Journal of Integer Sequences, Vol. 8 (2005),
Article 05.4.4

Primes in Fibonacci n-step and Lucas n-step Sequences

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Abstract

We search for primes in the Fibonacci n-step and Lucas n-step sequences, which are the natural generalizations of the Fibonacci and Lucas numbers. While the Fibonacci n-step sequences are nearly devoid of primes, the Lucas n-step sequences are prime-rich. We tabulate the occurrence of primes in the first 10000 terms for $n \le 100$. We also state two conjectures about Diophantine equations based on these sequences.

1 Introduction

As illustrated in the tome by Koshy [9], the Fibonacci and Lucas numbers are arguably two of the most interesting sequences in all of mathematics. The sequence of Fibonacci numbers F_k is defined by the second-order linear recurrence formula and initial terms:

$$F_{k+1} = F_k + F_{k-1}, \quad F_0 = 0, \quad F_1 = 1$$

Similarly, the sequence of Lucas numbers L_k is defined by

$$L_{k+1} = L_k + L_{k-1}, \quad L_0 = 2, \quad L_1 = 1$$

Simple generalizations of these sequences are $F_k^{(n)}$, the Fibonacci n-step sequence, and $L_k^{(n)}$, the Lucas n-step sequence, which are defined by linear recurrence formulas of order $n > 1$:

$$F_{k+1}^{(n)} = F_k^{(n)} + F_{k-1}^{(n)} + \cdots + F_{k-n+1}^{(n)} \tag{1}$$
$$L_{k+1}^{(n)} = L_k^{(n)} + L_{k-1}^{(n)} + \cdots + L_{k-n+1}^{(n)} \tag{2}$$

and n initial terms

$$F_{1-n}^{(n)} = 1, \quad F_k^{(n)} = 0, \qquad k = -n+2, \ldots, 0 \tag{3}$$
$$L_0^{(n)} = n, \quad L_k^{(n)} = -1, \quad k = -n+1, \ldots, -1 \tag{4}$$

The sequence generated by equations (1) and (3) is also known as the k-generalized Fibonacci numbers, which are discussed by Flores [7]. Observe that equations (1) and (2) are equivalent to the three-term recursions

$$F_{k+1}^{(n)} = 2F_k^{(n)} - F_{k-n}^{(n)}$$
$$L_{k+1}^{(n)} = 2L_k^{(n)} - L_{k-n}^{(n)}$$

which are computationally superior for large n, and which show that all of these n-step sequences grow at a rate less than 2^k. This recursion requires one more initial term, which we can take to be $F_1^{(n)} = L_1^{(n)} = 1$. Also observe that the Lucas n-step sequence can be obtained from the Fibonacci n-step sequence with the identity

$$L_k^{(n)} = F_k^{(n)} + 2F_{k-1}^{(n)} + \cdots + (n-1)F_{k-n+2}^{(n)} + nF_{k-n+1}^{(n)}$$

which follows from the generating functions of these two sequences.

Dickson [4] cites a long history of generalizations of the Fibonacci numbers. Miles [11] used equation (1) in 1960. Fielder [6] appears to be the first to generalize the Lucas numbers to the Lucas n-step sequences. Catalani [3] has also written about these sequences. More recently, Benjamin and Quinn [2] briefly discuss a combinatorial interpretation of these n-step sequences.

The usual Fibonacci and Lucas numbers are obtained for $n = 2$. For small values of n, these sequences are called tribonacci ($n = 3$), tetranacci or quadranacci ($n = 4$), pentanacci or pentacci ($n = 5$), hexanacci or esanacci ($n = 6$), heptanacci ($n = 7$), and octanacci ($n = 8$). Examples of these sequences and their A-numbers in Sloane's [12] OEIS database are given on the next page.

The unique primes in each table appear in boxes. Note that these abbreviated sequences have many primes and that there appear to be more Lucas primes than Fibonacci primes. Also note that every Mersenne prime — a prime of the form $2^p - 1$ — appears in these sequences as $F_{p+2}^{(p)}$ and $L_p^{(n)}$ for $n \geq p$.

The primary purpose of this paper is to tabulate the values of k that yield prime terms of the Fibonacci n-step and Lucas n-step sequences. Additionally, the paper describes how these primes were determined and makes conjectures about the distinct values in these sequences.

