide whether $\langle \alpha, n \rangle$is implied by the ontology or not. That is, we have to determine the degree of truth n here, compared with the reasoning problem in standard $\mathcal{EL}^+$ ontology. Currently, there exist two methods as follows.

One reasoning method is to develop a calculus which deals with the degrees of truth in every inference step, as shown in the paper by Stoilos et al [6]. The method is a modification of the classification algorithm of $\mathcal{EL}^+$ ontology. For each entry in the completion process, a value n is affiliated to reflect its degree of truth (considering this entry locally). For newly generated entry by the completion rule, it will get its affiliated value $k \in (0, 1]$ according to the input entries and their affiliated values. The completion rules will be repeatedly applied until there is no new binary tuple (entry, affiliated value) generated. For the resulting entries, their degrees of truth are the biggest ones among their affiliated values. Actually, this method is a classification algorithm, that is, it will return the degree of truth of any concept name subsumption. The method is similar to the axiom pinpointing algorithm based on monotone Boolean function [12], but much simpler. That is because in deciding whether to apply a completion rule, this method employs comparison between numerial values, in contrast with logical implication checking between formulae in pinpointing algorithm. Thus, with respect to complexity, this method sits strictly between classification algorithm and the axiom pinpointing algorithm.

From above reasoning method, we explore the property of the degree of truth of an entailment. An entailment has one or many MinAs, which is the minimal subset of the ontology implying that entailment [12]. From above inference steps, we know that the entailment has a *temporary degree of truth* from one MinA, called that *MinA's degree*, which is the minimal degree of truth of all the axioms in that MinA. The entailment's final degree of truth is the maximal among its temporary degrees of truth. Theorem 1 describes this formally.

Theorem 1. *Suppose that O is a fuzzy $\mathcal{EL}^+$ ontology, and $O \models \langle \alpha, n \rangle$. Obtain a classic O_{crisp} by deleting all the degrees of truth in O. Let $A_1 = \{\beta_{11}, \ldots, \beta_{1j_1}\}, \ldots, A_k = \{\beta_{k1}, \ldots, \beta_{kj_k}\}$ be all the MinAs of α in O_{crisp}. Then let $n_{11}, \ldots, n_{1j_1}, \ldots, n_{k1}, \ldots, n_{kj_k}$ be degrees of truth for $\beta_{11}, \ldots, \beta_{1j_1}, \ldots, \beta_{k1}, \ldots, \beta_{kj_k}$ in O respectively. Let $d(A_i)$ be $min_{1 \leq h \leq j_i}\{n_{ih}\}$, for $1 \leq i \leq k$. Then, we have $n = max_{1 \leq i \leq k}\{d(A_i)\}$.*

The other reasoning method treats the standard DL reasoner as an oracle in the computation of the degree of truth n of α, which is justified by the following result.

Theorem 2. *(from [10]) Suppose that O is a f_G - $\mathcal{EL}^+$ ontology. Then, $O \models \langle C_1 \sqsubseteq C_2, n \rangle$ if and only if $O_n \models C_1 \sqsubseteq C_2$, where $O_n = \{C \sqsubseteq D | \langle C \sqsubseteq D, m \rangle \in O, n \leq m\}$.*

Then, we can use binary search to compute degree of truth n [10]. This method enjoys the fast converging speed of binary search, as well as sophisticated existing $\mathcal{EL}^+$ reasoner [13].

Moreover, Theorem 2 shows that for an entailment with degree of truth n, only axioms of higher degree of truth contribute to its correctness. This property will be further used in subsection 4.2.

3 Logical Difference for Fuzzy DLs

We extend the logical difference to fuzzy DLs after listing that definition in classic DLs.

Definition 1. *(logical difference in classic DLs, from [2]) Suppose that O_1 and O_2 are two DL ontologies, and S is a signature, then their logical difference with respect to S is defined as $diffs_S(O_1, O_2) = \{C \sqsubseteq D | O_1 \models C \sqsubseteq D, O_2 \not\models C \sqsubseteq D$, symbols in $C \sqsubseteq D$ are from S.$\}$.*

Definition 2. *(logical difference for fuzzy DL ontologies) Suppose that O_1 and O_2 are two fuzzy DL ontologies, and S is a signature, then their logical difference is defined as f-$diffs_S(O_1, O_2) = \{h\alpha, n \rangle | O_1 \models \langle \alpha, n \rangle$ and $O_2 \not\models \langle \alpha, n \rangle$, symbols in α are from S.$\}$*

In the above two definitions, when the signature S is clear from context, we will drop it when mentioning the difference.

