Applied Mathematics and Nonlinear Sciences 4(2) (2019) 489–502

Applied Mathematics and Nonlinear Sciences

https://www.sciendo.com

Yang-Laplace Decomposition Method for Nonlinear System of Local Fractional Partial Differential Equations

Djelloul Ziane[1], Mountassir Hamdi Cherif[†], Carlo Cattani[3,4], Kacem Belghaba[5]

[1]Laboratory of mathematics and its applications (LAMAP), University of Oran1 Ahmed Ben Bella, Oran, Algeria

[2]Laboratory of mathematics and its applications (LAMAP), University of Oran1 Ahmed Ben Bella, Oran, Algeria

[3]Engineering School (DEIM), Tuscia University, 01100 Viterbo, Italy

[4]Ton Duc Thang University, Ho Chi Minh City, Vietnam

[5]Laboratory of mathematics and its applications (LAMAP), University of Oran1 Ahmed Ben Bella, Oran, Algeria

Submission Info

Communicated by Juan Luis García Guirao
Received April 15th 2019
Accepted June 2nd 2019
Available online December 24th 2019

Abstract

The basic motivation of the present study is to extend the application of the local fractional Yang-Laplace decomposition method to solve nonlinear systems of local fractional partial differential equations. The differential operators are taken in the local fractional sense. The local fractional Yang-Laplace decomposition method (LFLDM) can be easily applied to many problems and is capable of reducing the size of computational work to find non-differentiable solutions for similar problems. Two illustrative examples are given, revealing the effectiveness and convenience of the method.

Keywords: Local fractional derivative operator, local fractional Yang-Laplace decomposition method, nonlinear systems of local fractional partial differential equations.
AMS 2010 codes: 49K20.

1 Introduction

Fractional partial differential equations is a fundamental tool for the analysis of physical phenomena, such as, electromagnetic, acoustics, viscoelasticity, electrochemistry, and others. These physical and other phenomena

[†]Corresponding author, mountassir27@yahoo.fr

ISSN 2444-8656 doi:10.2478/AMNS.2019.2.00046

are expressed by fractional partial differential equations, which have been solved by several numerical-analytical methods ([13], [21], [25]). Among them, one of the most popular methods is the so-called Adomian decomposition method (ADM), which has been developed between the 1970s and 1990s by George Adomian ([1]- [5]).

With the new concepts of fractional derivative and fractional integral, as well as local fractional derivative and local fractional integral, researchers were able to use the ADM method to solve these new types of equations or systems which include, local fractional differential equations, local fractional partial differential equations and local fractional integro-differential equations ([6]- [10]).

Some more results were obtained by combining fractional differential operators with some known transforms, such as the Young-Laplace transform method and the Sumudu transform method, thus obtaining a solution of fractional equations or systems, even in the nonlinear case. Among these works we find the local fractional Laplace decomposition method [11] and the local fractional Sumudu decomposition method ([12], [14]). The theory and applications of local fractional derivative and integral operators, has been defined and deeply investigated by Xiao-Jun Yang (see e.g. [15], [16]).

Based on the concept of local fractional operator and on the Yang-Laplace decomposition method, Jassim has recently proposed a method [11] to solve linear local fractional partial differential equations. In this paper, we will generalize the Jassim method and extend it to solve nonlinear systems of local fractional partial differential equations. The Yang-Laplace decomposition will enable us to obtain exact solutions for nonlinear system of local fractional partial differential equations. Two examples are also given to show the effectiveness of this method.

This paper has been organized as follows: In Section 2 some basic definitions and properties of local fractional calculus and local fractional Yang-Laplace transform method are given. In section 3, we present an analysis of the modified method. In section 4 we apply the modified method (LFLDM) to nonlinear systems to obtain the analytical solution. Two applications are given in the same section.

2 Basic definitions

In this section, we will present the basic concepts of fractional local calculus, and in particular the local fractional derivative, local fractional integral, and local fractional Yang-Laplace transform.

2.1 Local fractional derivative

Definition 2.1. *The local fractional derivative of $\Phi(\varkappa)$ of order σ at $\varkappa = \varkappa_0$ is defined as ([15], [16])*

$$\Phi^{(\sigma)}(\varkappa) = \left.\frac{d^\sigma \Phi}{d\varkappa^\sigma}\right|_{\varkappa=\varkappa_0} = \lim_{\varkappa\to\varkappa_0} \frac{\Delta^\sigma(\Phi(\varkappa)-\Phi(\varkappa_0))}{(\varkappa-\varkappa_0)^\sigma}, \tag{1}$$

where

$$\Delta^\sigma(\Phi(\varkappa)-\Phi(\varkappa_0)) \cong \Gamma(1+\sigma)\left[(\Phi(\varkappa)-\Phi(\varkappa_0))\right]. \tag{2}$$

This operator can be extended to the interval (α,β) so that for any $\varkappa \in (\alpha,\beta)$, we can define

$$\Phi^{(\sigma)}(\varkappa) = D^\sigma_\varkappa \Phi(\varkappa),$$

denoted by

$$\Phi(\varkappa) \in D^\sigma_\varkappa(\alpha,\beta).$$

For the local fractional derivative of high order

$$\Phi^{(m\sigma)}(\varkappa) = \overbrace{D^{(\sigma)}_\varkappa \cdots D^{(\sigma)}_\varkappa}^{m\ times} \Phi(\varkappa), \tag{3}$$

and local fractional partial derivative of high order

$$\frac{\partial^{m\sigma}\Phi(\varkappa,\tau)}{\partial\varkappa^{m\sigma}} = \overbrace{\frac{\partial^{\sigma}}{\partial\varkappa^{\sigma}}\cdots\frac{\partial^{\sigma}}{\partial\varkappa^{\sigma}}}^{m\ times}\Phi(\varkappa,\tau). \tag{4}$$

2.2 Local fractional integral

Definition 2.2. *The local fractional integral of $\Phi(\varkappa)$ of order σ in the interval $[\alpha,\beta]$ is defined as ([15], [16])*

$$\begin{aligned} {}_{\alpha}I_{\beta}^{(\sigma)}\Phi(\varkappa) &= \frac{1}{\Gamma(1+\sigma)}\int_{\alpha}^{\beta}\Phi(\tau)(d\tau)^{\sigma} \\ &= \frac{1}{\Gamma(1+\sigma)}\lim_{\Delta\tau\longrightarrow 0}\sum_{j=0}^{N-1} f(\tau_j)(\Delta\tau_j)^{\sigma}, \end{aligned} \tag{5}$$

where $\Delta\tau_j = \tau_{j+1}-\tau_j$, $\Delta\tau = \max\{\Delta\tau_0,\Delta\tau_1,\Delta\tau_2,\cdots\}$ and $[\tau_j,\tau_{j+1}]$, $\tau_0=\alpha$, $\tau_N=\beta$, is a partition of the interval $[\alpha,\beta]$.

