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The Comparison Study of Hybrid Method with RDTM for Solving Rosenau-Hyman Equation

Derya Arslan †

Department of Mathematics, Faculty of Art and Science, Bitlis Eren University, 13200, Bitlis
Turkey

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Abstract

In this paper, the hybrid method (differential transform and finite difference methods) and the RDTM (reduced differential transform method) are implemented to solve Rosenau-Hyman equation. These methods give the desired accurate results in only a few terms and the approach procedure is rather simple and effective. An experiment is given to demonstrate the efficiency and reliability of these presented methods. The obtained numerical results are compared with each other and with exact solution. It seems that the results of the hybrid method and the RDTM show good performance as the other methods. The most important part of this study is that these methods are suitable to solve both some linear and nonlinear problems, and reduce the size of computation work.

Keywords: Rosenau-Hyman equation, hybrid method, RDTM, approximate solution.
AMS 2010 codes: 65Q05, 65Q10, 65R10, 35Q93.

1 Introduction

In the present paper, the hybrid method and the RDTM will be applied for solving Rosenau-Hyman equation that is given as

$$u_t - uu_{xxx} - uu_x - 3u_xu_{xx} = 0. \tag{1}$$

Special case of Gilson-Pickering equation is Rosenau-Hyman equation for $\varepsilon = 0, \kappa = 0, \alpha = 1, \beta = 3$. Gilson-Pickering equation is defined as follows:

$$u_t - \varepsilon u_{xxt} + 2\kappa u_x - uu_{xxx} - \alpha uu_x - \beta u_xu_{xx} = 0. \tag{2}$$

where $\varepsilon, \kappa, \alpha, \beta$ are arbitrary constants. The Eq. (2) was introduced by Gilson and Pickering [1]. The other special cases of Eq. (2) can be seen in the literature [1–5]. The Eq. (2) includes many other nonlinear equations

†Corresponding author.
Email address: ayredlanu@gmail.com

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except for the Rosenau-Hyman equation, such as Camassa-Holm equation and Fornberg-Whitham equation [1]. The Rosenau-Hyman equation was used as a simplified model for the study of nonlinear dispersion in pattern formation in liquid drops and also it has different applications in modelling of many problems in engineering and physics [2]. Various analytical methods have been discussed to obtain the solution of the Rosenau-Hyman equation. For example, q-homotopy analysis method, semi-analytical methods, dissociative perturbation methods, variational iteration method and homotopy perturbation method [7–12]. According to the literature review, the RDTM and the hybrid method for the solution of the Rosenau-Hyman equation have not been used until now. Therefore, this study is conducted. Recently, the hybrid method (differential transform and finite difference methods) and the RDTM have attracted many class of researcher in fields of science and engineering. For instance, the hybrid method has been used in the solution of several linear and nonlinear problems [25–31]. RDTM is used by various researchers in [13–24]. These important methods may also be potential approximate methods for in some partial differential equations [32–36].

In the target of this paper, by using the hybrid method and the RDTM, the approximate solution of the Rosenau-Hyman equation is obtained and determined the accuracy of this method for solving this equation. The remainder of this study is continued as follows: In Section 2, a brief introduction for the Rosenau-Hyman equation, the hybrid method and the RDTM are presented. In Section 3, the hybrid method and the RDTM are applied to the Rosenau-Hyman equation in a few steps. And then, the numerical results are obtained for the Rosenau-Hyman equation. The tables and graphs are given that the results are presented. Finally, this study ends with conclusion section.