Table 1: Fibonacci n-step Sequences

k / n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	A-number
2	1	1	$\boxed{2}$	$\boxed{3}$	$\boxed{5}$	8	$\boxed{13}$	21	34	55	$\boxed{89}$	144	$\boxed{233}$	377	A000045
3	1	1	2	4	$\boxed{7}$	$\boxed{13}$	24	44	81	$\boxed{149}$	274	504	927	1705	A000073
4	1	1	2	4	8	15	$\boxed{29}$	56	108	208	$\boxed{401}$	$\boxed{773}$	1490	2872	A000078
5	1	1	2	4	8	16	$\boxed{31}$	$\boxed{61}$	120	236	464	912	1793	3525	A001591
6	1	1	2	4	8	16	32	63	125	248	492	976	1936	3840	A001592
7	1	1	2	4	8	16	32	64	$\boxed{127}$	253	504	1004	2000	3984	A066178
8	1	1	2	4	8	16	32	64	128	255	$\boxed{509}$	1016	2028	4048	A079262
9	1	1	2	4	8	16	32	64	128	256	511	$\boxed{1021}$	2040	4076	A105753

Table 2: Lucas n-step Sequences

k / n	1	2	3	4	5	6	7	8	9	10	11	12	13	A-number
2	1	$\boxed{3}$	4	$\boxed{7}$	$\boxed{11}$	18	$\boxed{29}$	$\boxed{47}$	76	123	$\boxed{199}$	322	$\boxed{521}$	A000032
3	1	3	7	11	21	39	$\boxed{71}$	$\boxed{131}$	$\boxed{241}$	$\boxed{443}$	815	$\boxed{1499}$	2757	A001644
4	1	3	7	15	26	51	99	$\boxed{191}$	$\boxed{367}$	708	1365	2631	5071	A073817
5	1	3	7	15	$\boxed{31}$	57	$\boxed{113}$	$\boxed{223}$	$\boxed{439}$	$\boxed{863}$	1695	3333	$\boxed{6553}$	A074048
6	1	3	7	15	31	63	120	$\boxed{239}$	475	943	$\boxed{1871}$	3711	7359	A074584
7	1	3	7	15	31	63	$\boxed{127}$	247	493	$\boxed{983}$	1959	3903	7775	A104621
8	1	3	7	15	31	63	127	255	502	1003	$\boxed{2003}$	3999	7983	A105754
9	1	3	7	15	31	63	127	255	511	$\boxed{1013}$	2025	4047	$\boxed{8087}$	A105755

2 Tables of Primes and Probable Primes

Tables 3 and 4, at the end of this paper, list the n and k values that yield prime or probable prime (abbreviated *prp*) terms of the Fibonacci n-step sequences $F_k^{(n)}$ for $2 \le n \le 100$ and $1 \le k \le 10000$. Similarly, Tables 5, 6, and 7 list the n and k values that yield prime or probable prime terms of the Lucas n-step sequences $L_k^{(n)}$.

A striking difference between the $F_k^{(n)}$ and $L_k^{(n)}$ tables is the small number of Fibonacci primes and the relatively large number of Lucas primes. This difference is explained by the parity of the two types of sequences. The fraction of odd numbers in the sequence $F_k^{(n)}$ is $2/(n+1)$ while the fraction of odd numbers in the sequence $L_k^{(n)}$ is either 1 or $n/(n+1)$, depending on whether n is odd or even, respectively. Hence, because the $F_k^{(n)}$ sequence contains many fewer odd numbers, it contains many fewer prime numbers. In fact, using a probabilistic argument, such as in Hardy and Wright [8, § 2.5], and taking into account the parity of the numbers in the sequences, we estimate the number of primes in $F_k^{(n)}$ for $k \le N$:

$$\begin{aligned}\#\{k \le N : F_k^{(n)}\text{prime}\} &\approx \frac{4}{n+1}\sum_{i=1}^{N}\frac{1}{\log F_i^{(n)}} \\ &\approx \frac{4}{n+1}\sum_{i=1}^{N}\frac{1}{\log \alpha_n^i} = \frac{4}{n+1}\frac{H_N}{\log \alpha_n}\end{aligned}$$

where α_n is the growth rate of the sequence (the largest real root of $x^n(2-x)=1$) and

$$H_N = \sum_{i=1}^{N}\frac{1}{i}$$

is the harmonic sum. Similarly, an estimate for the number of primes in $L_k^{(n)}$ for $k \le N$ is

$$\#\{k \le N : L_k^{(n)}\text{prime}\} \approx \begin{cases} \frac{2H_N}{\log \alpha_n}, & \text{if } n \text{ is odd;} \\ \frac{n}{n+1}\frac{2H_N}{\log \alpha_n}, & \text{if } n \text{ is even.}\end{cases}$$