For any $\varkappa\in(\alpha,\beta)$, there exists

$$ {}_{\alpha}I_{\varkappa}^{(\sigma)}\Phi(\varkappa),$$

denoted by

$$\Phi(\varkappa)\in I_{\varkappa}^{(\sigma)}(\alpha,\beta).$$

2.3 Some properties of the local fractional operators

The local fractional operators fulfill some fundamental equations. In particular, starting from the Mittag-Leffler function we have the following

Definition 2.3. *The Mittag-Leffler function, the hyperbolic sine and hyperbolic cosine are defined as ([15], [16], [17])*

$$E_{\sigma}(\varkappa^{\sigma}) = \sum_{m=0}^{+\infty}\frac{\varkappa^{m\sigma}}{\Gamma(1+m\sigma)},\quad 0<\sigma\leqslant 1, \tag{6}$$

$$E_{\sigma}(\varkappa^{\sigma})E_{\sigma}(\upsilon^{\sigma}) = E_{\sigma}(\varkappa+\upsilon)^{\sigma},\quad 0<\sigma\leqslant 1, \tag{7}$$

$$E_{\sigma}(\varkappa^{\sigma})E_{\sigma}(-\upsilon^{\sigma}) = E_{\sigma}(\varkappa-\upsilon)^{\sigma},\quad 0<\sigma\leqslant 1, \tag{8}$$

$$\sin_{\sigma}(\varkappa^{\sigma}) = \sum_{m=0}^{+\infty}(-1)^m\frac{\varkappa^{(2m+1)\sigma}}{\Gamma(1+(2m+1)\sigma)},\quad 0<\sigma\leqslant 1, \tag{9}$$

$$\cos_{\sigma}(\varkappa^{\sigma}) = \sum_{m=0}^{+\infty}(-1)^m\frac{\varkappa^{2m\sigma}}{\Gamma(1+2m\sigma)},\quad 0<\sigma\leqslant 1. \tag{10}$$

By using the local fractional derivative (1) and the definitions (2.3) it can be easily shown that ([15], [16])

$$\frac{d^{\sigma}\varkappa^{m\sigma}}{d\varkappa^{\sigma}} = \frac{\Gamma(1+m\sigma)\varkappa^{(m-1)\sigma}}{\Gamma(1+(m-1)\sigma)}. \tag{11}$$

$$\frac{d^{\sigma}}{d\varkappa^{\sigma}}E_{\sigma}(\varkappa^{\sigma}) = E_{\sigma}(\varkappa^{\sigma}). \tag{12}$$

$$\frac{d^{\sigma}}{d\varkappa^{\sigma}}\sin_{\sigma}(\varkappa^{\sigma}) = \cos_{\sigma}(\varkappa^{\sigma}). \tag{13}$$

$$\frac{d^{\sigma}}{d\varkappa^{\sigma}}\cos_{\sigma}(\varkappa^{\sigma}) = -\sin_{\sigma}(\varkappa^{\sigma}). \tag{14}$$

$$ {}_{0}I_{\varkappa}^{(\sigma)}\frac{\varkappa^{m\sigma}}{\Gamma(1+m\sigma)} = \frac{\varkappa^{(m+1)\sigma}}{\Gamma(1+(m+1)\sigma)}. \tag{15}$$

2.4 Local fractional Yang-Laplace transform

We present here the definition of local fractional Yang-Laplace transform (denoted in this paper by ${}^{LF}L_{\sigma}[.]$) and some properties concerning this transformation.

Definition 2.4. *Let*

$$\frac{1}{\Gamma(1+\sigma)}\int_0^{\infty}|w(\varkappa,\tau)|(d\tau)^{\sigma}<k<\infty \qquad ,$$

the Yang-Laplace transform of $w(\varkappa,\tau)$ is defined as ([18], [19], [20]):

$$ {}^{LF}L_{\sigma}\{w(\varkappa,\tau)\}= W(\varkappa,s):=\frac{1}{\Gamma(1+\sigma)}\int_0^{\infty}E_{\sigma}(-s^{\sigma}\tau^{\sigma})w(\varkappa,\tau)(d\tau)^{\sigma};\ 0<\sigma\leqslant 1, \tag{16}$$

where the integral converges and $s\in\mathbb{R}$.

Definition 2.5. *The inverse Yang-Laplace transforms of $W(\varkappa,\tau)$ is defined as [19]:*

$$ {}^{LF}L_{\sigma}^{-1}\{W(\varkappa,s)\}= w(\varkappa,\tau):=\frac{1}{(2\pi)^{\sigma}}\int_{\beta-i\infty}^{\beta+i\infty}E_{\sigma}(s^{\sigma}\tau^{\sigma})W(x,s)(ds)^{\sigma};\ 0<\sigma\leqslant 1, \tag{17}$$

where $s^{\sigma}=\beta^{\sigma}+i^{\sigma}\infty^{\sigma}$, $(Re(s)=\beta>0)$ and i^{σ} is the fractal imaginary unit.

