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Components of Gini, Bonferroni, and Zenga Inequality Indexes for EU Income Data

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In this work we apply a new approach to assess contributions from factor components to income inequality. The new approach is based on the insight that most (synthetic) inequality indexes may be viewed as (weighted) averages of point inequality measures, which measure inequality between population subgroups identified by income. Assessing contributions of factor components to point inequality measures is usually an easy task, and based on these contributions it is straightforward to define contributions to the corresponding (synthetic) overall inequality indexes as well. As we shall show through an analysis of income data from Eurostat's European Community Household Panel Survey (ECHP), the approach based on point inequality measures gives rise to readily interpretable results, which, we believe, is an advantage over other methods that have been proposed in literature.

Key words: Inequality decomposition; factor components; point inequality measures; synthetic inequality index.

1. Introduction

A great deal of literature about income inequality is concerned with evaluation of contributions to inequality from factor components. A common approach to this problem is to express some given (synthetic) inequality index as sum of terms, with one term corresponding to each factor component, which are then interpreted as contributions to inequality. The interpretations are justified by showing that the terms representing the contributions are functions of some descriptive statistics for the joint distribution of the factor components and total income. In connection with the well-known Gini index, this approach has, for example, been applied by Rao (1969); Lerman and Yitzhaki (1984, 1985), and Radaelli and Zenga (2005).

Shorrocks (1982), on the other hand, explores an axiomatic approach. He considers a broad class of inequality indexes, but is faced with the problem that under a fairly general set of restrictions there exists an infinite number of potential decomposition rules for every given inequality index. To solve this nonuniqueness problem, he adds two further restrictions which imply that the relative contributions (or "proportional contributions" in his language) from the components are the same for all inequality indexes and are equal to those corresponding to what he calls the "natural decomposition rule" for the variance. In a

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later paper (Shorrocks 1983) Shorrocks acknowledges that not everyone might agree on the restrictions imposed to derive the "unique" decomposition rule, but he still defends that rule by showing that in applications to some empirical datasets it gives rise to reasonable results.

In the present article we illustrate, through an application to income data from the European Community Household Panel (ECHP), a new approach to factor component decomposition. This approach has been recently suggested by Zenga et al. (2012), and was originally developed for the inequality index I (Zenga 2007a). In a later paper it has been extended to the Gini and Bonferroni indexes as well (Zenga 2013). The new approach is based on the fact that these three inequality indexes are, *by their original definitions*, (weighted) averages of point inequality measures which measure inequality between population subgroups identified by income. Defining factor component contributions to the point inequality measures is, as we shall show below, an easy and straightforward task, and taking appropriate averages of these contributions yields decomposition rules for the (synthetic) inequality indexes as well.

The rest of this article is organized as follows. In Section 2 we recall the definitions of the Gini, Bonferroni, and Zenga indexes in terms of point inequality measures. In Section 3 we show how the decomposition rules based on the point inequality indexes are derived and in Section 4 we highlight some interesting relations between factor component contributions to inequality and shares on total population income. Since income distributions are usually available in the form of survey data with weights associated to each sample unit, we devoted Section 5 to estimation from survey data. Finally, in Section 6 we provide an application to data from the 2001 wave of the ECHP in order to give some insight into the range of possible outcomes. To help the reader to recall the meaning of certain symbols which we shall introduce in the course of this article, we added a list of notations at the end of the article.

2. The Gini, Bonferroni, and Zenga Indexes as Averages of Point Inequality Indexes

Let

$$y_1 \leq y_2 \leq \cdots \leq y_N \tag{1}$$

denote total income Y of individuals or families belonging to a given population, and let

$$p_i := \frac{i}{N}, \quad i = 1, 2, \ldots, N, \tag{2}$$

and

$$q_i := \frac{\sum_{\nu=1}^{i} y_\nu}{\sum_{\nu=1}^{N} y_\nu}, \quad i = 1, 2, \ldots, N, \tag{3}$$

denote the cumulative population and income shares, respectively. When Gini (1914) first proposed what later became the virtually most widely used inequality index, he set out from the fact that the cumulative income shares q_i can never exceed their corresponding cumulative population shares p_i. Thus, he proposed

$$R_i := \frac{p_i - q_i}{p_i}, \quad i = 1, 2, \ldots, N, \tag{4}$$

as basic point inequality measures from which he derived the definition of his well-known synthetic inequality index

$$R := \frac{\sum_{i=1}^{N-1} R_i p_i}{\sum_{i=1}^{N-1} p_i} = \frac{\sum_{i=1}^{N-1} (p_i - q_i)}{\sum_{i=1}^{N-1} p_i}. \tag{5}$$

Thereafter he showed that R is linked to the graph with the Lorenz curve (Lorenz 1905) in the sense that R is equal to the ratio between the "concentration area" and the area of the triangle with vertices in (0, 0), (($N - 1$)/N, 0) and (1, 1) (sometimes called the "maximum concentration area").

Starting from the observation that the mean income

$$M_i^-(Y) := \frac{1}{i} \sum_{\nu=1}^{i} y_\nu, \quad i = 1, 2, \ldots, N, \tag{6}$$

of the i "poorest" population members cannot exceed the mean income

$$M(Y) := M_N^-(Y) = \frac{1}{N} \sum_{\nu=1}^{N} y_\nu \tag{7}$$

of the whole population, Bonferroni (1930) proposed the inequality index

$$B := \frac{1}{N-1} \sum_{i=1}^{N-1} \frac{M(Y) - M_i^-(Y)}{M(Y)}. \tag{8}$$

As pointed out by DeVergottini (1940), B can also be viewed as unweighted average of the point inequality measures R_i proposed by Gini (1914). In fact,

$$\frac{M(Y) - M_i^-(Y)}{M(Y)} = \frac{\frac{\sum_{\nu=1}^{N} y_\nu}{N} - \frac{\sum_{\nu=1}^{i} y_\nu}{i}}{\frac{\sum_{\nu=1}^{N} y_\nu}{N}} = \frac{p_i - q_i}{p_i} =: R_i.$$

More recently, Zenga (1984, 2007a) introduced two new types of point inequality measures and put forward corresponding synthetic inequality indexes. In the present article we shall consider only the latter proposal. It is based on the point inequality measures given by

$$I_i := \frac{M_i^+(Y) - M_i^-(Y)}{M_i^+(Y)}, \quad i = N_1, N_2, \ldots, N_k, \tag{9}$$

where

$$M_i^+(Y) := \begin{cases} \dfrac{1}{N-i}\sum_{\nu=i+1}^{N} y_i & \text{if } i = 1, 2, \ldots, N-1 \\ y_N & \text{if } i = N \end{cases} \tag{10}$$

and where $N_1 < N_2 < \cdots < N_k = N$ are the cumulative frequencies corresponding to the *k different* values taken on by total income *Y*. Using the point inequality measures I_i, Zenga (2007a) defined the synthetic inequality index

$$I := \frac{1}{N}\sum_{s=1}^{k} I_{N_s} n_s \tag{11}$$

where $n_1, n_2, \ldots, n_k$ denote the absolute frequencies of the *k* different values observed for total income *Y*.

Notice that as opposed to the indexes proposed by Gini and Bonferroni, Zenga's synthetic inequality index *I* involves only the point inequality measures at $i = N_1, N_2, \ldots, N_k$, which, as will be seen in the next section, makes it easier to apply the approach to factor component decomposition based on point inequality measures.

Before moving on to factor component decomposition, we provide a brief list of references regarding the synthetic Zenga index *I*. Applications to real distributions may be found in Zenga (2007b), Zenga (2008), and Greselin et al. (2013). Polisicchio (2008), Polisicchio and Porro (2009), Porro (2008), and Porro (2011) deal with properties of the curve defined by the point inequality measures I_i and its relation with the Lorenz curve. Inferential problems related to the *I* index have been analyzed in Greselin and Pasquazzi (2009), Greselin et al. (2010), Langel and Tillé (2012), Antal et al. (2011), and Greselin et al. (2014). As for decomposition rules, Radaelli (2008a) proposed a subgroups decomposition for the point inequality indexes I_i and the synthetic *I* index that has been applied to income data in Radaelli (2007), Radaelli (2008b), and Greselin et al. (2009) and that has been compared with a subgroups decomposition rule for Gini's index in Radaelli (2010). Finally, as already mentioned above, the decomposition rule considered in the present work has been originally proposed in Zenga et al. (2012) and has been extended to the Gini and Bonferroni indexes in Zenga (2013).

3. Factor Component Contributions to Inequality

Assume

$$y_i := x_{i,1} + x_{i,2} + \cdots + x_{i,c}, \quad i = 1, 2, \ldots, N, \tag{12}$$

where $x_{i,j}$ denotes the income from factor component X_j of the *i*th individual or household. Obviously,

$$\sum_{\nu=1}^{i} y_\nu = \sum_{\nu=1}^{i} x_{\nu,1} + \sum_{\nu=1}^{i} x_{\nu,2} + \cdots + \sum_{\nu=1}^{i} x_{\nu,c}$$

so that

$$M(Y) = M(X_1) + M(X_2) + \cdots + M(X_c), \quad i = 1, 2, \ldots, N, \tag{13}$$

$$M_i^-(Y) = M_i^-(X_1) + M_i^-(X_2) + \cdots + M_i^-(X_c), \quad i = 1, 2, \ldots, N, \tag{14}$$

and

$$M_i^+(Y) = M_i^+(X_1) + M_i^+(X_2) + \cdots + M_i^+(X_c), \quad i = 1, 2, \ldots, N, \tag{15}$$

where $M(X_j)$, $M_i^-(X_j)$ and $M_i^+(X_j)$ are defined as $M(Y)$, $M_i^-(Y)$ and $M_i^+(Y)$, respectively, with $x_{i,j}$ in place of y_i. It is important to note that while $M_i^-(Y)$ is the mean of the i smallest values observed for total income Y, this is usually not the case for $M_i^-(X_j)$. In fact, $M_i^-(X_j)$ is the mean of the i smallest values observed for factor component X_j only if Y and X_j are perfectly rank correlated (the situation is analogous for $M_i^+(Y)$ and $M_i^+(X_j)$).