It is easy to verify that the logical difference on fuzzy DLs is the same as that on classic DLs if the underlying fuzzy DLs has only degrees of truth of 0 and 1. From the definition, we find that an element in the logical difference consists of a normal entailment, and its degree of truth. Thus, there are two sources to the contribution of the elements. One is the difference in normal entailments, which is the logical difference of two crisp DL ontologies. The other is the difference in degrees of truth, which illustrates the following situation. Two crisp DL ontologies imply the same set of entailments, but differ on the degrees of truth of some entailments.

Since we care more about the difference caused by modifications on some degrees of truth, the crisp parts of two DL ontologies are the same. Thus, the only source of difference is the degrees of truth of the entailments.

4 Compute the Logical Difference

In this section, we will discuss how to compute the logical difference of two fuzzy DL ontologies. If modelers of ontology are interested at the subsumption relationship between two concept names, which actually is the difference in taxonomy, we recommend the method in subsection 4.1. If modelers focus on the subsumption between concept descriptions, subsection 4.2 provides guidelines. If modelers expect more information in the difference, an approximation is proposed for fuzzy $\mathcal{EL}$ terminologies in subsection 4.3. Thus, we provide a range of methods for the choice of modelers according to requirements of application.

4.1 Difference in Taxonomy

In this subsection, we focus on computing the entailments of the form $\langle A \sqsubseteq B, n \rangle$in the difference of two fuzzy ontologies, where A and B are concept names. The method we propose in this subsection is an adaption of the method successfully applied in classic DL ontologies [14].

First, we recall some results in classic DL ontologies. In classic DL ontologies, module w.r.t a signature is a subset of an ontology which is indistinguishable with the ontology w.r.t that signature [15]. Thus, when reasoning on that signature, it is safe to use the module instead of the original ontology, which can speed up the reasoning when the module and the ontology are of different scales. For a concept name A, any entailment of the form $A \sqsubseteq C$, where C is concept description, can be implied from the locality-based module w.r.t $\{A\}$ [14].

Then, for a fuzzy $\mathcal{EL}^+$ ontology, we define its module with respect to a signature.

Definition 3. *(module for fuzzy ontology) For a fuzzy ontology O_f, its module with respect to signature S, denoted by $\mathcal{M}_{fS}$, is its subset which is indistinguishable with O with respect to S. That is, for any α which consists of symbols only from S and* $n \in (0, 1]$*, we always have* $O_f \models \langle n \rangle$*if and only if* $\mathcal{M}_{fS} \models \langle n \rangle$.

Moreover, for a fuzzy DL ontology O_f, its locality-based module $\mathcal{M}_f$ consists of all the axioms which also appear in locality-based module when their degree of truth discarded. That is, $\mathcal{M}_f = \{ \langle \alpha, n \rangle | \alpha \in \mathcal{M}_{crisp}, \mathcal{M}_{crisp}$ is a locality-based module of $O_{crisp} = \{\beta | \langle \beta, m \rangle \in O_f\} \}$.

Theorem 3 shows that the locality-based module is a module for fuzzy $\mathcal{EL}^+$ ontology.

Theorem 3. *For a fuzzy DL ontology O_f, its locality-based module $\mathcal{M}_f$ (with respect to Signature S) is a module (with respect to Signature S).*

Sketch of proof. For any entailment $\langle\alpha, n\rangle$implied by O_f, we have that α is implied by O_{crisp}, also by $\mathcal{M}_{crisp}$. It means that, any mininal explanation of α is contained in M. Since $\mathcal{M}$ is generated under the direction of $\mathcal{M}_{crisp}$, we say that any minimal explanation of $\langle\alpha, n\rangle$is contained in $\mathcal{M}_f$. Thus, $\langle\alpha, n\rangle$is implied by $\mathcal{M}_f$. That is, $\mathcal{M}_f$ is a module of O_f. Q.E.D.

Lemma 1. *(from [15]) For a classic DL ontology O, A is a concept name in it. Then any entailment of the form $A \sqsubseteq C$ implied by O can be implied from O's locality-based module with respect to $\{A\}$, where C is a concept description.*

Theorem 4. *For a fuzzy DL ontology O_f, A is a concept name in it. Then any entailment of the form $\langle A \sqsubseteq C, n\rangle$implied by O_f can be implied from O_f's locality-based module with respect to $\{A\}$, where C is a concept description.*

With these results at hand, our strategy (as in [14]) is for every concept name A, we keep a record of the module with respect to A. Then for two versions of fuzzy $\mathcal{EL}^+$ ontologies, we check whether the related module is changed or not for every A. If unchanged, it means that all the subsumptionhoods between A and other concept names are unchanged. If changed, it is enough to perform the fuzzy classification algorithm only on the module, which is much smaller, to find the new related subsumptionhoods. Thus our strategy, like the corresponding method in classic DLs, is especially suitable for large scale ontologies.