The Yang-Laplace transform is a linear operator ([18], [20])

$$ {}^{LF}L_{\sigma}\{af(\varkappa)+bg(\varkappa)\}=a^{LF}L_{\sigma}(f(\varkappa))+b^{LF}L_{\sigma}(g(\varkappa)), \tag{18}$$

moreover the following properties hold ([18], [20])

$$ {}^{LF}L_{\sigma}\left\{f^{(n\sigma)}(\varkappa)\right\}=s^{n\sigma}\,{}^{LF}L_{\sigma}\{f(\varkappa)\}-\sum_{k=1}^{n}s^{(k-1)\sigma}f(0)-f^{((n-k)\sigma)}(0), \tag{19}$$

$$\lim_{x\longrightarrow 0}f(\varkappa)=\lim_{\sigma\longrightarrow\infty}s^{\sigma}F(s), \tag{20}$$

$$\lim_{x\longrightarrow \infty}f(\varkappa)=\lim_{\sigma\longrightarrow 0}s^{\sigma}F(s), \tag{21}$$

$$ {}^{LF}L_{\sigma}\left\{\varkappa^{k\sigma}E_{\sigma}(a^{\sigma}\varkappa^{\sigma})\right\}=(\frac{\Gamma(1+k\sigma)}{(s-a)^{(k+1)\sigma}}, \tag{22}$$

$$ {}^{LF}L_{\sigma}\{\sin_{\sigma}(a^{\sigma}\varkappa^{\sigma})\}=\frac{a^{\sigma}}{s^{2\sigma}+a^{2\sigma}}, \tag{23}$$

$$ {}^{LF}L_{\sigma}\{\cos_{\sigma}(a^{\sigma}\varkappa^{\sigma})\}=\frac{s^{\sigma}}{s^{2\sigma}+a^{2\sigma}}, \tag{24}$$

$$ {}^{LF}L_{\sigma}\left\{\varkappa^{k\sigma}\right\}=\frac{\Gamma(1+k\sigma)}{s^{(k+1)\sigma}}. \tag{25}$$

3 Solution of a nonlinear fractional order differential system

Let us consider a general nonlinear system with local fractional derivative:

$$\begin{cases}\frac{\partial^{\sigma}X}{\partial\tau^{\sigma}}+\frac{\partial^{\sigma}T}{\partial\varkappa^{\sigma}}+N_{\sigma,1}(X,T)+R_{\sigma,1}(X,T)=\varphi(\varkappa,\tau),\\ \\ \frac{\partial^{\sigma}T}{\partial\tau^{\sigma}}+\frac{\partial^{\sigma}X}{\partial\varkappa^{\sigma}}+N_{\sigma,2}(X,T)+R_{\sigma,2}(X,T)=\psi(\varkappa,\tau),\end{cases} \tag{26}$$

where $\dfrac{\partial^{\sigma}}{\partial(\cdot)^{\sigma}}$ denotes linear local fractional derivative operator of order σ, $R_{\sigma,1}$, $R_{\sigma,2}$ are the linear local fractional operators, $N_{\sigma,1}$, $N_{\sigma,2}$ represent the nonlinear local fractional operators, and $\varphi(\varkappa,\tau)$, $\psi(\varkappa,\tau)$ are two given functions.

We will search an analytical solution of this system by the following steps.

Step 1 First we apply the local Yang-Laplace transform to both sides of each equation in system (26), so that:

$$\begin{cases} {}^{LF}L_\sigma\left[\frac{\partial^\sigma X}{\partial\tau^\sigma}\right] + {}^{LF}L_\sigma\left[\frac{\partial^\sigma T}{\partial\varkappa^\sigma}\right] + {}^{LF}L_\sigma\left[N_{\sigma,1}(X,T)\right] + {}^{LF}L_\sigma\left[R_{\sigma,1}(X,T)\right] = {}^{LF}L_\sigma\left[\varphi(\varkappa,\tau)\right] \\ {}^{LF}L_\sigma\left[\frac{\partial^\sigma T}{\partial\tau^\sigma}\right] + {}^{LF}L_\sigma\left[\frac{\partial^\sigma X}{\partial\varkappa^\sigma}\right] + {}^{LF}L_\sigma\left[N_{\sigma,2}(X,T)\right] + {}^{LF}L_\sigma\left[R_{\sigma,2}(X,T)\right] = {}^{LF}L_\sigma\left[\psi(\varkappa,\tau)\right] \end{cases}. \tag{27}$$

According to the properties of this transform, we have:

$$\begin{cases} {}^{LF}L_\sigma[X] = X(\varkappa,0) + s^{-\sigma}\left({}^{LF}L_\sigma\left[\varphi(\varkappa,\tau)\right]\right) - s^{-\sigma}\left(LFL_\sigma\left[\frac{\partial^\sigma T}{\partial\varkappa^\sigma} + N_{\sigma,1}(X,T) + R_{\sigma,1}(X,T)\right]\right) \\ {}^{LF}L_\sigma[T] = T(\varkappa,0) + s^{-\sigma}\left({}^{LF}L_\sigma\left[\psi(\varkappa,\tau)\right]\right) - s^{-\sigma}\left({}^{LF}L_\sigma\left[\frac{\partial^\sigma X}{\partial\varkappa^\sigma} + N_{\sigma,2}(X,T) + R_{\sigma,2}(X,T)\right]\right) \end{cases}. \tag{28}$$

By taking the inverse transformation on both sides of each system equation (28), there follows:

$$\begin{cases} X = X(\varkappa,0) + {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\varphi(\varkappa,\tau)\right]\right)\right) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\frac{\partial^\sigma T}{\partial\varkappa^\sigma} + N_{\sigma,1}(X,T) + R_{\sigma,1}(X,T)\right]\right)\right) \\ T = T(\varkappa,0) + {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\psi(\varkappa,\tau)\right]\right)\right) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\frac{\partial^\sigma X}{\partial\varkappa^\sigma} + N_{\sigma,2}(X,T) + R_{\sigma,2}(X,T)\right]\right)\right) \end{cases}. \tag{29}$$

Step 2 By using the Adomian decomposition method [1], we represent the two unknown functions X and T as infinite series:

$$X(\varkappa,\tau) = \sum_{n=0}^{\infty} X_n(\varkappa,\tau), \qquad T(\varkappa,\tau) = \sum_{n=0}^{\infty} T_n(\varkappa,\tau). \tag{30}$$

moreover, the nonlinear terms can be decomposed as:

$$N_{\sigma,1}(X,T) = \sum_{n=0}^{\infty} A_n, \qquad N_{\sigma,2}(X,T) = \sum_{n=0}^{\infty} B_n, \tag{31}$$

where A_n and B_n are Adomian polynomials [22].