Using relations (13), (14), and (15) yields simple decomposition rules for the point inequality indexes R_i and I_i. In fact, it is easily seen that

$$R_i := \frac{M(Y) - M_i^-(Y)}{M(Y)} = \sum_{j=1}^{c} \frac{M(X_j) - M_i^-(X_j)}{M(Y)}, \quad i = 1, 2, \ldots, N,$$

and

$$I_i := \frac{M_i^+(Y) - M_i^-(Y)}{M(Y)} = \sum_{j=1}^{c} \frac{M^+(X_j) - M_i^-(X_j)}{M(Y)}, \quad i = 1, 2, \ldots, N,$$

so that

$$\mathcal{R}_i(X_j) := \frac{M(X_j) - M_i^-(X_j)}{M(Y)}, \quad j = 1, 2, \ldots, c \tag{16}$$

and

$$\mathcal{I}_i(X_j) := \frac{M_i^+(X_j) - M_i^-(X_j)}{M_i^+(Y)}, \quad j = 1, 2, \ldots, c \tag{17}$$

can be interpreted as contributions from the factor components X_j to R_i and I_i, respectively. The corresponding relative contributions have a very neat interpretation:

$$\rho_i(X_j) := \frac{\mathcal{R}_i(X_j)}{R_i} = \frac{M(X_j) - M_i^-(X_j)}{M(Y) - M_i^-(Y)}, \quad i = 1, 2, \ldots, N-1, \tag{18}$$

and

$$\zeta_i(X_j) := \frac{\mathcal{I}_i(X_j)}{I_i} = \frac{M_i^+(X_j) - M_i^-(X_j)}{M_i^+(Y) - M_i^-(Y)}, \quad i = 1, 2, \ldots, N, \tag{19}$$

are simply the contributions from factor component X_j to $M(Y) - M_i^-(Y)$ and to $M_i^+(Y) - M_i^-(Y)$, respectively (observe that ρ_N is not defined because $R_N = 0$). The interpretations of $\rho_i(X_j)$ and $\zeta_i(X_j)$ can actually be interchanged since for $i = 1, 2, \ldots, N-1$ these relative contributions are always the same. This perhaps unexpected result follows immediately from the fact that

$$\frac{N-i}{N}\left(M_i^+(\cdot)-M_i^-(\cdot)\right)=M(\cdot)-M_i^-(\cdot),\quad i=1,2,\ldots,N-1. \tag{20}$$

Based on the contributions $\mathcal{R}_i(X_j)$ and $\mathcal{I}_i(X_j)$, it is straightforward to define contributions to the corresponding synthetic inequality indexes as well. In fact,

$$R:=\frac{\sum_{i=1}^{N-1}R_i\,p_i}{\sum_{i=1}^{N-1}p_i}=\frac{\sum_{i=1}^{N-1}\sum_{j=1}^{c}\mathcal{R}_i(X_j)\,p_i}{\sum_{i=1}^{N-1}p_i}=\sum_{j=1}^{c}\frac{\sum_{i=1}^{N-1}\mathcal{R}_i(X_j)\,p_i}{\sum_{i=1}^{N-1}p_i},$$

$$B:=\frac{1}{N-1}\sum_{i=1}^{N-1}R_i=\frac{1}{N-1}\sum_{i=1}^{N-1}\sum_{j=1}^{c}\mathcal{R}_i(X_j)=\sum_{j=1}^{c}\frac{1}{N-1}\sum_{i=1}^{N-1}\mathcal{R}_i(X_j)$$

and

$$I:=\frac{1}{N}\sum_{s=1}^{k}I_{N_s}n_s=\frac{1}{N}\sum_{s=1}^{k}\sum_{j=1}^{c}\mathcal{I}_{N_s}(X_j)\,n_s=\sum_{j=1}^{c}\frac{1}{N}\sum_{s=1}^{k}\mathcal{I}_{N_s}(X_j)\,n_s,$$

and the expressions on the far right suggest to consider

$$\mathcal{R}(X_j):=\frac{\sum_{i=1}^{N-1}\mathcal{R}_i(X_j)\,p_i}{\sum_{i=1}^{N-1}p_i},\quad j=1,2,\ldots,c, \tag{21}$$

$$\mathcal{B}(X_j):=\frac{1}{N-1}\sum_{i=1}^{N-1}\mathcal{R}_i(X_j),\quad j=1,2,\ldots,c, \tag{22}$$

and

$$\mathcal{I}(X_j):=\frac{1}{N}\sum_{s=1}^{k}\mathcal{I}_{N_s}(X_j)\,n_s,\quad j=1,2,\ldots,c, \tag{23}$$

as contributions to the synthetic inequality indexes R, B, and I, respectively. The corresponding relative contributions are then given by

$$\rho(X_j):=\frac{\mathcal{R}(X_j)}{R}=\frac{\sum_{i=1}^{N-1}\mathcal{R}_i(X_j)\,p_i}{\sum_{i=1}^{N-1}R_i\,p_i}=\frac{\sum_{i=1}^{N-1}\rho_i(X_j)\,R_i\,p_i}{\sum_{i=1}^{N-1}R_i\,p_i} \tag{24}$$

$$\beta(X_j):=\frac{\mathcal{B}(X_j)}{B}=\frac{\sum_{i=1}^{N-1}\mathcal{R}_i(X_j)}{\sum_{i=1}^{N-1}R_i}=\frac{\sum_{i=1}^{N-1}\rho_i(X_j)\,R_i}{\sum_{i=1}^{N-1}R_i} \tag{25}$$

$$\zeta(X_j) := \frac{\mathcal{I}(X_j)}{I} = \frac{\sum_{s=1}^{k} \mathcal{I}_{N_s}(X_j)\, n_s}{\sum_{s=1}^{k} I_{N_s}\, n_s} = \frac{\sum_{i=1}^{N-1} \zeta_{N_s}(X_j)\, I_{N_s}\, n_s}{\sum_{i=1}^{N-1} I_{N_s}\, n_s}, \tag{26}$$

and are thus nothing else than weighted averages, with different sets of weights, of essentially the same relative contributions (recall $\rho_i(X_j) = \zeta_i(X_j)$ for $i = 1, 2, \ldots, N-1$ and for $j = 1, 2, \ldots, c$). $\rho(X_j)$, $\beta(X_j)$ and $\zeta(X_j)$ can thus be interpreted as average values of the contributions of the factor components X_j to the N differences $M(Y) - M_i^-(Y)$ or $M_i^+(Y) - M_i^-(Y)$.

However, there might be a nonuniqueness problem in the definitions of the contributions. The problem occurs if there are several population members with the same total income Y and with different incomes from two or more factor components X_j. In this case, the values of $M_i^-(X_j)$ and $M_i^+(X_j)$ for $i \neq N_1, N_2, \ldots, N_k$ depend on the i index assigned to the population members with same total income Y, and thus the corresponding contributions $\mathcal{R}_i(X_j)$ and $\mathcal{I}_i(X_j)$ depend on this assignment as well. It follows that $\mathcal{R}(X_j)$ and $\mathcal{B}(X_j)$ depend on the way in which the i indexes are assigned, while for $\mathcal{I}(X_j)$ this is not the case, because $\mathcal{I}(X_j)$ depends only on the contributions $\mathcal{I}_i(X_j)$ for $i = N_1, N_2, \ldots, N_k$. Even though in large populations with few repeated values for total income Y this dependence has little impact on the results, we propose an easy way to neutralize it: instead of the original definitions, one might consider modified versions of the Gini and Bonferroni indexes that are weighted averages of the point inequality measures R_i and B_i just for $i = N_1, N_2, \ldots, N_k$. A convenient modified version of the Gini index is for example given by

$$R' := \frac{\sum_{s=1}^{k} R_{N_s}\, r_s}{\sum_{s=1}^{k} r_s} \tag{27}$$

where

$$r_s := \begin{cases} N_s\,(n_s + n_{s+1}) & \text{if } 1 \leq s < k \\ N\, n_k & s = k \end{cases} \tag{28}$$

while for the Bonferroni index we suggest

$$B' := \frac{1}{N} \sum_{s=1}^{k} R_{N_s}\, n_s. \tag{29}$$

A few comments are due regarding the definitions of R' and B'. In first place we observe that in large populations with few repeated values R' and B' are close to R and B, respectively. Second, it is worth noting that the definitions of R' and B', as opposed to those of R and B, include the point inequality measure R_N even though $R_N = 0$ for every income distribution: we made this choice for ease of comparison with the Zenga index which depends on k point inequality measures as well. Finally, regarding the definition of R', it is not difficult to show that it coincides with the ratio between the "concentration area" and the area of triangle with vertices in (0, 0), (1, 0), and (1, 1) (see the proof in the Appendix).