If we only care about the subsumptionhood between A and a specific concept name, we might be able to avoid the fuzzy classification algorithm in some cases. We will explain this in the next subsection.

4.2 Concept Subsumption Difference

In this subsection, we focus on the entailment of the form $C \sqsubseteq D$ in the logical difference, where C and D are concept descriptions. We consider the simplest case: When the degree of truth of an axiom in the first ontology is increased, we want to know the new degree of truth of one entailment of the form $C \sqsubseteq D$. Formally, suppose that O is a fuzzy $\mathcal{EL}^+$ ontology, $\langle\alpha, n\rangle$is an axiom in its TBox, and $\langle\beta, d\rangle$is one of entailments of O of interest. Let O' be obtained by replacing $\langle\alpha, n\rangle$with $\langle\alpha, m\rangle$, where $m \in (n, 1]$. Then we want to calculate the value of d' such that O' implies $\langle\beta, d'\rangle$. We discuss the problem according to the degree of truth d.

Theorem 5. *If $d < n$, then $d' = d$.*

The theorem says that, for entailment whose degree of truth is less than n, then its uncertainty degree in the new ontology is the same as that in the previous ontology.

Proof of sketch. From Theorem 1, we know that the degree of truth of α contributes to β's degree of truth, if and only if α is in one of β's MinA, and α's degree is the smallest compared with other axioms' in that MinA. Moreover, α's degree is the biggest compared with the degrees returned by other MinAs. The statement *dn* in the precondition shows that, even α is in one of β's MinA, there exists another axiom with degree of truth $d < n$. That axiom decides the degree of β to be d. That is, the degree of α has no effection on β's degree. Thus, increase on α's degree will leave β's degree unchanged. Q.E.D.

Theorem 6. *If $n \leq d < m$, then let $\mathcal{O}_{>d}$ be $\{\gamma | \langle \gamma, k \rangle \in \mathcal{O}' and k > d\}$.*

1. *if $\mathcal{O}'_{>d} \models \beta$, then β's degree of truth $d' \in (d, m]$.*
2. *otherwise, β's degree of truth stays unchanged, i.e. $d' = d$.*

In this case, we can first perform some reasoning in classic $\mathcal{EL}^+$ ontologies, then we know whether β's degree of truth has to be recomputed or not. Even the degree has to be recomputed, we know the range of its new value.

Proof of sketch. In the first case, $d' > d$ can be derived from Theorem 2. If α is in one MinA, and its degree m is the smallest in that MinA, then d' possibly equals m. It is not possible that d' is bigger than m, which means that there exists a MinA A, the smallest degree in A is bigger than m. Thus, α must not be in A. Since $\langle \alpha, m \rangle$is the only difference between two ontologies, the MinA A is also in the first ontology $\mathcal{O}$. Then the degree of β in $\mathcal{O}$ is bigger than m. Conflict.

For the second case, we know that $\mathcal{O}'_{\geq d} = \mathcal{O}_{\geq d}$. From Theorem 2 we have $\mathcal{O}_{\geq d} \models \beta$. Thus $\mathcal{O}'_{\geq d} \models \beta$. With $\mathcal{O}'_{>d} \not\models \beta$ from the precondition, we conclude that β's degree of truth is d. Q.E.D.

Theorem 7. *If $d \geq m$, then $d' = d$.*

That is, for entailment whose degree of truth is not less than m, then that degree in the new ontology is the same as in the previous ontology.

Proof of Sketch. Suppose that α is in a MinA of β, and its new degree m is the smallest in that MinA. Since $d \geq m$, it means that there exists another axiom with degree of d. Thus, β's degree will stay at d. Q.E.D.

From above analysis, when the degree of truth of one axiom is increased, for an entailment of interest, we know the new range of its degree of truth before performing concrete computation. Sometimes, the new range might be enough for requirements of users, then the effort in performing concrete computation will be saved.