Substituting (30) and (31) in (29), we get:

$$\begin{cases} \sum_{n=0}^{\infty} X_n(\varkappa,\tau) = X(\varkappa,0) + {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\varphi(\varkappa,\tau)\right]\right)\right) \\ -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\frac{\partial^\sigma}{\partial\varkappa^\sigma}\left(\sum_{n=0}^{\infty} T_n\right) + \sum_{n=0}^{\infty} A_n + R_{1,\sigma}\left(\sum_{n=0}^{\infty} X_n, \sum_{n=0}^{\infty} T_n\right)\right]\right)\right), \\ \sum_{n=0}^{\infty} T_n(\varkappa,\tau) = T(\varkappa,0) + {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\psi(\varkappa,\tau)\right]\right)\right) \\ -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\frac{\partial^\sigma}{\partial\varkappa^\sigma}\left(\sum_{n=0}^{\infty} X_n\right) + \sum_{n=0}^{\infty} B_n + R_{2,\sigma}\left(\sum_{n=0}^{\infty} X_n, \sum_{n=0}^{\infty} T_n\right)\right]\right)\right). \end{cases} \tag{32a}$$

By comparing both sides of (32a), we have:

$$\begin{bmatrix} X_0(\varkappa,\tau)=X(\varkappa,0)+\ LFL_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\varphi(\varkappa,\tau)\right]\right)\right), \\ X_1(\varkappa,\tau)=-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\frac{\partial^{\sigma}T_0}{\partial\varkappa^{\sigma}}+A_0+R_{1,\sigma}(X_0,T_0)\right]\right)\right), \\ X_2(\varkappa,\tau)=-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\frac{\partial^{\sigma}T_1}{\partial\varkappa^{\sigma}}+A_1+R_{1,\sigma}(X_1,T_1)\right]\right)\right), \\ X_3(\varkappa,\tau)=-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\frac{\partial^{\sigma}T_2}{\partial\varkappa^{\sigma}}+A_2+R_{1,\sigma}(X_2,T_2)\right]\right)\right), \\ \vdots \end{bmatrix} \tag{33}$$

and

$$\begin{bmatrix} T_0(\varkappa,\tau)=T(\varkappa,0)+{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\psi(\varkappa,\tau)\right]\right)\right), \\ T_1(\varkappa,\tau)=-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\frac{\partial^{\sigma}X_0}{\partial\varkappa^{\sigma}}+B_0+R_{2,\sigma}(X_0,T_0)\right]\right)\right), \\ T_2(\varkappa,\tau)=-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\frac{\partial^{\sigma}X_1}{\partial\varkappa^{\sigma}}+B_1+R_{2,\sigma}(X_1,T_1)\right]\right)\right), \\ T_3(\varkappa,\tau)=-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[\frac{\partial^{\sigma}X_2}{\partial\varkappa^{\sigma}}+B_2+R_{2,\sigma}(X_2,T_2)\right]\right)\right), \\ \vdots \end{bmatrix} \tag{34}$$

Step 3 At the last step, we get the solution by a limit of the analytical solution (X,T) of the system (26):

$$\begin{cases} X(\varkappa,\tau)=\lim_{N\to\infty}\sum_{n=0}^{N}X_n(\varkappa,\tau) \\ T(\varkappa,\tau)=\lim_{N\to\infty}\sum_{n=0}^{N}T_n(\varkappa,\tau) \end{cases}. \tag{35}$$

4 Applications

In this section, we will implement the proposed method based on local fractional Yang-Laplace decomposition method (LFLDM) [11] and Adomian decomposition for solving two nonlinear systems of local fractional partial differential equations.

Example 4.1. *Consider the following coupled nonlinear system of local fractional Burger equations*

$$\begin{cases} \frac{\partial^{\sigma}X}{\partial\tau^{\sigma}}-\frac{\partial^{2\sigma}X}{\partial\varkappa^{2\sigma}}-2XX_{\varkappa}^{(\sigma)}+(XT)_{\varkappa}^{(\sigma)}=0 \\ \frac{\partial^{\sigma}T}{\partial\tau^{\sigma}}-\frac{\partial^{2\sigma}T}{\partial\varkappa^{2\sigma}}-2TT_{\varkappa}^{(\sigma)}+(XT)_{\varkappa}^{(\sigma)}=0 \end{cases},\ 0<\sigma\leqslant 1, \tag{36}$$

under the initial conditions:

$$X(\varkappa,0)=\sin_{\sigma}(\varkappa^{\sigma}),\ T(\varkappa,0)=\sin_{\sigma}(\varkappa^{\sigma}). \tag{37}$$

From the equation (29), we obtain:

$$\begin{cases} X(\varkappa,\tau)=\sin_{\sigma}(\varkappa^{\sigma})-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[-\frac{\partial^{2\sigma}T}{\partial\varkappa^{2\sigma}}-2XX_{\varkappa}^{(\sigma)}+(XT)_{\varkappa}^{(\sigma)}\right]\right)\right) \\ T(\varkappa,\tau)=\sin_{\sigma}(\varkappa^{\sigma})-{}^{LF}L_{\sigma}^{-1}\left(s^{-\sigma}\left({}^{LF}L_{\sigma}\left[-\frac{\partial^{2\sigma}X}{\partial\varkappa^{2\sigma}}-2TT_{\varkappa}^{(\sigma)}+(XT)_{\varkappa}^{(\sigma)}\right]\right)\right) \end{cases}. \tag{38}$$

So that by using the Adomian decomposition [1], each function of the solution (X,T) can be decomposed as infinite series:

$$\begin{aligned} X(\varkappa,\tau) &= \sum_{n=0}^{\infty} X_n(\varkappa,\tau), \\ T(\varkappa,\tau) &= \sum_{n=0}^{\infty} T_n(\varkappa,\tau), \end{aligned} \tag{39}$$

and the nonlinear terms can be decomposed as:

$$XX_{\varkappa}^{(\sigma)} = \sum_{n=0}^{\infty} A_n(X), \tag{40}$$

$$(XT)_{\varkappa}^{(\sigma)} = \sum_{n=0}^{\infty} B_n(X,T), \tag{41}$$

and

$$TT_{\varkappa}^{(\sigma)} = \sum_{n=0}^{\infty} C_n(X). \tag{42}$$

Substituting (39), (40), (41) and (42) in (38), we get:

$$\begin{cases} \sum_{n=0}^{\infty} X_n(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\begin{array}{c} -\frac{\partial^{2\sigma}}{\partial\varkappa^{2\sigma}}\left(\sum_{n=0}^{\infty} X_n(\varkappa,\tau)\right) \\ -2\sum_{n=0}^{\infty} A_n(X) + \sum_{n=0}^{\infty} B_n(X,T) \end{array}\right]\right)\right) \\ \sum_{n=0}^{\infty} T_n(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\begin{array}{c} -\frac{\partial^{2\sigma}}{\partial\varkappa^{2\sigma}}\left(\sum_{n=0}^{\infty} T_n(\varkappa,\tau)\right) \\ -2\sum_{n=0}^{\infty} C_n(X) + \sum_{n=0}^{\infty} B_n(X,T) \end{array}\right]\right)\right) \end{cases}. \tag{43}$$