These estimates are evaluated for each n and appear in the last column of the tables. In many cases, the estimates are close to the actual number of primes that we found.

3 Prime Search and Prime Proving

Each probable prime was determined using the PrimeQ function in *Mathematica*®. PrimeQ, which Wagon [13] discusses at length, is based on the Miller-Rabin strong pseudoprime test base 2 and base 3, and a Lucas test. Testing $F_k^{(n)}$ and $L_k^{(n)}$ with PrimeQ for $2 \le n \le 100$ and $1 \le k \le 10000$ required approximately 32 hours on a dual-processor 1.8 GHz G5 Macintosh® computer.

For small n, the search for probable primes has been more extensive than the limit $k \le 10000$ used here. For instance, searches by Dubner and Keller [5] and more recently by Lifchitz [10] have found probable prime values of $F_k^{(2)}$ and $L_k^{(2)}$ for $k > 500000$.

For the probable primes up to 1000 digits (and a few prps with more digits), we proved primality using an elliptic curve primality algorithm, as described by Atkin and Morain [1]. This computation required approximately 60 hours on a 2.81 GHz Pentium® 4 processor. All of the tested probable primes proved to be prime. For the Fibonacci and Lucas numbers, $F_k^{(2)}$ and $L_k^{(2)}$, Dubner and Keller [5] have proved the primality for those entries in row $n = 2$ of Tables 3 and 5. Probable primes in Tables 3–7 are marked by an asterisk.

4 Duplicate Primes?

Are there duplicate primes in these tables? Let us ignore the initial terms of these n-step sequences because m-step and n-step sequences with $m < n$ have the initial m or $m+1$ terms in common. For Fibonacci n-step sequences, the prime 13 is common to both the 2-step and 3-step sequences. For Lucas n-step sequences, the primes 7 and 11 are in the 2-step and 3-step sequences. This observation leads us to two conjectures:

Conjecture 1 *The Diophantine equation*

$$F_r^{(m)} = F_s^{(n)}, \quad r > m+1, \quad s > n+1$$

has only two solutions:

$$13 = F_7^{(2)} = F_6^{(3)} \quad \textit{and} \quad 504 = F_{12}^{(3)} = F_{11}^{(7)}.$$

Conjecture 2 *The Diophantine equation*

$$L_r^{(m)} = L_s^{(n)}, \quad r > m, \quad s > n$$

has only three solutions:

$$7 = L_4^{(2)} = L_3^{(3)}, \quad 11 = L_5^{(2)} = L_4^{(3)}, \quad \textit{and} \quad 5071 = L_{14}^{(3)} = L_{13}^{(4)}.$$

We have confirmed these conjectures for all terms whose magnitude is less than 2^{2000}. Here we offer an heuristic argument showing that the expected number of solutions is small. Let t be a positive integer and consider all the Fibonacci (or Lucas) n-step numbers (for any $n \le t+1$) that are between 2^t and 2^{t+1}. It is easy to see that any Fibonacci (or Lucas) n-step sequence has at most two numbers in this range for each t. Assuming that these numbers are randomly distributed in the range 2^t to 2^{t+1}, the probability that at least two numbers are the same is

$$1 - \prod_{d=1}^{2t-1} (1 - d/2^t)$$

Hence, an upper bound for the number of solutions is

$$\sum_{t=1}^{\infty} \left(1 - \prod_{d=1}^{2t-1} (1 - d/2^t) \right) = 6.26505\ldots$$

5 Acknowledgment

The authors wish to acknowledge Eric Weisstein [14] for inspiring this research.

References

[1] A. O. L. Atkin and F. Morain, Elliptic Curves and Primality Proving, *Math. Comp.* **61** (1993), 29–68.