The factor component contributions to R' and B' are obviously defined as

$$\mathcal{R}'(X_j) := \frac{\sum_{s=1}^{k} \mathcal{R}_{N_s}(X_j)\, r_s}{\sum_{s=1}^{k} r_s}, \quad j = 1, 2, \ldots, c, \tag{30}$$

and

$$\mathcal{B}'(X_j) := \frac{1}{N} \sum_{s=1}^{k} \mathcal{R}_{N_s}(X_j)\, n_s, \quad j = 1, 2, \ldots, c, \tag{31}$$

and the corresponding relative contributions are then given by

$$\rho'(X_j) := \frac{\mathcal{R}'(X_j)}{R'} = \frac{\sum_{s=1}^{k} \mathcal{R}_{N_s}(X_j)\, r_s}{\sum_{s=1}^{k} R_{N_s}\, r_s} = \frac{\sum_{s=1}^{k} \rho_{N_s}(X_j)\, R_{N_s}\, r_s}{\sum_{s=1}^{k} R_{N_s}\, r_s} \tag{32}$$

and

$$\beta'(X_j) := \frac{\mathcal{B}'(X_j)}{B'} = \frac{\sum_{s=1}^{k} \mathcal{R}_{N_s}(X_j)\, n_s}{\sum_{s=1}^{k} R_{N_s}\, n_s} = \frac{\sum_{s=1}^{k} \rho_{N_s}(X_j)\, R_{N_s}\, n_s}{\sum_{s=1}^{k} R_{N_s}\, n_s}. \tag{33}$$

4. Contributions to Inequality and Shares on Population Income

As suggested by Zenga et al. (2012), it is instructive to compare the relative contributions $\rho_i(X_j)$ and $\zeta_i(X_j)$ and their weighted averages $\rho(X_j)$, $\beta(X_j)$, and $\zeta(X_j)$ (as well as $\rho'(X_j)$ and $\beta'(X_j)$) with the share

$$\gamma(X_j) := \frac{\sum_{i=1}^{N} x_{i,j}}{\sum_{i=1}^{N} y_i} \tag{34}$$

of their corresponding factor component X_j on total population income. In fact, in the hypothetical case, the so-called *scale transformation hypothesis*, where

$$x_{i,j} = \gamma(X_j)\, y_i \quad \text{for every } i = 1, 2, \ldots, N,$$

one would have

$$M_i^-(X_j) = \gamma(X_j)\, M_i^-(Y) \quad \text{and} \quad M_i^+(X_j) = \gamma(X_j)\, M_i^+(Y)$$

for all $i = 1, 2, \ldots, N$, so that

$$\rho_i(X_j) = \zeta_i(X_j) = \gamma(X_j) \quad \text{for } i = 1, 2, \ldots, N-1 \text{ and } \zeta_N = \gamma(X_j).$$

In this case it follows that

$$\rho(X_j) = \beta(X_j) = \zeta(X_j) = \gamma(X_j).$$

In real income distributions one should obviously expect that

$$x_{i,j} \neq \gamma(X_j)\, y_i$$

for most population members i, but since the deviations $x_{i,j} - \gamma(X_j)\, y_i$ must sum (over i) to zero, the scale transformation hypothesis provides a useful benchmark against which to compare the actual distribution of the factor components. For illustrative purposes we shall next describe two types of deviations from the scale transformation hypothesis that are helpful for the interpretation of the relative contributions:

- First, consider the case where

  $$x_{i,j} < \gamma(X_j)\, y_i \quad \text{for } i = 1, 2, \ldots, i^* < N$$

  and

  $$x_{i,j} \geq \gamma(X_j)\, y_i \quad \text{for } i = i^* + 1, i^* + 2, \ldots, N.$$

  Since y_i is nondecreasing in i, we can describe this as a situation where *all* population members with total income Y below a given threshold value y_{i^*} have less income from factor component X_j than they would have under the scale transformation hypothesis, while *all* other (more fortunate) population members have at least as much income from X_j as they would have under the scale transformation hypothesis. It is not difficult to show that in this case

  $$M_i^-(X_j) < \gamma(X_j)\, M_i^-(Y) \quad \text{and} \quad M_i^+(X_j) > \gamma(X_j)\, M_i^+(Y) \tag{35}$$

  for all $i = 1, 2, \ldots, N$, so that

  $$\rho_i(X_j) = \zeta_i(X_j) > \gamma(X_j) \ \text{ for } i = 1, 2, \ldots, N-1 \quad \text{and} \quad \zeta_N > \gamma(X_j).$$

  From these inequalities it follows that

  $$\rho(X_j) > \gamma(X_j), \quad \beta(X_j) > \gamma(X_j) \quad \text{and} \quad \zeta(X_j) > \gamma(X_j). \tag{36}$$

  The first two inequalities hold also with $\rho'(X_j)$ and $\beta'(X_j)$ in place of $\rho(X_j)$ and $\beta(X_j)$, respectively.
- The second case is opposite to the first one. It occurs when

  $$x_{i,j} > \gamma(X_j)\, y_i \quad \text{for } i = 1, 2, \ldots, i^* < N$$

  and

  $$x_{i,j} \leq \gamma(X_j)\, y_i \quad \text{for } i = i^* + 1, i^* + 2, \ldots, N.$$

  In this case,

  $$M_i^-(X_j) > \gamma(X_j)\, M_i^-(Y) \quad \text{and} \quad M_i^+(X_j) < \gamma(X_j)\, M_i^+(Y) \tag{37}$$

  for all $i = 1, 2, \ldots, N$, so that

  $$\rho_i(X_j) = \zeta_i(X_j) < \gamma(X_j) \quad \text{for } i = 1, 2, \ldots, N-1 \ \text{ and } \quad \zeta_N < \gamma(X_j).$$

  Therefore it follows that

  $$\rho(X_j) < \gamma(X_j), \quad \beta(X_j) < \gamma(X_j) \quad \text{and} \quad \zeta(X_j) < \gamma(X_j). \tag{38}$$

  Also here, the first two inequalities hold also with $\rho'(X_j)$ and $\beta'(X_j)$ in place of $\rho(X_j)$ and $\beta(X_j)$, respectively.

The two cases described above are somewhat artificial in that they require that *all* population members with total income below (above) a certain threshold value have smaller (larger) income from factor component X_j than they would have under the scale transformation hypothesis. Nevertheless, we can regard the inequalities in (36) (and in (38)) as symptomatic for situations where income from a given factor component X_j tends to be more concentrated among population members with large (small) total income Y than total income Y itself. In fact, if $\gamma(X_j)$ is positive (which is usually the case), the inequalities in (35) imply that

$$\frac{\sum_{\nu=1}^{i} x_{\nu,j}}{\sum_{\nu=1}^{N} x_{\nu,j}} < \frac{\sum_{\nu=1}^{i} y_\nu}{\sum_{\nu=1}^{N} y_\nu}$$

and

$$\frac{\sum_{\nu=i+1}^{N} x_{\nu,j}}{\sum_{\nu=1}^{N} x_{\nu,j}} > \frac{\sum_{\nu=i+1}^{N} y_\nu}{\sum_{\nu=1}^{N} y_\nu}$$

for $1 \leq i \leq N - 1$, while those in (37) imply that

$$\frac{\sum_{\nu=1}^{i} x_{\nu,j}}{\sum_{\nu=1}^{N} x_{\nu,j}} > \frac{\sum_{\nu=1}^{i} y_\nu}{\sum_{\nu=1}^{N} y_\nu}$$

and

$$\frac{\sum_{\nu=i+1}^{N} x_{\nu,j}}{\sum_{\nu=1}^{N} x_{\nu,j}} < \frac{\sum_{\nu=i+1}^{N} y_\nu}{\sum_{\nu=1}^{N} y_\nu}$$

for $1 \leq i \leq N - 1$.

5. Estimation from Survey Data

The definitions of the Gini, Bonferroni, and Zenga indexes and the decomposition rules outlined in Section 3 can be directly applied to *population data*. In this section we propose estimators which can be applied to *survey data* and which should be reasonably well-behaved for a broad class of sample designs. So let

$$S = \{i_1, i_2, \ldots, i_d\} \tag{39}$$

denote a set of indexes corresponding to a sample of d units drawn from the population $\mathcal{U} = \{1, 2, \ldots, N\}$ and let

$$w_{i_1}, w_{i_2}, \ldots, w_{i_d} \tag{40}$$

denote survey weights corresponding to the d sample units in S. In what follows we shall assume that the survey weights w_i are strictly positive and that they are scaled so that

$$\sum_{i \in S} w_i = N. \tag{41}$$

The estimators we shall propose below do not actually depend on how the survey weights are scaled. Assumption (41) is only needed to make the estimators look more similar to their corresponding population quantities.

Now, suppose there are $\hat{k} \leq d$ different values for total income Y among the d observed values in the sample, and denote these values by

$$\tilde{y}_1 < \tilde{y}_2 < \cdots < \tilde{y}_{\hat{k}}. \tag{42}$$

For $s = 1, 2, \ldots, \hat{k}$, let

$$\hat{n}_s := \sum_{i \in S: y_i = \tilde{y}_s} w_i \tag{43}$$

denote the sum of the survey weights w_i corresponding to the sample units with total income Y equal to $\tilde{y}_s$. Moreover, let

$$\hat{N}_s := \sum_{\nu=1}^{s} \hat{n}_\nu, \quad s = 1, 2, \ldots, \hat{k}, \tag{44}$$

denote the corresponding cumulative weights. Obviously, $\hat{N}_{\hat{k}} = N$. Based on the cumulative weights define

$$\sigma(p) := \min\{s : \hat{N}_{\hat{s}} \geq Np\}, \quad p \in [0, 1]. \tag{45}$$

Then, use $\sigma(p)$ to define

$$\hat{M}_p^-(Y) := \frac{\sum_{s=1}^{\sigma(p)} \tilde{y}_s \hat{n}_s}{\sum_{s=1}^{\sigma(p)} \hat{n}_s}, \tag{46}$$

$$\hat{M}_p^+(Y) := \begin{cases} \dfrac{\sum_{s=\sigma(p)+1}^{\hat{k}} \tilde{y}_s \hat{n}_s}{\sum_{s=\sigma(p)+1}^{\hat{k}} \hat{n}_s} & \text{if } \sigma(p) < \hat{k}, \\ \tilde{y}_{\hat{k}} & \text{if } \sigma(p) = \hat{k}, \end{cases} \tag{47}$$

and observe that $\hat{M}_p^-(Y)$ and $\hat{M}_p^+(Y)$ at $p = i/N$ can be taken as estimators for $M_i^-(Y)$ and $M_i^+(Y)$, respectively. Note, however, that the estimators $\hat{M}_p^-(Y)$ and $\hat{M}_p^+(Y)$ are defined for every $p \in [0, 1]$ and that they give rise to right continuous step functions with discontinuities at $p = \hat{N}_s/N$ for $s = 1, 2, \ldots, \hat{k}$. Obviously, $\hat{M}_p^-(Y)$ at $p = 1$ is equal to the weighted sample mean

$$\hat{M}(Y) := \frac{\sum_{s=1}^{\hat{k}} \tilde{y}_s \hat{n}_s}{\sum_{s=1}^{\hat{k}} \hat{n}_s}. \tag{48}$$

On the other hand, $\hat{M}_p^+(Y)$ at $p = 0$ is larger than the weighted sample mean $\hat{M}(Y)$, unless there are no different values for total income Y in the sample in which case $\hat{M}_p^+(Y)$ would not be defined for any $p \in [0, 1]$. The latter case is obviously not of interest in applications.