By comparing both sides of (43), it is:

$$\begin{aligned} X_0(\varkappa,\tau) &= \sin_\sigma(\varkappa^\sigma), \\ X_1(\varkappa,\tau) &= -{}^{LF}L_\sigma^{-1}\left(u^{-\sigma}\left({}^{LF}L_\sigma\left[-\frac{\partial^{2\sigma}T_0}{\partial\varkappa^{2\sigma}} - 2A_0(X) + B_0(X,T)\right]\right)\right), \\ X_2(\varkappa,\tau) &= -{}^{LF}L_\sigma^{-1}\left(u^{-\sigma}\left({}^{LF}L_\sigma\left[-\frac{\partial^{2\sigma}T_1}{\partial\varkappa^{2\sigma}} - 2A_1(X) + B_1(X,T)\right]\right)\right), \\ X_3(\varkappa,\tau) &= -{}^{LF}L_\sigma^{-1}\left(u^{-\sigma}\left({}^{LF}L_\sigma\left[-\frac{\partial^{2\sigma}T_2}{\partial\varkappa^{2\sigma}} - 2A_2(X) + B_2(X,T)\right]\right)\right), \\ &\vdots \end{aligned} \tag{44}$$

$$\begin{aligned} T_0(\varkappa,\tau) &= \sin_\sigma(\varkappa^\sigma), \\ T_1(\varkappa,\tau) &= -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[-\frac{\partial^{2\sigma}X_0}{\partial\varkappa^{2\sigma}} - 2C_0(T) + B_0(X,T)\right]\right)\right), \\ T_2(\varkappa,\tau) &= -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[-\frac{\partial^{2\sigma}X_0}{\partial\varkappa^{2\sigma}} - 2C_1(T) + B_1(X,T)\right]\right)\right), \\ T_3(\varkappa,\tau) &= -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[-\frac{\partial^{2\sigma}X_0}{\partial\varkappa^{2\sigma}} - 2C_2(T) + B_2(X,T)\right]\right)\right), \\ &\vdots \end{aligned} \tag{45}$$

and so on.

For example, The first few components of $A_n(X)$, $B_n(X,t)$ and $C(T)$ polynomials [22], are given by:

$$\begin{aligned} A_0(X) &= X_0 X^{(\sigma)}_{0,\varkappa}, \\ A_1(X) &= X_0 X^{(\sigma)}_{1,\varkappa} + X_1 X^{(\sigma)}_{0,\varkappa}, \\ A_2(X) &= X_0 X^{(\sigma)}_{2,\varkappa} + X_2 X^{(\sigma)}_{0,\varkappa} + X_1 X^{(\sigma)}_{1,\varkappa}, \vdots \end{aligned} \tag{46}$$

$$\begin{aligned} B_0(X,T) &= (X_0 T_0)^{(\sigma)}_{\varkappa}, \\ B_1(X,T) &= (X_0 T_1 + X_1 T_0)^{(\sigma)}_{\varkappa}, \\ B_2(X,T) &= (X_1 T_1 + X_0 T_2 + X_2 T_0)^{(\sigma)}_{\varkappa}, \\ &\vdots \end{aligned} \tag{47}$$

and

$$\begin{aligned} C_0(T) &= T_0 T^{(\sigma)}_{0,\varkappa}, \\ C_1(T) &= T_0 T^{(\sigma)}_{1,\varkappa} + T_1 T^{(\sigma)}_{0,\varkappa}, \\ C_2(T) &= T_0 T^{(\sigma)}_{2,\varkappa} + T_2 T^{(\sigma)}_{0,\varkappa} + T_1 T^{(\sigma)}_{1,\varkappa}, \\ &\vdots \end{aligned} \tag{48}$$

According to the equations (44)-(45) and formulas (46)-(48), the first terms of local fractional Yang-Laplace decomposition method of the system (36), is given by:

$$\begin{aligned} X_0(\varkappa,\tau) &= \sin_\sigma(\varkappa^\sigma), \\ X_1(\varkappa,\tau) &= -\sin_\sigma(\varkappa^\sigma)\tfrac{\tau^\sigma}{\Gamma(1+\sigma)}, \\ X_2(\varkappa,\tau) &= \sin_\sigma(\varkappa^\sigma)\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}, \\ X_3(\varkappa,\tau) &= -\sin_\sigma(\varkappa^\sigma)\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}, \\ &\vdots \end{aligned} \tag{49}$$

and

$$\begin{aligned} T_0(\varkappa,\tau) &= \sin_\sigma(\varkappa^\sigma), \\ T_1(\varkappa,\tau) &= -\sin_\sigma(\varkappa^\sigma)\tfrac{\tau^\sigma}{\Gamma(1+\sigma)}, \\ T_2(\varkappa,\tau) &= \sin_\sigma(\varkappa^\sigma)\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}, \\ T_3(\varkappa,\tau) &= -\sin_\sigma(\varkappa^\sigma)\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}, \\ &\vdots \end{aligned} \tag{50}$$

So that, the local fractional series solution (X,T), is:

$$\begin{cases} X(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)\left(1 - \frac{\tau^\sigma}{\Gamma(1+\sigma)} + \frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)} - \frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)} + \cdots\right), \\ T(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)\left(1 - \frac{\tau^\sigma}{\Gamma(1+\sigma)} + \frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)} - \frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)} + \cdots\right), \end{cases} \tag{51}$$

and in a closed form, we obtain the non-differentiable solution (X,T):

$$\begin{cases} X(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)E_\sigma(-\tau^\sigma), \\ T(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)E_\sigma(-\tau^\sigma). \end{cases} \tag{52}$$

By letting $\sigma = 1$ into (52), we have:

$$\begin{cases} X(\varkappa,\tau) = \sin(\varkappa)e^{-\tau}, \\ T(\varkappa,\tau) = \sin(\varkappa)e^{-\tau}. \end{cases} \tag{53}$$

It should be noticed that, solution (52) satisfies the initial conditions (37), and in the case $\sigma = 1$, we have the same solution obtained in [23] by homotopy perturbation method, and in [24] by the natural decomposition method.