[2] A. T. Benjamin and J. J. Quinn, The Fibonacci numbers – exposed more discretely, *Math. Mag.* **76** (2003), 182–192.

[3] M. Catalani, Polymatrix and generalized polynacci numbers, published electronically at www.arXiv.org/math/0210201

[4] L. E. Dickson, *History of the Theory of Numbers*, Vol. 1, AMS Chelsea, 1919.

[5] H. Dubner and W. Keller, New Fibonacci and Lucas primes, *Math. Comp.* **68** (1999), 417–427.

[6] D. C. Fielder, Certain Lucas-like sequences and their generation by partitions of numbers, *Fibonacci Quart.*, **5** (1967), 319

[7] I. Flores, Direct calculation of k-generalized Fibonacci numbers, *Fibonacci Quart.*, **5** (1967), 259–266.

[8] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, fourth ed., Oxford University Press, 1960.

[9] T. Koshy, *Fibonacci and Lucas Numbers with Applications*, John Wiley and Sons, NY, 2001.

[10] H. Lifchitz, *Probable primes top 10000*, published electronically at www.primenumbers.net/prptop/prptop.php

[11] E. P. Miles, Jr., Generalized Fibonacci numbers and associated matrices, *Amer. Math. Monthly* **67** (1960), 745–752.

[12] N. J. A. Sloane, *The On-Line Encyclopedia of Integer Sequences*, published electronically at www.research.att.com/∼njas/sequences.

[13] S. Wagon, *Mathematica® in Action*, second ed., Springer-Telos, New York, 2000.

[14] E. W. Weisstein, *Fibonacci n-step number*, published electronically at mathworld.wolfram.com/Fibonaccin-StepNumber.html

Table 3: Primes of the form $F_k^{(n)}$

n	k for which $F_k^{(n)}$ is prime or prp(*)	number of primes for $k \leq 10000$	est. # of primes for $k \leq 10000$
2	3, 4, 5, 7, 11, 13, 17, 23, 29, 43, 47, 83, 131, 137, 359, 431, 433, 449, 509, 569, 571, 2971, 4723, 5387, 9311, 9677 A001605 A005478	26	27
3	3, 5, 6, 10, 86, 97, 214, 801, 4201 A092835 A092836	9	16
4	3, 7, 11, 12, 36, 56, 401, 2707, 8417* A104534 A104535	9	12
5	3, 7, 8, 25, 146, 169, 182, 751, 812, 1507, 1591, 3157, 3752 A105756 A105757	13	10
6	3, 36, 37, 92, 660, 6091*, 8415* A105758 A105759	7	8
7	3, 9, 17 A105760 A105761	3	7
8	3, 11, 19, 119, 344, 1316	6	6
9	3, 12, 871	3	6
10	3, 353, 365, 2432	4	5
11	3, 14, 50, 325, 2510	5	5
12	3, 80, 1237	3	4
13	3, 15, 16, 8051*	4	4
14	3, 361, 1471	3	4
15	3, 34, 82, 657, 5361*, 7026*	6	4
16	3, 274, 4098, 4506	4	3
17	3, 19, 37, 1459	4	3
18	3, 59, 3079	3	3
19	3, 21, 22, 2982, 4382*	5	3
20	3, 233, 484	3	3
21	3, 24, 46, 4072*	4	3
22	3, 47, 232, 393	4	2
23	3, 26	2	2
24	3, 52, 451	3	2
25	3, 287	2	2
26	3, 487	2	2
27	3	1	2
28	3, 31	2	2
29	3	1	2
30	3, 1118	2	2
31	3, 33, 7233*	3	2
32	3, 167	2	2
33	3	1	2
34	3, 9942	2	2
35	3, 254, 3745, 6013*	4	2
36	3	1	2
37	3, 723	2	1
38	3	1	1
39	3	1	1
40	3, 247	2	1
41	3	1	1
42	3, 518, 1205	3	1
43	3	1	1
44	3	1	1
45	3	1	1
46	3	1	1
47	3, 194, 865, 2401	4	1
48	3, 6322*	2	1
49	3	1	1
50	3	1	1