2 Description of the Methods

Here, the hybrid method and the RDTM which are analyzed the numerical solution of the Rosenau-Hyman equation are introduced, respectively. The differential transform method was studied in the solution of linear and nonlinear problems by Zhou in 1986 [17]. Two-dimensional differential transform method was applied to two dimensional partial differential equations by Chen and Ho in 1999 [16]. The definition and properties of the differential transform method are introduced as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U(i,k)t^k = U(i,0) + U(i,1)t + U(i,2)t^2 + \ldots \tag{3}$$

The differential transform of $u(x,t)$ based on $t-$ time variable is defined as

$$U(i,k) = \frac{1}{k!}\left[\frac{d^k u(x,t)}{dt^k}\right]_{t=0}. \tag{4}$$

The reverse of the $U(i,k)$ differential function based on $t-$ time variable is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U(i,k)t^k. \tag{5}$$

The differential transform and finite difference method apply to $t-$ and $x-$ variables. Linear and nonlinear partial differential equations are solved by using hybrid method with a few iteration and with obtaining rapid convergence. Some properties of differential transform and finite difference method, which using to solve the Rosenau-Hyman equation, are presented the Table 1 and 2 as follows:

Using the definition of differential transform method and its some properties, the form of RDTM is given as [13–15] and Table 3:

$u(x,t)$ and $U_k(x)$ are defined as original function and the transformed function in the RDTM, respectively. The transformed function is defined as

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0}$$

Table 1 Some properties of differential transform based on $t-$ variable.

Function	Transform
$\frac{dw(x,t)}{dt}$	$W(i,k)=(k+1)W(i,k+1)$
$w(x,t)=\alpha w(x,t)$	$W(i,k)=\alpha W(i,k)$

Table 2 Some properties of central difference and differential transform based on $x-$variable.

Function	Transform
$\frac{\partial w(x,t)}{\partial x}$	$W(i,k)=\frac{W(i+1,k)-W(i-1,k)}{2h}$
$\frac{\partial^2 w(x,t)}{\partial x^2}$	$W(i,k)=\frac{W(i+1,k)-2W(i,k)+W(i-1,k)}{h^2}$
$\frac{\partial^3 w(x,t)}{\partial x^3}$	$W(i,k)=\frac{W(i+2,k)-2W(i+1,k)+2W(i-1,k)-W(i-2,k)}{2h^3}$
$u(x,t)\frac{\partial^3 w(x,t)}{\partial x^3}$	$W(i,k)=\sum_{m=0}^{k}W(i,k-m)\frac{W(i+2,k)-2W(i+1,k)+2W(i-1,k)-W(i-2,k)}{2h^3}$
$\frac{\partial w(x,t)}{\partial x}\frac{\partial x^2 w(x,t)}{\partial x^2}$	$W(i,k)=\sum_{m=0}^{k}\frac{W(i+1,k)-2W(i,k)+W(i-1,k)}{h^2}\frac{W(i+1,k-m)-W(i-1,k-m)}{2h}$
$u(x,t)\frac{\partial w(x,t)}{\partial x}$	$W(i,k)=\sum_{m=0}^{k}U(i,k-m)\frac{W(i+1,k)-W(i-1,k)}{2h}$
$w(x,t)=sinh(x)$	$W(x,t)=sinh(x_i)$
$w(x,t)=cosh(x)$	$W(x,t)=cosh(x_i)$

The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t)=\sum_{k=0}^{\infty}U_k(x)t^k.$$

Table 3 Some properties of the RDTM.

Function	Transform
$u(x,t)$	$U_k(x)=\frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0}$
$w(x,t)=u(x,t)\pm v(x,t)$	$W_k(x)=U_k(x)\pm V_k(x)$
$w(x,t)=\alpha u(x,t),\alpha$ is a constant.	$W_k(x)=\alpha U_k(x)$
$w(x,t)=u(x,t)v(x,t)$	$W_k(x)=\sum_{r=0}^{k}U_r(x)V_{k-r}(x)$
$w(x,t)=\frac{\partial u(x,t)}{\partial t}$	$W_k(x)=(k+1)U_{k+1}(x)$

3 Implementation and Comparison of the Hybrid Method and the RDTM to the Rosenau-Hyman Equation

In the present study, the Rosenau-Hyman equation is analyzed by the hybrid method and the RDTM. The obtained numerical solutions are compared with the exact solution and each other.