As a second example, let us now consider the following:

Example 4.2. *Let*

$$\begin{cases} X_\tau^{(\sigma)} + T_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)} - T_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)} = -X \\ T_\tau^{(\sigma)} + X_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)} = T \qquad , 0 < \sigma \leqslant 1, \\ Z_\tau^{(\sigma)} + X_\varkappa^{(\sigma)}T_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)}T_\varkappa^{(\sigma)} = Z \end{cases} \tag{54}$$

be a nonlinear system of local fractional partial differential equations given under the initial conditions:

$$X(\varkappa,\upsilon,0) = E_\sigma(\varkappa^\sigma + \upsilon^\sigma),\ T(\varkappa,\upsilon,0) = E_\sigma(\varkappa^\sigma - \upsilon^\sigma),\ Z(\varkappa,\upsilon,0) = E_\sigma(-\varkappa^\sigma + \upsilon^\sigma). \tag{55}$$

Also in this case we will search for solutions by first applying the local fractional Yang-Laplace transform on both sides of each equation of the system (54):

$$\begin{cases} {}^{LF}S_\sigma[X(\varkappa,\upsilon,\tau)] = X(\varkappa,\upsilon,0) - s^{-\sigma}\left({}^{LF}L_\sigma\left[T_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)} - T_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)} + X\right]\right) \\ {}^{LF}S_\sigma[T(\varkappa,\upsilon,\tau)] = T(\varkappa,\upsilon,0) - s^{-\sigma}\left({}^{LF}L_\sigma\left[X_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)} - T\right]\right) \\ {}^{LF}S_\sigma[Z(\varkappa,\upsilon,\tau)] = Z(\varkappa,\upsilon,0) - s^{-\sigma}\left({}^{LF}L_\sigma\left[X_\varkappa^{(\sigma)}T_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)}T_\varkappa^{(\sigma)} - Z\right]\right) \end{cases}. \tag{56}$$

From here, by taking the inverse local fractional Yang-Laplace transform on both sides of each equation of (56) and taking into account the initial conditions (55), we have:

$$\begin{cases} X(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma + \upsilon^\sigma) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[T_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)} - T_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)} + X\right]\right)\right) \\ T(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma - \upsilon^\sigma) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[X_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)} - T\right]\right)\right) \\ Z(\varkappa,\upsilon,\tau) = E_\sigma(-\varkappa^\sigma + \upsilon^\sigma) - {}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[X_\varkappa^{(\sigma)}T_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)}T_\varkappa^{(\sigma)} - Z\right]\right)\right) \end{cases}. \tag{57}$$

Then by using the Adomian decomposition method [1] each function of the solution (X,T,Z) can be decomposed as an infinite series:

$$X(\varkappa,\upsilon,\tau)=\sum_{n=0}^{\infty}X_n(\varkappa,\upsilon,\tau),$$
$$T(\varkappa,\upsilon,\tau)=\sum_{n=0}^{\infty}T_n(\varkappa,\upsilon,\tau), \tag{58}$$
$$Z(\varkappa,\upsilon,\tau)=\sum_{n=0}^{\infty}Z_n(\varkappa,\upsilon,\tau),$$

and the nonlinear terms can be decomposed as:

$$T_{\varkappa}^{(\sigma)}Z_{\upsilon}^{(\sigma)}=\sum_{n=0}^{\infty}A_n(T,Z),\quad T_{\upsilon}^{(\sigma)}Z_{\varkappa}^{(\sigma)}=\sum_{n=0}^{\infty}A_n'(T,Z), \tag{59}$$

$$X_{\varkappa}^{(\sigma)}Z_{\upsilon}^{(\sigma)}=\sum_{n=0}^{\infty}B_n(X,Z),\quad X_{\upsilon}^{(\sigma)}Z_{\varkappa}^{(\sigma)}=\sum_{n=0}^{\infty}B_n'(X,Z), \tag{60}$$

and

$$X_{\varkappa}^{(\sigma)}T_{\upsilon}^{(\sigma)}=\sum_{n=0}^{\infty}C_n(X,T),\quad X_{\upsilon}^{(\sigma)}T_{\varkappa}^{(\sigma)}=\sum_{n=0}^{\infty}C_n'(X,T). \tag{61}$$

Substituting (58), (59), (60) and (67) into (57), we get:

$$\begin{cases} \sum_{n=0}^{\infty}X_n(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma) \\ -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\sum_{n=0}^{\infty}A_n(T,Z)-\sum_{n=0}^{\infty}A_n'(T,Z)+\sum_{n=0}^{\infty}X_n(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \sum_{n=0}^{\infty}T_n(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma) \\ -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\sum_{n=0}^{\infty}B_n(X,Z)+\sum_{n=0}^{\infty}B_n'(X,Z)-\sum_{n=0}^{\infty}T_n(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \sum_{n=0}^{\infty}Z_n(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma) \\ -{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[\sum_{n=0}^{\infty}C_n(X,T)+\sum_{n=0}^{\infty}C_n'(X,T)-\sum_{n=0}^{\infty}Z_n(\varkappa,\upsilon,\tau)\right]\right)\right). \end{cases} \tag{62}$$

By comparing both sides of (62), we have:

$$\begin{aligned} X_0(\varkappa,\upsilon,\tau)&=E_\sigma(\varkappa^\sigma+\upsilon^\sigma),\\ X_1(\varkappa,\upsilon,\tau)&=-{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[A_0(T,Z)-A_0'(T,Z)+X_0(\varkappa,\upsilon,\tau)\right]\right)\right),\\ X_2(\varkappa,\upsilon,\tau)&=-{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[A_1(T,Z)-A_0'(T,Z)+X_1(\varkappa,\upsilon,\tau)\right]\right)\right),\\ X_3(\varkappa,\upsilon,\tau)&=-{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[A_2(T,Z)-A_0'(T,Z)+X_2(\varkappa,\upsilon,\tau)\right]\right)\right),\\ &\vdots \end{aligned} \tag{63}$$