To estimate the point inequality measures, let

$$\hat{R}_p := \frac{\hat{M}_p^-(Y) - \hat{M}(Y)}{\hat{M}(Y)} \tag{49}$$

and

$$\hat{I}_p := \frac{\hat{M}_p^+(Y) - \hat{M}_p^-(Y)}{\hat{M}_{,p}^+(Y)}, \tag{50}$$

and, as before, put $p = i/N$ to get estimators for R_i and I_i, respectively.

Next, consider the synthetic inequality indexes. To define an estimator for R', let

$$\hat{r}_s := \begin{cases} \hat{N}_s\,(\hat{n}_s + \hat{n}_{s+1}) & \text{if } 1 \leq s < \hat{k} \\ \hat{N}_{\hat{k}}\,\hat{n}_k & \text{if } s = \hat{k} \end{cases} \tag{51}$$

and use $\hat{r}_s$ in place of the weights r_s and $\hat{R}_p$ at $p = \hat{N}_s/N$ in place of R_{N_s} in the definition of R'. The resulting estimator is then given by

$$\hat{R}' := \frac{\sum_{s=1}^{\hat{k}} \hat{R}_{\hat{N}_s/N}\,\hat{r}_s}{\sum_{s=1}^{\hat{k}} \hat{r}_s} \tag{52}$$

and, under suitable conditions, it can be used to estimate R as well. Similar reasoning suggests that B' and B can be estimated by

$$\hat{B}' := \frac{1}{\hat{N}_{\hat{k}}} \sum_{s=1}^{\hat{k}} \hat{R}_{\hat{N}_s/N}\,\hat{n}_s \tag{53}$$

and that

$$\hat{I} := \frac{1}{\hat{N}_{\hat{k}}} \sum_{s=1}^{\hat{k}} \hat{I}_{\hat{N}_s/N}\,\hat{n}_s. \tag{54}$$

can be used to estimate I.

Now, consider the population quantities involving the factor components X_j. For their estimation we shall employ the weighted averages given by

$$\tilde{x}_{s,j} := \frac{\sum_{i \in S: y_i = \tilde{y}_s} x_{i,j}\,w_i}{\sum_{i \in S: y_i = \tilde{y}_s} w_i} = \frac{1}{\hat{n}_s} \sum_{i \in S: y_i = \tilde{y}_s} x_{i,j}\,w_i, \tag{55}$$

for $s = 1, 2, \ldots, \hat{k}$ and $j = 1, 2, \ldots, c$. Note that $\tilde{x}_{s,j}$ is the weighted average of income from factor component X_j among the sample units with total income equal to $\tilde{y}_s$. Using $\hat{M}_p^-(X_j)$ and $\hat{M}_p^+(X_j)$ to indicate $\hat{M}_p^-(Y)$ and $\hat{M}_p^+(Y)$ with $\tilde{x}_{s,j}$ in place of $\tilde{y}_s$, we define the step functions

$$\hat{\mathcal{R}}_p(X_j) := \frac{\hat{M}_p^-(X_j) - \hat{M}(X_j)}{\hat{M}(Y)} \tag{56}$$

and

$$\hat{\mathcal{I}}_p(X_j) := \frac{\hat{M}_p^+(X_j) - \hat{M}_p^-(X_j)}{\hat{M}_p^+(Y)}, \tag{57}$$

which, at $p = i/N$, provide estimates for the contributions $\mathcal{R}_i(X_j)$ and $\mathcal{I}_i(X_j)$. Based on the step functions $\hat{\mathcal{R}}_p(X_j)$ and $\hat{\mathcal{I}}_p(X_j)$ we further construct estimators for the contributions $\mathcal{R}'(X_j)$, $\mathcal{B}'(X_j)$ and $\mathcal{I}(X_j)$. These are given by

$$\hat{\mathcal{R}}'(X_j) := \frac{\sum_{s=1}^{\hat{k}} \hat{\mathcal{R}}_{\hat{N}_s/N}(X_j)\,\hat{r}_s}{\sum_{s=1}^{\hat{k}} \hat{r}_s}, \tag{58}$$

$$\hat{\mathcal{B}}'(X_j) := \frac{1}{\hat{N}_{\hat{k}}} \sum_{s=1}^{\hat{k}} \hat{\mathcal{R}}_{\hat{N}_s/N}(X_j)\,\hat{n}_s \tag{59}$$

and

$$\hat{\mathcal{I}}(X_j) := \frac{1}{\hat{N}_{\hat{k}}} \sum_{s=1}^{\kappa} \hat{\mathcal{I}}_{\hat{N}_s/N}(X_j)\,\hat{n}_s \tag{60}$$

Also here, under suitable conditions, we can regard $\hat{\mathcal{R}}'(X_j)$ and $\hat{\mathcal{B}}'(X_j)$ as well as estimators of $\mathcal{R}(X_j)$ and $\mathcal{B}(X_j)$, respectively.

Since

$$\sum_{j=1}^{c} \hat{M}_p^-(X_j) = \hat{M}_p^-(Y) \quad \text{and} \quad \sum_{j=1}^{c} \hat{M}_p^+(X_j) = \hat{M}_p^+(Y)$$

for every $p \in [0, 1]$ (as for the corresponding population quantities), it follows that the relations

$$\sum_{j=1}^{c} \hat{\mathcal{R}}_p(X_j) = \hat{R}_p, \quad \sum_{j=1}^{c} \hat{\mathcal{R}}'(X_j) = \hat{R}', \quad \sum_{j=1}^{c} \hat{\mathcal{B}}'(X_j) = \hat{B}'$$

and

$$\sum_{j=1}^{c} \hat{\mathcal{I}}_p(X_j) = \hat{I}_p, \quad \sum_{j=1}^{c} \hat{\mathcal{I}}(X_j) = \hat{I}.$$

hold true for the estimators as well.

To estimate the relative contributions to the point inequality measures we can use the values taken on by the step functions

$$\hat{\rho}_p(X_j) := \frac{\hat{\mathcal{R}}_p(X_j)}{\hat{R}_p} = \frac{\hat{M}_p^-(X_j) - \hat{M}(X_j)}{\hat{M}_p^-(Y) - \hat{M}(Y)} \tag{61}$$

and

$$\hat{\zeta}_p(X_j) := \frac{\hat{\mathcal{I}}_p(X_j)}{\hat{I}_p} = \frac{\hat{M}_p^+(X_j) - \hat{M}_p^-(X_j)}{\hat{M}_p^+(Y) - \hat{M}_p^-(Y)} \tag{62}$$

at $p = i/N$ for $i = 1, 2, \ldots, N$. Note that, as for the corresponding population quantities, $\hat{\rho}_p(X_j)$ is not defined for $p \in (\hat{N}_{\hat{k}-1}/N, 1]$, and that for $p \in [0, \hat{N}_{\hat{k}-1}/N]$

$$\hat{\rho}_p(X_j) = \hat{\zeta}_p(X_j), \quad j = 1, 2, \ldots, c,$$

since an obvious generalization of relation (20) holds for weighted means as well. Taking appropriate averages finally yields

$$\hat{\rho}'(X_j) := \frac{\hat{\mathcal{R}}'(X_j)}{\hat{R}'} = \frac{\sum_{s=1}^{\hat{k}} \hat{\rho}_{\hat{N}_s/N}(X_j)\, \hat{R}_{\hat{N}_s/N}\, \hat{r}_s}{\sum_{s=1}^{\hat{k}} \hat{R}_{\hat{N}_s/N}\, \hat{r}_s}, \tag{63}$$

$$\hat{\beta}'(X_j) := \frac{\hat{\mathcal{B}}'(X_j)}{\hat{B}} = \frac{\sum_{s=1}^{\hat{k}} \hat{\rho}_{\hat{N}_s/N}(X_j)\, \hat{R}_{\hat{N}_s/N}\, \hat{n}_s}{\sum_{s=1}^{\hat{k}} \hat{R}_{\hat{N}_s/N}\, \hat{n}_s} \tag{64}$$

and

$$\hat{\zeta}(X_j) := \frac{\hat{\mathcal{R}}(X_j)}{\hat{R}} = \frac{\sum_{s=1}^{\hat{k}} \hat{\rho}_{\hat{N}_s/N}(X_j)\, \hat{R}_{\hat{N}_s/N}\, \hat{n}_s}{\sum_{s=1}^{\hat{k}} \hat{R}_{\hat{N}_s/N}\, \hat{n}_s} \tag{65}$$

as estimators of $\rho'(X_j)$, $\beta'(X_j)$ and $\zeta(X_j)$. Again, under suitable conditions, the former two estimators can be used to estimate $\rho(X_j)$ and $\beta(X_j)$ as well.

Finally, to estimate the shares $\gamma(X_j)$, one can simply use

$$\hat{\gamma}(X_j) := \frac{\sum_{i \in S} x_{i,j}\, w_i}{\sum_{i \in S} w_i} = \frac{\sum_{s=1}^{\hat{k}} \tilde{x}_{s,j}\, \hat{n}_s}{\sum_{s=1}^{\hat{k}} \hat{n}_s}, \quad j = 1, 2, \ldots, c. \tag{66}$$

It is not difficult to check that the relations between the relative contributions and the shares outlined in Section 4 hold for the estimates obtained from the estimators defined in the present section as well.

6. Application to ECHP Income Data

The European Community Household Panel (ECHP) is a multi-purpose annual longitudinal survey covering the time span between 1994 and 2001. Its aim is to provide comparable information from EU countries. It is centrally designed and coordinated by Eurostat and covers topics such as demographics, labor force behavior, income, health, education and training, housing, migration, and so on. The objective of the ECHP is to represent the population of the EU at individual and household level. More information about this survey may be found in the accompanying documentation (see Eurostat 1996; Eurostat 2003a; Eurostat 2003b; Eurostat 2002; Eurostat 2003c; Eurostat 2003d; Eurostat 2003e).