Example 1. *To demonstrate the basic idea of the hybrid method and the RDTM, we consider the following Rosenau-Hyman equation:*

$$u_t-uu_{xxx}-uu_x-3u_xu_{xx}=0. \tag{6}$$

subject to the initial condition

$$u(x,0)=-\frac{8}{3}cos^2(\frac{x}{4}) \tag{7}$$

where $\varepsilon=0,\kappa=0,\alpha=1,\beta=3$, *the exact solution of problem (6)-(7) is given as*

$$u(x,t)=-\frac{8}{3}cos^2(\frac{x-t}{4}).$$

Now, let us solve the Rosenau-Hyman equation using the hybrid method and the RDTM respectively as follows:

The solution with the hybrid method:

Using the hybrid method in the (6)-(7) Rosenau-Hyman equation, differential transformations corresponding to both each term and initial condition are as follows:

$$\frac{\partial u(x,t)}{\partial t}\to U(i,k)=(k+1)U(i,k+1),$$

$$\frac{\partial u(x,t)}{\partial x}\to U(i,k)=\frac{U(i+1,k)-U(i-1,k)}{2h},$$

$$\frac{\partial^2 u(x,t)}{\partial x^2}\to U(i,k)=\frac{U(i+1,k)-2U(i,k)+U(i-1,k)}{h^2},$$

$$\frac{\partial^3 u(x,t)}{\partial x^3}\to U(i,k)=\frac{U(i+2,k)-2U(i+1,k)+2U(i-1,k)-U(i-2,k)}{2h^3},$$

$$\frac{\partial u(x,t)}{\partial x}\frac{\partial^2 u(x,t)}{\partial x^2}\to U(i,k)=\sum_{m=0}^{k}\frac{U(i+1,k)-2U(i,k)+U(i-1,k)}{h^2}\frac{U(i+1,k-m)-U(i-1,k-m)}{2h},$$

$$u(x,t)\frac{\partial u(x,t)}{\partial x}\to U(i,k)=\sum_{m=0}^{k}U(i,k-m)\frac{U(i+1,k)-U(i-1,k)}{2h},$$

$$u(x,0)\to -\frac{8}{3}cos^2(\frac{x_i}{4}),\ \ x_i=ih,\ \ i=0,1,2,...,\ \ k=0,1,2,....$$

According to the hybrid method procedure, the presented differential transforms in the above can be obtained the following recurrence relation by putting in its place on equation (1).

$$\begin{aligned}U(k+1)=&\frac{1}{(k+1)}(\sum_{m=0}^{k}\frac{U(i,k-m)}{2h^3}(\frac{U(i+2,k)-2U(i+1,k)+2U(i-1,k)-U(i-2,k)}{2h^3})\\&-\sum_{m=0}^{k}\frac{U(i,k-m)}{2h}(U(i+1,k)-U(i-1,k))\\&-\frac{3}{2h^3}\sum_{m=0}^{k}(U(i+1,k)-2U(i,k)+U(i-1,k))((U(i+1,k-m)-U(i-1,k-m))),\end{aligned}$$

where for $k=0,1,2,3,...$ differential transform coefficients $U(i,0),U(i,1),U(i,2),...$ are obtained. After this, approximate solutions for $t=0.01$ are found with the help of (5) and demonstrated in Table 3.

$$\begin{aligned}x_i=0,\ \ u(0,t)=&\sum_{k=0}^{\infty}U(0,k)t^k=U(0,0)+U(0,1)t+U(0,2)t^2)+...\\=&-2.666666667-2.503443416t-1.25000000t^2-...-1.333333333t^{10}.\end{aligned}$$

$$x_i = 0.1, \quad u(0.1,t) = \sum_{k=0}^{\infty} U(0.1,k)t^k = U(0.1,0) + U(0.1,1)t + U(0.1,2)t^2 + \ldots$$
$$= -2.665000347 - 2.533929470t - 1.250104145t^2 - \ldots - 1.333333333t^{10}.$$