$$\begin{aligned} T_0(\varkappa,\upsilon,\tau)&=E_\sigma(\varkappa^\sigma-\upsilon^\sigma),\\ T_1(\varkappa,\upsilon,\tau)&=-{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[B_0(X,Z)+B_0'(X,Z)-T_0(\varkappa,\upsilon,\tau)\right]\right)\right),\\ T_2(\varkappa,\upsilon,\tau)&=-{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[B_1(X,Z)+B_1'(X,Z)-T_1(\varkappa,\upsilon,\tau)\right]\right)\right),\\ T_3(\varkappa,\upsilon,\tau)&=-{}^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left({}^{LF}L_\sigma\left[B_2(X,Z)+B_2'(X,Z)-T_2(\varkappa,\upsilon,\tau)\right]\right)\right),\\ &\vdots \end{aligned} \tag{64}$$

and

$$\begin{aligned} Z_0(\varkappa,\upsilon,\tau) &= E_\sigma(-\varkappa^\sigma+\upsilon^\sigma),\\ Z_1(\varkappa,\upsilon,\tau) &= -^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left(^{LF}L_\sigma\left[C_0(X,T)+C_0'(X,T)-Z_0(\varkappa,\upsilon,\tau)\right]\right)\right),\\ Z_2(\varkappa,\upsilon,\tau) &= -^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left(^{LF}L_\sigma\left[C_1(X,T)+C_1'(X,T)-Z_1(\varkappa,\upsilon,\tau)\right]\right)\right),\\ Z_3(\varkappa,\upsilon,\tau) &= -^{LF}L_\sigma^{-1}\left(s^{-\sigma}\left(^{LF}L_\sigma\left[C_2(X,T)+C_2'(X,T)-Z_2(\varkappa,\upsilon,\tau)\right]\right)\right),\\ &\vdots \end{aligned} \tag{65}$$

and so on.

For example, the first few components of $A_n(T,Z)$, $B_n(X,Z)$ and $C_n(X,T)$ polynomials [22], are:

$$\begin{aligned} A_0(T,Z) &= T_{0\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)},\\ A_1(T,Z) &= T_{1\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)}+T_{0\varkappa}^{(\sigma)}Z_{1\upsilon}^{(\sigma)},\\ A_2(T,Z) &= T_{0\varkappa}^{(\sigma)}Z_{2\upsilon}^{(\sigma)}+T_{2\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)}+T_{1\varkappa}^{(\sigma)}Z_{1\upsilon}^{(\sigma)},\\ &\vdots \end{aligned} \tag{66}$$

$$\begin{aligned} B_0(X,Z) &= X_{0\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)},\\ B_1(X,Z) &= X_{1\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)}+X_{0\varkappa}^{(\sigma)}Z_{1\upsilon}^{(\sigma)},\\ B_2(X,Z) &= X_{0\varkappa}^{(\sigma)}Z_{2\upsilon}^{(\sigma)}+X_{2\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)}+X_{1\varkappa}^{(\sigma)}Z_{1\upsilon}^{(\sigma)},\\ &\vdots \end{aligned} \tag{67}$$

and

$$\begin{aligned} C_0(X,T) &= X_{0\varkappa}^{(\sigma)}T_{0\upsilon}^{(\sigma)},\\ C_1(X,T) &= X_{1\varkappa}^{(\sigma)}T_{0\upsilon}^{(\sigma)}+X_{0\varkappa}^{(\sigma)}T_{1\upsilon}^{(\sigma)},\\ C_2(X,T) &= X_{0\varkappa}^{(\sigma)}T_{2\upsilon}^{(\sigma)}+X_{2\varkappa}^{(\sigma)}T_{0\upsilon}^{(\sigma)}+X_{1\varkappa}^{(\sigma)}T_{1\upsilon}^{(\sigma)},\\ &\vdots \end{aligned} \tag{68}$$

The other polynomials A_n', B_n' and C_n', can be computed in the same way.

From the equations (63)-(65) and formulas of the polynomial terms, the first terms of local fractional Yang-Laplace decomposition method of the system (54), is given by:

$$\begin{aligned} X_0(\varkappa,\upsilon,\tau) &= E_\sigma(\varkappa^\sigma+\upsilon^\sigma),\\ X_1(\varkappa,\upsilon,\tau) &= -E_\sigma(\varkappa^\sigma+\upsilon^\sigma)\tfrac{\tau^\sigma}{\Gamma(1+\sigma)},\\ X_2(\varkappa,\upsilon,\tau) &= E_\sigma(\varkappa^\sigma+\upsilon^\sigma)\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)},\\ X_3(\varkappa,\upsilon,\tau) &= -E_\sigma(\varkappa^\sigma+\upsilon^\sigma)\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)},\\ &\vdots \end{aligned} \tag{69}$$

$$\begin{aligned}
&T_0(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma),\\
&T_1(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)\tfrac{\tau^\sigma}{\Gamma(1+\sigma)},\\
&T_2(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)},\\
&T_3(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)},\\
&\vdots
\end{aligned} \tag{70}$$

and

$$\begin{aligned}
&Z_0(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma),\\
&Z_1(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)\tfrac{\tau^\sigma}{\Gamma(1+\sigma)},\\
&Z_2(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)},\\
&Z_3(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)},\\
&\vdots
\end{aligned} \tag{71}$$

Each function of the solution (X,T,Z) is defined by an approximate series as follows:

$$\begin{cases}
X(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma)(1-\tfrac{\tau^\sigma}{\Gamma(1+\sigma)}+\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}-\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}+\cdots),\\
T(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)(1+\tfrac{\tau^\sigma}{\Gamma(1+\sigma)}+\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}+\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}+\cdots),\\
Z(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)(1+\tfrac{\tau^\sigma}{\Gamma(1+\sigma)}+\tfrac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}+\tfrac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}+\cdots)\,.
\end{cases} \tag{72}$$

So that in closed form, the non-differentiable solution (X,T,Z) can be written as:

$$\begin{cases}
X(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma)E_\sigma\left(-\tau^\sigma\right),\\
T(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)E_\sigma\left(\tau^\sigma\right),\\
Z(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)E_\sigma\left(\tau^\sigma\right),
\end{cases} \tag{73}$$

and, according to [17],

$$\begin{cases}
X(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma-\tau^\sigma),\\
T(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma+\tau^\sigma),\\
Z(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma+\tau^\sigma).
\end{cases} \tag{74}$$

In particular, by letting $\sigma=1$, from (74), we get

$$\begin{cases}
X(\varkappa,\upsilon,\tau)=e^{\varkappa+\upsilon-\tau},\\
T(\varkappa,\upsilon,\tau)=e^{\varkappa-\upsilon+\tau},\\
Z(\varkappa,\upsilon,\tau)=e^{-\varkappa+\upsilon+\tau}.
\end{cases} \tag{75}$$

Let us notice that, our solution (74) fulfills the initial conditions (55), and in the case $\sigma = 1$, we re-obtain the same solution already obtained both in [26] by the projected differential transform method and Elzaki transform, and in [24] by the natural decomposition method. Thus showing that the method proposed in this paper is the more general approach to the solution of nonlinear fractional differential system.