In the present work we analyze data about household income from the Users' Database (UDB) referring to the 2001 wave of the ECHP. Information on income is collected very detailed in the ECHP questionnaire. Some of the income components are collected at household level, while others are collected for each individual in sample households. In order to have complete information at both household and individual level, household income components are shared among its members aged over 16, and personal income components are aggregated for the whole household. To be specific, income components collected at household level are: property and rental income, social assistance and housing allowances. All other income components are collected individually among persons aged over 16 who reside in sample households. As for taxes, some of the income components are collected net and others gross of taxes. To allow for the computation of comparable net values, the survey provides net/gross ratios for each household (variable *HI020* in the Household-file of the UDB; except for the country-specific informations provided in Table 2, all other variables listed in this work are included in the Household-file).

Below we shall apply the estimators of Section 5 to evaluate the contributions from several income components to inequality in the distribution of total net household income (variable *HI100*). To avoid excessive scattering of the contributions among a large number of income components, we shall aggregate the latter into four main components:

- ***Wage and salary income*** (X_1 := variable *HI111*). This income component includes wages and salary payments and any other form of pay for work as an employee or apprentice.
- ***Self-employment income*** (X_2 := variable *HI112*). This includes any income from self-employment such as own business, professional practice or farm, working as free-lance or subcontractor, providing services or selling goods on own account.
- ***Other income components*** (X_3 := the sum of variables *HI121*, *HI122*, *HI123* and *HI140*). This includes capital income (variable *HI121*), income from property and rents (variable *HI122*), private transfers (variable *HI123*) and adjustments for within household non-response (variable *HI140*).
- ***Social transfers*** (X_4 := variable *HI130*). This includes unemployment related benefits, pension or benefit relating to old-age or retirement, survivor's pension or benefits for widows or orphans, family related benefits, benefits relating to sickness or invalidity, education related allowances and any other social benefits.

Except for the samples from France and Finland, the variables *HIxxx* in the UDB contain amounts of income net of taxes. For households where these variables are filled (the variables referring to the income components are always filled if the net household income variable *HI100* is filled; however, for all countries, except Luxembourg, there are a few households where the value of the net household income variable is missing), the reported net values are consistent in the sense that

$$\begin{aligned}
&\text{net household income } (Y := \mathit{HI100}) := \\
&:= \mathit{wage\ and\ salary\ income}\ (X_1 := \mathit{HI111}) + \\
&\quad + \mathit{self\ employment\ income}\ (X_2 := \mathit{HI112}) + \\
&\quad + \mathit{other\ income\ components} \\
&\qquad (X_3 := \mathit{HI121} + \mathit{HI122} + \mathit{HI123} + \mathit{HI140}) + \\
&\quad + \mathit{social\ transfers\ income}\ (X_4 := \mathit{HI130}).
\end{aligned}$$

For households belonging to the samples from France and Finland, the variables *HI111*, *HI112*, *HI130*, *HI121*, *HI122*, and *HI123* report gross values, which must be converted into net values through multiplication by variable *HI020* (the household net/gross ratio), while all other variables *HIxxx* still contain net values. Thus, for the households included in the samples from France and Finland,

$$\begin{aligned}
&\text{net household income } (Y := \mathit{HI100}) := \\
&:= \mathit{wage\ and\ salary\ income}\ (X_1 := \mathit{HI111}) + \\
&\quad + \mathit{self\ employment\ income}\ (X_2 := \mathit{HI020} \times \mathit{HI112}) + \\
&\quad + \mathit{other\ income\ components} \\
&\quad (X_3 := \mathit{HI020} \times (\mathit{HI121} + \mathit{HI122} + \mathit{HI123}) + \mathit{HI140}) \\
&\quad + \mathit{social\ transfers\ income}\ (X_4 := \mathit{HI020} \times \mathit{HI130}).
\end{aligned}$$

Finally, as for the sample weights w_i, we shall follow a suggestion given in Eurostat (2003a) and use the cross-sectional household weights provided in the Household-file of the UDB (variable *HG004*). In fact, in the ECHP each household with completed household interview has its own nonnegative cross-sectional household weight *HG004*, and these weights are scaled to make sure that their sum over all interviewed households in each country equals the number d^* of interviewed households within the country. However, since for all countries except Luxembourg there are some sample households for which the net household income variable $Y := HI100$ is not filled, the final samples S we shall use for estimation comprise $d \leq d^*$ households. Table 1 reports the values of d^*, d and the relative weight θ of the sample households for which the total net household income *HI100* is missing (i.e., θ is the ratio between the sum of the cross-sectional household weights for sample households where the total net income variable *HI100* is missing and d^*). Note that there is no country for which θ exceeds two percent.

Now, consider Table 2. For each of the 15 countries included in the ECHP, Table 2 reports the population size, the number of households and the average household size as from the Country-file included in the UDB provided by Eurostat. Besides this general informations, Table 2 reports also the final sample sizes d used for estimation and some estimates regarding the distribution of net household income Y. The estimates for the

Table 1. Sample sizes in the 2001 wave of the ECHP.

Country	d^*	d	$(d^* - d)/d^*$	θ
Ireland	1,760	1,757	0.002	0.001
Denmark	2,283	2,279	0.002	0.001
Belgium	2,362	2,342	0.008	0.010
Luxembourg	2,428	2,428	0.000	0.000
Austria	2,544	2,535	0.004	0.002
Finland	3,115	3,106	0.003	0.002
Greece	3,916	3,895	0.005	0.006
Portugal	4,614	4,588	0.006	0.005
UK	4,819	4,779	0.008	0.009
Netherlands	4,851	4,824	0.006	0.005
Spain	4,966	4,950	0.003	0.003
Sweden	5,680	5,085	0.105	0.020
France	5,345	5,247	0.018	0.015
Italy	5,606	5,525	0.014	0.012
Germany	5,563	5,559	0.001	0.003

Legend: d^* is the number of interviewed households which coincides with the sum of the cross-sectional household weights $HG004$; d is the number of households used for estimation of the inequality indexes and the contributions to inequality from the four factor components, that is, number of households for which the net household income variable $Y := HI100$ is filled; θ is the ratio between the sum of the cross-sectional household weights for which Y is not filled and d^*.

median were obtained from the estimator

$$\widehat{Median}(Y) := \tilde{y}_{s^*},$$

where, in the notation of Section 5, s^* is the smallest integer s, $1 \leq s \leq \hat{k}$, such that $\hat{N}_{s^*}/N \geq 0.5$. Observe that the countries in Table 2 are ordered according to the estimates $\hat{R}'$ of the Gini index.

Next, consider the contributions in Table 3:

- ***Wage and salary income***, with shares $\hat{\gamma}(X_1)$ between 0.482 in Greece and 0.680 in Denmark, accounts for the largest share on total population income Y in all 15 countries. To understand how this factor component affects inequality, we first observe that the contributions $\hat{\rho}'(X_1)$, $\hat{\beta}'(X_1)$ and $\hat{\zeta}(X_1)$ are clearly larger than $\hat{\gamma}(X_1)$ which suggests that wage and salary income tends to be more concentrated among high income households than total income Y itself.

  To assess the impact on inequality at different levels p of the income distribution, we shall next examine the relative contributions $\hat{\rho}_p(X_1)$: we find that $\hat{\rho}_p(X_1) > \hat{\gamma}(X_1)$ for all countries for all values of p reported in Table 3, and that the trend of $\hat{\rho}_p(X_1)$ is quite similar in all countries: $\hat{\rho}_p(X_1)$ tends to increase for $0 < p \leq 0.25$ and to decrease for $p > 0.75$. For the interpretation of the relative contributions, recall that $\hat{\rho}_p(X_1)$ is the ratio between $M_p^+(X_1) - M_p^-(X_1)$ and $M_p^+(Y) - M_p^-(Y)$. In Italy, for example, $\hat{\rho}_{0.50}(X_1) = 0.661$ indicates that the difference between the means of wage and salary income among the households belonging to the upper half of the income distribution an those belonging to the lower half is equal to 0.661 times the difference between the corresponding means of total income Y.

Table 2. General information about countries included in the 2001 wave of the ECHP.

Country	Population size (in milions)	Number of households (in milions)	Average household size	Sample size d	$\widehat{Median}(Y)$	$\hat{M}(Y)$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
Denmark	5.368	2.456	2.19	2279	33561	34597	0.302	0.435	0.646
Netherlands	15.773	6.889	2.29	4824	22331	24788	0.303	0.428	0.643
Luxembourg	0.433	0.172	2.52	2428	38333	44729	0.304	0.414	0.631
Austria	7.986	3.3	2.42	2535	25058	28543	0.328	0.456	0.672
France	57.949	24.523	2.36	5247	24408	28053	0.329	0.457	0.674
Sweden	8.663	4.576	1.89	5085	20389	23651	0.331	0.459	0.677
Germany	81.569	37.711	2.16	5559	24554	28486	0.336	0.460	0.679
Italy	57.388	21.967	2.61	5525	18179	21210	0.342	0.471	0.688
Finland	5.12	2.381	2.15	3106	21067	24801	0.350	0.481	0.697
Belgium	10.263	4.278	2.4	2342	25558	30374	0.354	0.472	0.694
United Kingdom	59.063	25.564	2.31	4779	26893	32151	0.369	0.499	0.717
Greece	10.354	3.993	2.59	3895	12208	14853	0.382	0.517	0.734
Ireland	3.839	1.291	2.97	1757	25457	30685	0.388	0.524	0.740
Spain	39.137	13.281	2.95	4950	16810	21453	0.399	0.526	0.745
Portugal	10.024	3.391	2.96	4588	12362	15661	0.402	0.530	0.749

The estimates for the median and the mean of net household income Y are expressed in euros. They have been obtained using the fixed conversion rates for Germany, Denmark, Netherlands, Luxembourg, France, UK, Ireland, Italy, Greece, Spain, Portugal, and Austria and using the conversion rates for the year 2001 as given in the Country-file of the ECHP for Belgium, Finland, and Sweden.