$$\ldots$$

$$x_i = 1, \quad u(1,t) = \sum_{k=0}^{\infty} U(1,k)t^k = U(1,0) + U(1,1)t + U(1,2)t^2) + \ldots$$
$$= -2.503443416 - 2.666666667t - 1.260201453t^2 - \ldots - 1.333333333t^{10}.$$

where $x_i = ih$ mesh points for $h = 0.1, \quad i = 0,1,2,.....$

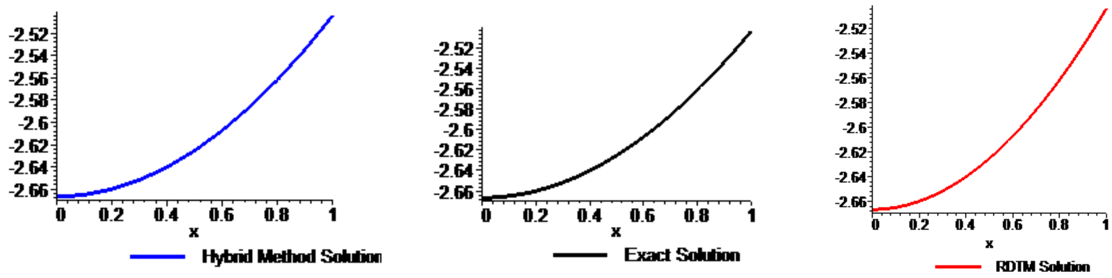

Fig. 1 2D curves of Hybrid method, Exact and RDTM solutions for $t = 0{,}0001$.

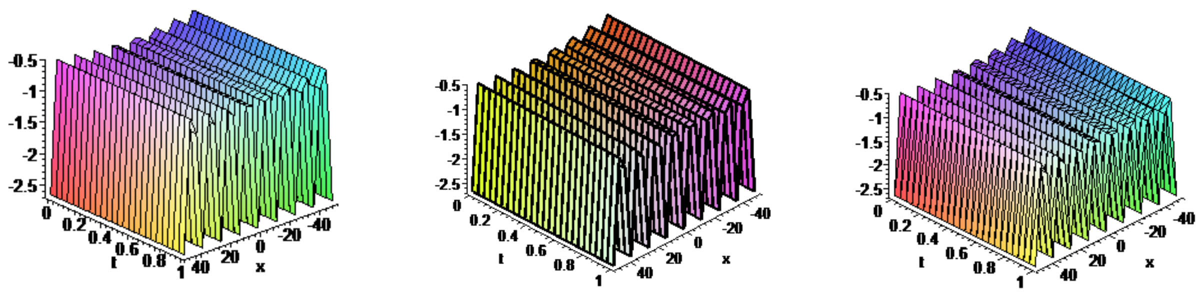

Fig. 2 3D curves of Hybrid method, Exact and RDTM solutions for $t = 0{,}0001$, respectively.

The solution with the RDTM:

According to $t-$ time variable and Table 3, we take differential transform of the equation (6) and the initial condition (7), respectively. If these differential transformations are written in the Eq. (6), the following recurrence relation is obtained:

$$\frac{\partial u(x,t)}{\partial t} \to (k+1)U_{k+1}(x),$$

$$\frac{\partial u(x,t)}{\partial x} \to \frac{\partial U_k(x)}{\partial x},$$

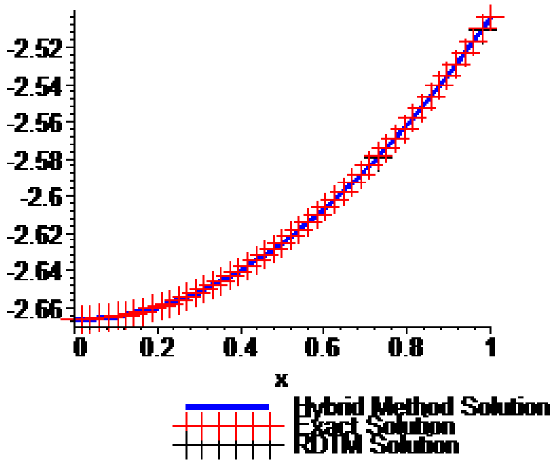

Fig. 3 The Comparison of Exact, Hybrid Method and RDTM for $t = 0,0001$.