5 Conclusions

The local fractional Yang-Laplace transform decomposition method (LFLDM) has been used to solve nonlinear systems of local fractional partial differential equations. It has been shown that combined with the Adomian decomposiition method, LFLDM enables us to establish an efficient algorithm. This algorithm provides the solution in a series form that converges rapidly to the exact solution, as shown by the results obtained through the two non-trivial examples given in this paper. From the obtained results, it can be concluded that this algorithm is powerful and effective and it can be used to explore some more complicated nonlinear systems with local fractional derivative.

Author contributions

This is an author contribution text.

Financial disclosure

None reported.

Conflict of interest

The authors declare no potential conflict of interests. lisis non, adipiscing quis, ultrices a, dui.

References

[1] Adomian G. Nonlinear Stochastic Systems Theory and Applications to Physics. *Kluwer Academic Publishers Netherlands*; 1989.
[2] Adomian G, Rach R. Equality of partial solutions in the decomposition method for linear or nonlinear partial differential equations.*Comput Math Appl*. 1990;10:9-12.
[3] Adomian G. Solving Frontier Problems of Physics The Decomposition Method. *Kluwer Academic Publishers Boston*; 1994.
[4] Adomian G. Solution of physical problems by decomposition. *Comput Math Appl*. 1994;27:145-154.
[5] Adomian G. Solutions of nonlinear P.D.E. *Appl Math Lett*. 1998;11:121-123.
[6] Yang XJ, Baleanu D, Zhon WP. Approximate Solutions for Diffusion Equations on Cantor Space-Time, *Proc of the Romanian Aca Series A*. 2013;14(2):127-133.
[7] Jafari H, Jassim HK. Local Fractional Adomain Decomposition Method for Solving Two Dimensional Heat conduction Equations with Local Fractional Operators. *J of Adv in Math*. 2014;9(4):2574-2582.
[8] Yan SP, Jafari H, Jassim HK. Local Fractional Adomian Decomposition and Function Decomposition Methods for Laplace Equation within Local Fractional Operators. *Adv in Math Phy*. 2014;A ID 161580:1-7.
[9] Ahmad J, Mohyud-Din ST, Yang XJ. Applications of Local Fractional Adomian Decomposition Method to Integral Equations. *J of Sci Arts*. 2014;1(26):73-82.
[10] Yang XJ, Baleanu D, Lazarević MP, Cajić MS. Fractal Boundary Value Problems for Integral and Differential Equations with Local Fractional Operators. *Ther Sci*. 2015;19(3):959-966.
[11] Jassim HK. Local fractional Laplace decomposition method for nonhomogeneous heat equations arising in fractal heat flow with local fractional derivative. *Int J Adv Appl Math Mech*. 2015;2(4):1-7.
[12] Ziane D, Baleanu D, Belghaba K, Hamdi Cherif M. Local fractional Sumudu decomposition method for linear partial differential equations with local fractional derivative. *J King Saud Univ Sci*. 2017;http://dx.doi.org/10.1016/j.jksus.2017.05.002.
[13] Goncalves E, Zeidan D. Numerical Study of Turbulent Cavitating Flows in Thermal Regime. *Int J Num Meth Heat-Fluid Flow*. 2017;27(7):1487-1503.
[14] Ziane D, Belghaba K, Hamdi Cherif M. Exact solutions for linear systems of local fractional partial differential equations. *Mal J of Mat*. 2018;6(1):53-60.
[15] Yang XJ. Fractional Functional Analysis and Its Applications. *Asian Academic Hong Kong*; 2011.
[16] Yang XJ. Local Fractional Calculus and Its Applications. *World Science Publisher New York USA*; 2012.
[17] Hu MS, Agarwal RP, Yang XJ. Local Fractional Fourier Series with Application to Wave Equation in Fractal Vibrating

String. *Abs and Appl Anal*. 2012;A ID567401:1-15.
[18] Srivastava HM, Golmankhaneh AK, Baleanu D, Yang XJ. Local Fractional Sumudu Transform with Application to IVPs on Cantor Sets., *Abs Appl Anal*. 2014;A:ID620529:7.
[19] Yanga YJ, Yang C, Jin XF. The Yang Laplace transform- DJ iteration method for solving the local fractional differential equation. *J Nonl Sci Appl*. 20;(10):3023-3029.
[20] Zhao CG, Yang AM, Jafari H, Haghbin A. The Yang-Laplace Transform for Solving the IVPs with Local Fractional Derivative. *Abs Appl Anal*. 2014;A:ID386459:5pp.
[21] Bira B, Raja Sekhar T, Zeidan D. Exact Solutions for Some Time-Fractional Evolution Equations Using Lie Group Theory. *Math Meth Appl Scie*. 2018;41(16):6717-6725.
[22] Zhu Y, Chang Q, Wu S. A new algorithm for calculating Adomian polynomials. *Appl Math Comput*. 2005;169:402-416.
[23] Sweilam NH, Khader MM. Exact solutions of some coupled nonlinear partial differential equations using the homotopy perturbation method. *Comput Math with Appl*.2009;58:2134-2141.
[24] Rawashdeh M S, Al-JammalLiu H. New approximate solutions to fractional nonlinear systems of partial differential equations using the FNDM. *Advan in Diff Equat*. 2016:235; DOI 10.1186/s13662-016-0960-x, 1-19.
[25] Zeidan D. The Riemann Problem for a Hyperbolic Model of Two-Phase Flow in Conservative Form. *Int J Comput Fluid Dyn*. 2011;25(6):299–318.
[26] Elzaki TM, Alamri BAS. Projected Differential Transform Method and Elzaki Transform for Solving System of Nonlinear Partial Differential Equations. *W Appl Sci J*. 2014;32(9):1974-1979.