Table 3. Contributions to inequality from income factor components.

	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
						Austria					
X_1	0.622	0.740	0.802	0.834	0.857	0.817	0.811	0.761	0.834	0.818	0.818
X_2	0.061	0.063	0.075	0.086	0.094	0.118	0.111	0.132	0.097	0.087	0.095
X_3	0.032	0.006	0.020	0.025	0.031	0.046	0.056	0.056	0.036	0.028	0.033
X_4	0.284	0.191	0.103	0.055	0.018	0.019	0.022	0.050	0.033	0.066	0.053
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
$\hat{R}_p$		0.810	0.751	0.630	0.460	0.280	0.147	0.091	0.328	0.456	–
$\hat{I}_p$		0.817	0.770	0.694	0.630	0.609	0.633	0.664	–	–	0.672
						Belgium					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.550	0.672	0.724	0.776	0.787	0.696	0.544	0.431	0.720	0.731	0.692
X_2	0.071	0.081	0.086	0.103	0.130	0.172	0.230	0.301	0.149	0.122	0.152
X_3	0.108	0.129	0.135	0.139	0.150	0.172	0.219	0.260	0.163	0.149	0.166
X_4	0.271	0.118	0.055	−0.019	−0.067	−0.040	0.007	0.008	−0.032	−0.001	−0.009
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
$\hat{R}_p$		0.787	0.733	0.635	0.488	0.310	0.182	0.126	0.354	0.472	–
$\hat{I}_p$		0.795	0.752	0.699	0.656	0.643	0.689	0.742	–	–	0.694
						Denmark					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.680	0.762	0.822	0.928	1.018	0.961	0.773	0.675	0.945	0.908	0.890
X_2	0.051	0.061	0.066	0.079	0.095	0.110	0.175	0.217	0.102	0.086	0.108
X_3	0.043	0.039	0.038	0.040	0.036	0.038	0.067	0.064	0.039	0.038	0.042
X_4	0.226	0.138	0.073	−0.047	−0.149	−0.109	−0.014	0.044	−0.086	−0.032	−0.040
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
$\hat{R}_p$		0.816	0.748	0.619	0.437	0.240	0.120	0.075	0.302	0.435	–
$\hat{I}_p$		0.823	0.768	0.684	0.608	0.558	0.577	0.619	–	–	0.646

Table 3. Continued.

					Finland						
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.612	0.676	0.725	0.808	0.814	0.809	0.707	0.608	0.791	0.772	0.758
X_2	0.070	0.078	0.084	0.094	0.110	0.137	0.172	0.211	0.119	0.103	0.117
X_3	0.053	0.053	0.060	0.065	0.075	0.099	0.161	0.231	0.090	0.075	0.098
X_4	0.266	0.193	0.132	0.034	0.000	−0.045	−0.040	−0.051	0.001	0.050	0.027
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}_p$	0.821	0.763	0.666	0.500	0.298	0.157	0.098	0.350	0.481	–
	$\hat{I}_p$	0.829	0.781	0.727	0.666	0.629	0.649	0.685	–	–	0.697
					France						
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.604	0.695	0.748	0.780	0.830	0.815	0.777	0.744	0.803	0.777	0.779
X_2	0.063	0.069	0.072	0.083	0.090	0.111	0.148	0.182	0.100	0.087	0.102
X_3	0.044	0.030	0.031	0.036	0.036	0.041	0.036	0.038	0.038	0.036	0.037
X_4	0.290	0.207	0.148	0.101	0.045	0.033	0.0390	0.036	0.060	0.100	0.082
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}'_p$	0.823	0.749	0.623	0.458	0.284	0.153	0.096	0.329	0.457	–
	$\hat{I}_p$	0.831	0.768	0.688	0.628	0.614	0.643	0.679	–	–	0.674
					Germany						
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.529	0.622	0.658	0.687	0.712	0.648	0.555	0.454	0.667	0.667	0.641
X_2	0.090	0.109	0.120	0.128	0.161	0.212	0.268	0.327	0.178	0.151	0.183
X_3	0.076	0.065	0.076	0.091	0.096	0.128	0.129	0.133	0.108	0.095	0.105
X_4	0.305	0.204	0.146	0.094	0.031	0.012	0.048	0.085	0.047	0.088	0.071
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}_p$	0.797	0.738	0.628	0.467	0.292	0.162	0.105	0.336	0.460	–
	$\hat{I}_p$	0.805	0.758	0.692	0.637	0.622	0.658	0.698	–	–	0.679

Table 3. Continued.

						Greece					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.482	0.539	0.569	0.649	0.681	0.654	0.588	0.587	0.647	0.627	0.621
X_2	0.210	0.226	0.230	0.250	0.220	0.217	0.251	0.280	0.233	0.233	0.239
X_3	0.058	0.054	0.054	0.052	0.065	0.074	0.100	0.098	0.070	0.062	0.071
X_4	0.250	0.181	0.147	0.050	0.034	0.054	0.061	0.035	0.050	0.079	0.069
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}_p$	0.878	0.821	0.706	0.532	0.336	0.182	0.116	0.382	0.517	–
	$\hat{I}_p$	0.884	0.836	0.762	0.695	0.669	0.689	0.723	–	–	0.734
						Ireland					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.615	0.720	0.732	0.784	0.755	0.691	0.611	0.588	0.732	0.742	0.715
X_2	0.137	0.153	0.162	0.167	0.200	0.240	0.311	0.361	0.214	0.189	0.219
X_3	0.059	0.063	0.067	0.067	0.072	0.071	0.075	0.056	0.068	0.067	0.066
X_4	0.190	0.064	0.039	−0.019	−0.028	−0.002	0.003	−0.004	−0.014	0.002	0.000
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}'_p$	0.847	0.820	0.734	0.548	0.336	0.178	0.111	0.388	0.524	–
	$\hat{I}_p$	0.853	0.835	0.786	0.708	0.669	0.683	0.713	–	–	0.740
						Italy					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.505	0.586	0.617	0.688	0.661	0.631	0.542	0.479	0.642	0.639	0.621
X_2	0.162	0.182	0.189	0.197	0.220	0.229	0.306	0.347	0.226	0.208	0.230
X_3	0.041	0.022	0.029	0.040	0.053	0.063	0.080	0.098	0.055	0.045	0.054
X_4	0.293	0.211	0.166	0.075	0.067	0.077	0.072	0.077	0.077	0.108	0.095
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}_p$	0.831	0.769	0.644	0.480	0.297	0.159	0.098	0.342	0.471	–
	$\hat{I}_p$	0.838	0.787	0.707	0.648	0.626	0.653	0.685	–	–	0.689

Table 3. Continued.

						Luxembourg					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}^{l}(\cdot)$	$\hat{\beta}^{l}(\cdot)$	$\zeta(\cdot)$
X_1	0.635	0.733	0.771	0.859	0.882	0.847	0.737	0.674	0.843	0.825	0.812
X_2	0.042	0.054	0.060	0.065	0.078	0.115	0.143	0.155	0.092	0.077	0.091
X_3	0.050	0.063	0.062	0.064	0.071	0.085	0.125	0.169	0.079	0.071	0.085
X_4	0.273	0.151	0.107	0.012	−0.032	−0.047	−0.005	0.002	−0.015	0.028	0.012
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}^{l}$	$\hat{B}^{l}$	$\hat{I}$
	$\hat{R}^{l}_{p}$	0.714	0.662	0.563	0.423	0.269	0.144	0.089	0.304	0.414	–
	$\hat{I}_p$	0.725	0.685	0.632	0.594	0.596	0.626	0.658	–	–	0.631
						Netherlands					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}^{l}(\cdot)$	$\hat{\beta}^{l}(\cdot)$	$\zeta(\cdot)$
X_1	0.635	0.693	0.788	0.871	0.883	0.839	0.804	0.743	0.856	0.831	0.827
X_2	0.038	0.045	0.046	0.057	0.074	0.098	0.118	0.160	0.080	0.065	0.080
X_3	0.058	0.034	0.049	0.068	0.064	0.060	0.048	0.052	0.062	0.059	0.059
X_4	0.269	0.228	0.117	0.004	−0.020	0.003	0.031	0.045	0.002	0.045	0.034
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}^{l}$	$\hat{B}^{l}$	$\hat{I}$
	$\hat{R}_p$	0.804	0.714	0.589	0.423	0.253	0.136	0.088	0.303	0.428	–
	$\hat{I}_p$	0.812	0.735	0.656	0.595	0.576	0.611	0.657	–	–	0.643
						Portugal					
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}^{l}(\cdot)$	$\hat{\beta}^{l}(\cdot)$	$\zeta(\cdot)$
X_1	0.629	0.716	0.750	0.778	0.747	0.745	0.747	0.714	0.755	0.754	0.746
X_2	0.124	0.131	0.141	0.148	0.148	0.121	0.088	0.108	0.133	0.137	0.132
X_3	0.034	0.035	0.037	0.039	0.048	0.059	0.071	0.093	0.052	0.045	0.055
X_4	0.214	0.117	0.073	0.035	0.057	0.075	0.094	0.085	0.061	0.065	0.067
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}^{l}$	$\hat{B}^{l}$	$\hat{I}$
	$\hat{R}^{l}_{p}$	0.869	0.821	0.710	0.544	0.364	0.216	0.142	0.402	0.530	–
	$\hat{I}_p$	0.874	0.836	0.765	0.705	0.696	0.733	0.768	–	–	0.749

Table 3. Continued.