Similarly, we give the following results when the degree of truth of one axiom is decreased. The problem is described the same as in the beginning of this subsection. The only exception is that when $\langle \alpha, n \rangle$is replaced by $\langle \alpha, m \rangle$, we require that $m \in [0, n)$. Thus, for $\langle \beta, d \rangle$implied in the previous ontology $\mathcal{O}$, if $d \in (0, m] \cup (n, 1]$, then β's degree of truth in the new ontology $\mathcal{O}'$ is still d. When the range of d is $(m, n]$, if $\mathcal{O}'_{\geq d} \models \beta$, then β's degree in $\mathcal{O}'$ is still d. Otherwise, its degree will be in $[m, d)$.

Now, we relax the conditions in our discussion a bit. We allow that in the first ontology, several axioms have their degrees of truth increased. For an entailment of interest, to know its degree of truth in the modified ontology, we can first apply Theorem 5 and Theorem 7, to eliminate the modifications having no effect on the entailment of interest. For the modifications falling in the category of Theorem 6, we can get an estimation of the resulting degree before complete computation. Similar methods can deal with the case when some axioms have their degrees decreased, or even increased and decreased modifications co-exist.

4.3 More Difference

What we have done in the previous subsections is: first, set some entailments (either elements from taxonomy or concept subsumptions of interest), then perform reasoning to find the change of their degrees of truth. The results obtained therefore are only part of the logical difference between ontologies. In this subsection, we try to compute as much information as possible of the logical difference, instead of focusing on some pre-fixed entailments.

Before coming to our solution, we briefly go through the related work in classic $\mathcal{EL}$ [4, 2]. Given two $\mathcal{EL}$ terminologies, any entailment in their logical difference, which is of form $C \sqsubseteq D$, can be derived from either $E \sqsubseteq A$ or $B \sqsubseteq E$, where C, D, E are concept descriptions and A and B are concept names. Thus, the lists of such As and Bs form a reasonable approximation of the logical difference. There exist polynomial algorithms respectively to return the above two lists. However, this kind of approximation in $\mathcal{EL}^+$ remains an open problem due to some role inclusion axioms. Thus, our work described below is only for $\mathcal{EL}$ terminologies, not ontologies, nor $\mathcal{EL}^+$. Terminologies are special kind of ontologies, where every concept is defined (when it occurs, as the only concept, on the left side of an axiom) at most once, and does not refers to itself directly or indirectly in the definition.

Theorem 8 describes the composition of fuzzy logical difference, thus points out one way of computation.

Theorem 8. *Suppose that O and O' are two fuzzy DL ontologies, and f-diff(O, O') is the logical difference between them. Let $n \in (0, 1]$. Then f-diff$_{\geq n}(O, O')$ = diff$(O_{\geq n}, O'_{\geq n})$.*

Proof of sketch. Use Theorem 2 and Definition 1 and 2. Q.E.D.

Theorem 8 points out that, the cut set of the fuzzy logical difference by n, equals to the classic logical difference between cut sets of the fuzzy ontologies. Since we can compute approximation for the classic logical difference, then the result can be used as approximation of the cut set of the fuzzy logical difference by n. Thus, with Theorem 8, we can *build* approximation of the logical difference of two fuzzy $\mathcal{EL}$ terminologies gradually.

This result can be further strengthened in the following case. Suppose that two fuzzy $\mathcal{EL}$ terminologies only differ on degrees of some axioms. Let n be the maximal value among the degrees on which two terminologies differ. Then, from Theorem 2, we know that the cut set of the fuzzy logical difference by n is empty. That is, two terminologies imply, to the degree of n, the same entailments.

5 Conclusion and Future Work

Because of the applications of fuzzy $\mathcal{EL}^+$ and its similarly to ontology with access control, we investigate the logical difference problem in fuzzy $\mathcal{EL}^+$. We define the logical difference of two fuzzy $\mathcal{EL}^+$ ontologies. We investigate strategies of computing some forms of the difference, from the simplest one of taxonomy difference to the approximation. Thus, modellers of fuzzy ontologies can choose a suitable solution according to requirements.

We discuss two interesting problems for future investigation. The first one is in computing concept subsumption difference. In subsection 4.2, our discussion is under the assumption that two fuzzy ontologies share the same set of axioms, and only differ on the degrees of truth of some axioms. If we drop this assumption, and let the two fuzzy ontologies be arbitrary, can we still find similar heuristics to avoid fuzzy classification?

The second problem is about deciding conservative extensions between two fuzzy ontologies. We can try the method described in subsection 4.3. However, it is interesting to develop algorithms which deal with degrees of truth on inference step level, instead of converting to classic DLs.

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