$$\frac{\partial^2 u(x,t)}{\partial x^2} \to \frac{\partial^2 U_k(x)}{\partial x^2},$$

and

$$U_{k+}(x) = \frac{1}{k+1}\left\{\sum_{m=0}^{k} \frac{\partial^3 U_{k-m}(x)}{\partial x^3} U_m(x) - \sum_{m=0}^{k} \frac{\partial U_{k-m}(x)}{\partial x} U_m(x) - 3\sum_{m=0}^{k} \left(\frac{\partial^2 U_{k-m}(x)}{\partial x^2}\right)\left(\frac{\partial U_k(x)}{\partial x}\right)\right\}.$$

Here, the following differential transform coefficients are found by using above iteration procedure and two iterations for $k = 0,1,2,....$

$$u_0(x) = -\frac{8}{3}cos^2(\frac{x}{4}),$$

$$u_1(x) = \frac{40}{9}cos^3(\frac{x}{4})sin(\frac{x}{4}) - \frac{4}{3}cos^2(\frac{x}{4})sin(\frac{x}{4})cos(\frac{x}{4}),$$

These differential transform coefficients are written in the equation (7), the approximate solution is found as

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k = U_0(x) + U_1(x)t + ... = -\frac{8}{3}cos^2(\frac{x}{4}) + (\frac{40}{9}cos^3(\frac{x}{4})sin(\frac{x}{4}) - \frac{4}{3}cos^2(\frac{x}{4})sin(\frac{x}{4})cos(\frac{x}{4}))t + ...$$

One can see that the obtained approximate solutions by the hybrid method and the RDTM are quite close to their exact solutions.

The exact, the approximate solutions and error values for different values for $t = 0.0001$, x are presented in Table 4 as.

The approximate solution curves are presented in Figure 1, Figure 2 and Figure 3. Exact, hybrid and RDTM solutions are compared in Table 1 for various values of x and especially $t = 0.0001$

4 Conclusion

In this study, we have successfully acquired approximate solution of the Rosenau-Hyman equation using the hybrid method and the RDTM. The obtained numerical solutions by these method were compared with the exact solution and with each other. These solutions are of high accuracy and also the results show that the proposed methods were strongly and efficient for solving the Rosenau-Hyman equation. In conclusion, It was revealed these methods are as good as the other methods in terms of fast convergence. As suggestion, these methods are a potential important approximate method for further works in strongly nonlinear partial differential equations.

Table 4 Comparison of the approximate solutions (hybrid method and RDTM) with exact solution $u(x,t)$ and the error values for $t = 0.0001$.

$x(x_i)$	Hybrid Method	RDTM	Exact	Error of RDTM	Error of Hybrid Method
0.0	-2.666917024	-2.666666656	-2.666666665	$0.9x10^{-8}$	0.000250359
0.1	-2.665253753	-2.664992564	-2.665003678	0.000011114	0.000250075
0.2	-2.660261708	-2.659990035	-2.660012208	0.000022173	0.000249500
0.3	-2.651953366	-2.651671588	-2.651704732	0.000033144	0.000248635
0.4	-2.640349496	-2.640058033	-2.640102014	0.000043981	0.000247483
0.5	-2.625479097	-2.625178409	-2.625233055	0.000054646	0.000246043
0.6	-2.607379341	-2.607069925	-2.607135019	0.000065094	0.000244322
0.7	-2.586095466	-2.585777858	-2.585853142	0.000075284	0.000242324
0.8	-2.561680672	-2.561355438	-2.561440619	0.000085181	0.000240054
0.9	-2.534195983	-2.533863725	-2.533958466	0.000094741	0.000237517
1.0	-2.503710096	-2.503371438	-2.503475376	0.000103938	0.000234720

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