					Spain						
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.573	0.634	0.679	0.715	0.686	0.625	0.585	0.491	0.655	0.668	0.643
X_2	0.145	0.159	0.173	0.181	0.196	0.222	0.255	0.319	0.210	0.192	0.212
X_3	0.061	0.058	0.065	0.068	0.081	0.104	0.142	0.162	0.093	0.080	0.091
X_4	0.221	0.149	0.084	0.036	0.037	0.049	0.018	0.028	0.042	0.060	0.053
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{G}_p$	0.857	0.803	0.703	0.543	0.360	0.210	0.144	0.399	0.526	–
	$\hat{I}_p$	0.863	0.819	0.759	0.703	0.692	0.727	0.766	–	–	0.745
					Sweden						
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.609	0.658	0.718	0.839	0.829	0.900	0.865	0.827	0.845	0.803	0.817
X_2	0.018	0.003	0.000	0.013	0.013	0.007	0.016	0.024	0.011	0.010	0.012
X_3	0.050	0.055	0.058	0.070	0.083	0.105	0.139	0.156	0.092	0.077	0.091
X_4	0.323	0.284	0.224	0.078	0.075	−0.012	−0.020	−0.006	0.052	0.110	0.080
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}_p$	0.835	0.753	0.624	0.465	0.282	0.154	0.100	0.331	0.459	–
	$\hat{I}_p$	0.842	0.772	0.688	0.635	0.611	0.645	0.689	–	–	0.677
					United Kingdom						
	$\hat{\gamma}(\cdot)$	$\hat{\rho}_{0.05}(\cdot)$	$\hat{\rho}_{0.10}(\cdot)$	$\hat{\rho}_{0.25}(\cdot)$	$\hat{\rho}_{0.50}(\cdot)$	$\hat{\rho}_{0.75}(\cdot)$	$\hat{\rho}_{0.90}(\cdot)$	$\hat{\rho}_{0.95}(\cdot)$	$\hat{\rho}'(\cdot)$	$\hat{\beta}'(\cdot)$	$\zeta(\cdot)$
X_1	0.556	0.633	0.668	0.737	0.787	0.734	0.670	0.588	0.741	0.722	0.709
X_2	0.076	0.085	0.091	0.104	0.117	0.145	0.161	0.193	0.127	0.112	0.130
X_3	0.132	0.138	0.145	0.162	0.178	0.187	0.222	0.244	0.181	0.167	0.178
X_4	0.236	0.144	0.096	−0.003	−0.082	−0.066	−0.053	−0.025	−0.049	−0.001	−0.017
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$\hat{R}'$	$\hat{B}'$	$\hat{I}$
	$\hat{R}_p$	0.838	0.784	0.680	0.514	0.322	0.178	0.115	0.369	0.499	–
	$\hat{I}_p$	0.844	0.802	0.739	0.679	0.655	0.684	0.721	–	–	0.717

- ***Self-employment income***. The share $\hat{\gamma}(X_2)$ of self-employment income on total population income may vary a lot from country to country. In fact, it ranges from $\hat{\gamma}(X_2) = 0.018$ in Sweden to $\hat{\gamma}(X_2) = 0.210$ in Greece. Apart from Greece, the group of countries with large shares $\hat{\gamma}(X_2)$ includes Italy ($\hat{\gamma}(X_2) = 0.162$), Spain ($\hat{\gamma}(X_2) = 0.145$), Ireland ($\hat{\gamma}(X_2) = 0.137$) and Portugal ($\hat{\gamma}(X_2) = 0.124$). The contributions $\hat{\rho}'(X_2)$, $\hat{\beta}'(X_2)$ and $\hat{\zeta}(X_2)$ do clearly exceed $\hat{\gamma}(X_2)$ in all countries except for Sweden, indicating that also this factor component tends to be more concentrated among high income households than total income Y. The relative contributions $\hat{\rho}_p(X_2)$ are, except for Sweden, clearly larger than $\hat{\gamma}(X_2)$ at all levels of p reported in Table 3, and they tend to increase as p gets larger. In many countries the increasing trend is quite marked starting from $p = 0.5$.
- ***Other income components***. The share of income from this component is about $\hat{\gamma}(X_3) = 0.050$ in all countries except for Belgium and the United Kingdom, where $\hat{\gamma}(X_3) = 0.108$ and $\hat{\gamma}(X_3) = 0.132$, respectively. The contributions $\hat{\rho}'(X_3)$, $\hat{\beta}'(X_3)$, and $\hat{\zeta}(X_3)$ do slightly exceed $\hat{\gamma}(X_3)$ in most countries, indicating that, like for the former two factor components, the distribution of the other income components X_3 tends to exacerbate inequality in total income Y as well. The largest contributions $\hat{\rho}'(X_3)$, $\hat{\beta}'(X_3)$, and $\hat{\zeta}(X_3)$ are observed in those countries where the share $\hat{\gamma}(X_3)$ is also largest, that is, Belgium and the United Kingdom. Inspection of the relative contributions $\hat{\rho}_p(X_3)$ reveals an increasing trend in most countries. In some countries like Belgium, Finland, Sweden, and the United Kingdom the increasing trend is quite marked in the final part of the income distribution (i.e., for $p \geq 0.75$).
- ***Social transfers***, with shares $\hat{\gamma}(X_4)$ between 0.190 in Ireland, and 0.323 in Sweden, is the second largest factor component in all considered countries. As expected, the relative contributions $\hat{\rho}'(X_4)$, $\hat{\beta}'(X_4)$, and $\hat{\zeta}(X_4)$ are clearly smaller than $\hat{\gamma}(X_4)$, confirming that the distribution of this income component has an offsetting impact on inequality. In Belgium, Denmark, Ireland, Luxembourg, and the United Kingdom some of the relative contributions $\hat{\rho}'(X_4)$, $\hat{\beta}'(X_4)$, and/or $\hat{\zeta}(X_4)$ are even negative. As for the relative contributions $\hat{\rho}_p(X_4)$, they are for all countries smaller than $\hat{\gamma}(X_4)$ at all levels of p reported in Table 3, and they exhibit a decreasing trend in the initial part of the income distribution up to $p = 0.50$, and are thereafter almost constant, except for Sweden, where the decreasing trend holds on up to $p = 0.75$, and for Denmark, where $\hat{\rho}_p(X_4)$ increases after $p = 0.500$.

Appendix

In this appendix we prove that R' as defined in (27) and (28) is the ratio between the concentration area (i.e., the area between the Lorenz curve and the straight line which joins the origin (0, 0) with the point (1, 1)) and the area of the triangle with vertices in (0, 0), (1, 0) and (1, 1).

So let $P_s := p_{N_s}$ and $Q_s := q_{N_s}$, $s = 1, 2, \ldots, k-1$, be the abscissa and ordinate values of the points at which the slope of the Lorenz curve changes. It is not difficult to see that the conentration area is given by the sum of

- the area of the triangle with vertices in (0, 0), (P_1, P_1) and (P_1, Q_1), which is given by

$$\begin{aligned} A_1 &= \frac{(P_1 - Q_1)\, P_1}{2} \\ &= R_{N_1} \frac{P_1^2}{2} \end{aligned}$$

- the sum of areas of the $k-2$ trapezoids with vertices in (P_{s-1}, Q_{s-1}), (P_{s-1}, P_{s-1}), (P_s, Q_s) and (P_s, P_s), $s = 2, 3, \ldots, k-1$, which are given by

$$\begin{aligned} A_s &= \frac{[(P_{s-1} - Q_{s-1}) + (P_s - Q_s)]\,(P_s - P_{s-1})}{2} \\ &= R_{N_{s-1}} \frac{P_{s-1}\,(P_s - P_{s-1})}{2} + R_{N_s} \frac{P_s\,(P_s - P_{s-1})}{2} \end{aligned}$$

- the area of the triangle with vertices in (P_{k-1}, Q_{k-1}), (P_{k-1}, P_{k-1}) and (1, 1), which is given by

$$\begin{aligned} A_k &= \frac{(P_{k-1} - Q_{k-1})\,(1 - P_{k-1})}{2} \\ &= R_{N_{k-1}} \frac{P_{k-1}\,(1 - P_{k-1})}{2} \end{aligned}$$

Thus, the concentration area is given by

$$\begin{aligned} \sum_{s=1}^{k} A_s = R_{N_1} \frac{P_1^2}{2} + \sum_{s=2}^{k-1} R_{N_{s-1}} \frac{P_{s-1}\,(P_s - P_{s-1})}{2} + \\ + \sum_{s=2}^{k-1} R_{N_s} \frac{P_s\,(P_s - P_{s-1})}{2} + R_{N_{k-1}} \frac{P_{k-1}\,(1 - P_{k-1})}{2}. \end{aligned}$$

Setting $P_0 := 0$ and $P_k := 1$, the concentration area can also be written as

$$\sum_{s=1}^{k} A_s = \sum_{s=2}^{k} R_{N_{s-1}} \frac{P_{s-1}\,(P_s - P_{s-1})}{2} + \sum_{s=1}^{k-1} R_{N_s} \frac{P_s\,(P_s - P_{s-1})}{2}.$$

Using the fact that

$$\sum_{s=2}^{k} R_{N_{s-1}} \frac{P_{s-1}(P_s - P_{s-1})}{2} = \sum_{s=1}^{k-1} R_{N_s} \frac{P_s(P_{s+1} - P_s)}{2},$$

it is easily seen that

$$\sum_{s=1}^{k} A_s = \sum_{s=1}^{k-1} R_{N_s} r_s^*, \tag{67}$$

with

$$r_s^* := \frac{P_s(P_{s+1} - P_{s-1})}{2}, \quad s = 1, 2, \ldots, k-1,$$

Next, consider the hypothetical case where

$$Q_1 = Q_2 = \cdots = Q_{k-1} = 0.$$

In this case the concentration area would be given by the area of the triangle with vertices in (0, 0), (P_{k-1}, 0) and (1, 1), which is

$$\sum_{s=1}^{k} A_s = \frac{P_{k-1}}{2}, \tag{68}$$

and since we would have

$$R_{N_1} = R_{N_2} = \cdots = R_{N_{k-1}} = 1,$$

it follows from (67) and (68) that

$$\sum_{s=1}^{k-1} r_s^* = \frac{P_{k-1}}{2}.$$

Thus, if we set

$$r_k^* := \frac{1 - P_{k-1}}{2},$$

we get

$$\sum_{s=1}^{k} r_s^* = \frac{1}{2}, \tag{69}$$

and since $R_{N_k} = R_N = 0$ for every income distribution, it follows that the ratio between the concentration area and the area of triangle with vertices in (0, 0), (1, 0) and (1, 1) is given by (use (67) and (69))

$$2\sum_{s=1}^{k} A_s = 2\sum_{s=1}^{k} R_{N_s} r_s^* = \frac{\sum_{s=1}^{k} R_{N_s} r_s^*}{\sum_{s=1}^{k} r_s^*}.$$

Rescaling the weights r_s^* through multiplication by $2N^2$ yields finally the definition of R' in (27) and (28).

List of Notations

Symbol	Equation	Meaning
N	(1)	Number of population members
y_i for $i = 1, 2, \ldots, N$	(1)	Total incomes of the population members
Y	(1)	Symbol to indicate the total income variable
p_i for $i = 1, 2, \ldots, N$	(2)	Cumulative population shares
q_i for $i = 1, 2, \ldots, N$	(3)	Cumulative income shares
R_i for $i = 1, 2, \ldots, N$	(4)	Gini's point inequality measures
R	(5)	Gini's synthetic inequality index
$M_i^-(Y)$ for $i = 1, 2, \ldots, N$	(6)	Mean income of the i "poorest" population members, i.e., the i population members with smallest total income Y
$M(Y)$	(7)	Mean income of the whole population
B	(8)	Bonferroni's synthetic inequality index
I_i for $i = 1, 2, \ldots, N$	(9)	Zenga's point inequality indexes
k	(9)	Number of different values among $y_1, y_2, \ldots, y_N$
N_j for $j = 1, 2, \ldots, k$	(9)	Cumulative frequencies corresponding to different values among $y_1, y_2, \ldots, y_N$
$M_i^+(Y)$ for $i = 1, 2, \ldots, N$	(10)	Mean income of the $n - i$ "richest" population members, i.e., the $n - i$ population members with largest total income Y
I	(11)	Zenga's synthetic inequality index
n_j for $j = 1, 2, \ldots, k$	(11)	Absolute frequencies corresponding to different values among $y_1, y_2, \ldots, y_N$
c	(12)	Number of factor components
$x_{i,j}$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(12)	Incomes from the c factor components
X_j for $j = 1, 2, \ldots, c$	(13)	Symbols to indicate factor components
$M(X_j)$ for $j = 1, 2, \ldots, c$	(13)	Population means of the factor components
$M_i^-(X_j)$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(14)	Mean incomes from the factor components among the i "poorest" population members, that is, among the i population members with smallest total income Y
$M_i^+(X_j)$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(15)	Mean incomes from the factor components among the $n - i$ "richest" population members, that is, among the $n - i$ population members with largest total income Y
$\mathcal{R}_i(X_j)$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(16)	Contribution to the Gini point inequality index R_i from factor component X_j
$\mathcal{I}_i(X_j)$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(17)	Contribution to the Zenga point inequality index I_i from factor component X_j
$\rho_i(X_j)$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(18)	Relative contribution to the Gini point inequality index R_i from factor component X_j
$\zeta_i(X_j)$ for $i = 1, 2, \ldots, N$ and for $j = 1, 2, \ldots, c$	(19)	Relative contribution to the Zenga point inequality index I_i from factor component X_j
$\mathcal{R}(X_j)$ for $j = 1, 2, \ldots, c$	(21)	Contribution to Gini's synthetic inequality index R from factor component X_j
$\mathcal{B}(X_j)$ for $j = 1, 2, \ldots, c$	(22)	Contribution to Bonferroni's synthetic inequality index B from factor component X_j
$\mathcal{I}(X_j)$ for $j = 1, 2, \ldots, c$	(23)	Contribution to Zenga's synthetic inequality index I from factor component X_j

Symbol	Equation	Meaning
$\rho(X_j)$ for $j = 1, 2, \ldots, c$	(24)	Relative contribution to Gini's synthetic inequality index R from factor component X_j
$\beta(X_j)$ for $j = 1, 2, \ldots, c$	(25)	Relative contribution to Bonferroni's synthetic inequality index B from factor component X_j
$\zeta(X_j)$ for $j = 1, 2, \ldots, c$	(26)	Relative contribution to Zenga's synthetic inequality index I from factor component X_j
R'	(27)	Modified version of Gini's synthetic inequality index
r_s	(28)	Weights in Gini's synthetic inequality index
B'	(29)	Modified version of Bonferroni's synthetic inequality index
$\mathcal{R}'(X_j)$ for $j = 1, 2, \ldots, c$	(30)	Contribution to the modified version R' of Gini's synthetic inequality index from factor component X_j
$\mathcal{B}'(X_j)$ for $j = 1, 2, \ldots, c$	(31)	Contribution to the modified version B' of Bonferroni's synthetic inequality index from factor component X_j
$\rho'(X_j)$ for $j = 1, 2, \ldots, c$	(32)	Relative contribution from factor component X_j to the modified version of Gin's synthetic inequality index
$\beta'(X_j)$ for $j = 1, 2, \ldots, c$	(33)	Relative contribution from factor component X_j to the modified version of Bonferroni's synthetic inequality index
$\gamma(X_j)$ for $j = 1, 2, \ldots, c$	(34)	Share of factor component X_j on total population income
S	(39)	Set of indexes i identifying population units belonging to a sample
d	(39)	Sample size, i.e., number of indexes i in S. Note that in the application of Section 6 we considered for estimation only sample households for which the net household income variable $Y := HI100$ is filled. Thus, the samples S used for estimation do not comprise all interviewed households: in fact, for every country there are some interviewed households for which the net household income variable $Y := HI100$ is not filled (see Table 1).
w_i for $i \in S$	(40)	Survey weights corresponding to the sample units $i \in S$
$\hat{k}$	(42)	Number of sample units with different total income Y
$\tilde{y}_1 < \tilde{y}_2 < \cdots < \tilde{y}_{\hat{k}}$	(42)	Different values of total income Y among sample units
$\hat{n}_s$ for $s = 1, 2, \ldots, \hat{k}$	(43)	Sum of survey weights corresponding to the sample units with total income Y equal to $\tilde{y}_s$
$\hat{N}_s$ for $s = 1, 2, \ldots, \hat{k}$	(44)	Cumulative survey weights corresponding to different values of total income Y in the sample
$\sigma(p)$ for $p \in [0, 1]$	(45)	$\sigma(p) := \min\{s : \hat{N}_s \geq Np\}$, that is, number of different values $\tilde{y}_s$ of total income Y among sample units with total income not larger the pth sample quantile of total income
$\hat{M}_p^-(Y)$ for $p \in [0, 1]$	(46)	Weighted mean of total income Y among sample units with total income not larger than the pth sample quantile $\tilde{y}_{\sigma(p)}$
$\hat{M}_p^+(Y)$ for $p \in [0, 1]$	(47)	Weighted mean of total income Y among sample units with total income larger than the pth sample quantile $\tilde{y}_{\sigma(p)}$
$\hat{M}(Y)$ for $p \in [0, 1]$	(48)	Weighted sample mean of total income Y
$\hat{R}_p$ for $p \in [0, 1]$	(49)	Estimates for Gini's point inequality measures
$\hat{I}_p$ for $p \in [0, 1]$	(50)	Estimates for Zenga's point inequality measures

Symbol	Equation	Meaning
$\hat{r}_s$ for $s = 1, 2, \ldots, \hat{k}$	(51)	Estimates for the weights in the modified version of Gini's synthetic inequality index R'
$\hat{R}'$	(52)	Estimate for the modified version R' of Gini's synthetic inequality
$\hat{B}'$	(53)	Estimate for the modified version B' of Bonferroni's synthetic inequality
$\hat{I}$	(54)	Estimate for Zenga's synthetic inequality index I
$\tilde{x}_{s,j}$ for $s = 1, 2, \ldots, \hat{k}$ and for $j = 1, 2, \ldots, c$	(55)	Weighted average of income from factor component X_j among the sample units with total income equal to $\tilde{y}_s$
$\hat{\mathcal{R}}_p(X_j)$ for $p \in [0, 1]$ and for $j = 1, 2, \ldots, c$	(56)	Sample estimate for the contribution $\mathcal{R}_i(X_j)$ at $i = \lceil Np \rceil$
$\hat{\mathcal{I}}_p(X_j)$ for $p \in [0, 1]$ and for $j = 1, 2, \ldots, c$	(57)	Sample estimate for the contribution $\mathcal{I}_p(X_j)$ at $i = \lceil Np \rceil$
$\hat{\mathcal{R}}(X_j)$ for $j = 1, 2, \ldots, c$	(58)	Sample estimate for the contribution $\mathcal{R}'(X_j)$
$\hat{\mathcal{B}}(X_j)$ for $j = 1, 2, \ldots, c$	(59)	Sample estimate for the contribution $\mathcal{B}'(X_j)$
$\hat{\mathcal{I}}(X_j)$ for $j = 1, 2, \ldots, c$	(60)	Sample estimate for the contribution $\mathcal{I}(X_j)$
$\hat{\rho}_p(X_j)$ for $p \in [0, \hat{N}_{\hat{k}-1}/N]$ and for $j = 1, 2, \ldots, c$	(61)	Sample estimate for the relative contribution $\rho_i(X_j)$ at $i = \lceil Np \rceil$
$\hat{\zeta}_p(X_j)$ for $p \in [0, \hat{N}_{\hat{k}-1}/N]$ and for $j = 1, 2, \ldots, c$	(62)	Sample estimate for the relative contribution $\zeta_i(X_j)$ at $i = \lceil Np \rceil$
$\hat{\rho}(X_j)$ for $j = 1, 2, \ldots, c$	(63)	Sample estimate for the relative contribution $\rho'(X_j)$
$\hat{\beta}(X_j)$ for $j = 1, 2, \ldots, c$	(64)	Sample estimate for the relative contribution $\beta'(X_j)$
$\hat{\zeta}(X_j)$ for $j = 1, 2, \ldots, c$	(65)	Sample estimate for the relative contribution $\zeta(X_j)$
$\hat{\gamma}(X_j)$ for $j = 1, 2, \ldots, c$	(66)	Sample estimate for the share of factor component X_j on total population income

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