Table 4: Primes of the form $F_k^{(n)}$, continued

n	k for which $F_k^{(n)}$ is prime or prp(*)	number of primes for $k \leq 10000$	est. # of primes for $k \leq 10000$
51	3	1	1
52	3	1	1
53	3, 2809, 4052*, 4592*	4	1
54	3, 2366	2	1
55	3	1	1
56	3	1	1
57	3	1	1
58	3, 120, 3718	3	1
59	3	1	1
60	3, 1405	2	1
61	3, 63, 374	3	1
62	3, 8695*	2	1
63	3, 5057*	2	1
64	3	1	1
65	3, 925	2	1
66	3, 269	2	1
67	3, 341	2	1
68	3	1	1
69	3	1	1
70	3, 1564	2	1
71	3	1	1
72	3	1	1
73	3	1	1
74	3, 302, 2476	3	1
75	3	1	1
76	3	1	1
77	3, 4916*	2	1
78	3	1	1
79	3	1	1
80	3	1	1
81	3	1	1
82	3	1	1
83	3	1	1
84	3	1	1
85	3	1	1
86	3, 263, 8701*	3	1
87	3	1	1
88	3	1	1
89	3, 91, 811	3	1
90	3	1	1
91	3	1	1
92	3, 1024	2	1
93	3, 96	2	1
94	3	1	1
95	3	1	1
96	3	1	1
97	3	1	1
98	3, 299	2	1
99	3	1	1
100	3	1	1

Table 5: Primes of the form $L_k^{(n)}$

n	k for which $L_k^{(n)}$ is prime or prp(*)	number of primes for $k \le 10000$	est. # of primes for $k \le 10000$
2	2, 4, 5, 7, 8, 11, 13, 16, 17, 19, 31, 37, 41, 47, 53, 61, 71, 79, 113, 313, 353, 503, 613, 617, 863, 1097, 1361, 4787, 4793, 5851, 7741, 8467 A001606 A005479	32	27
3	2, 3, 4, 7, 8, 9, 10, 12, 20, 30, 33, 66, 76, 77, 82, 87, 98, 180, 205, 360, 553, 719, 766, 1390, 1879, 1999, 4033*, 5620* A104576 A105762	28	32
4	2, 3, 8, 9, 16, 19, 24, 27, 46, 68, 71, 78, 107, 198, 309, 377, 477, 1057, 1631, 2419, 3974*, 4293*, 8247* A104577 A105763	23	24
5	2, 3, 5, 7, 8, 9, 10, 13, 30, 35, 77, 98, 126, 160, 192, 810, 1086, 1999, 2021, 3157, 3426*, 3471* A105764 A105765	22	29
6	2, 3, 5, 8, 11, 32, 37, 46, 123, 237, 332, 408, 772, 827, 1523, 5610* A105766 A105767	16	24
7	2, 3, 5, 7, 10, 17, 24, 25, 26, 28, 38, 40, 49, 62, 79, 89, 114, 140, 145, 182, 248, 353, 437, 654, 702, 784, 921, 931, 986, 1206, 2136, 2137, 3351*, 5411* A104622 A105768	34	28
8	2, 3, 5, 7, 11, 16, 17, 112, 140, 159, 186, 347, 425, 565, 2643, 2931, 3314, 4767*, 9015*	19	25
9	2, 3, 5, 7, 10, 13, 23, 27, 38, 41, 56, 84, 107, 112, 123, 203, 209, 758, 765, 895, 1830, 3172, 6763*, 7709*	24	28
10	2, 3, 5, 7, 48, 59, 75, 79, 156, 167, 245, 335, 370, 412, 970, 1358, 1479, 1652	18	26
11	2, 3, 5, 7, 15, 31, 32, 39, 41, 47, 66, 89, 111, 173, 182, 295, 334, 350, 357, 480, 548, 662, 681, 853, 1100, 1128, 1171, 1328, 2203, 2427, 2817, 2891, 6818*	33	28
12	2, 3, 5, 7, 15, 16, 46, 69, 112, 120, 144, 411, 574, 1009, 1140, 1191, 1257, 1358, 2809, 3223, 3512*, 3759*, 4441*, 6099*, 8304*, 9673*, 9734*	27	26
13	2, 3, 5, 7, 13, 14, 22, 27, 53, 55, 92, 242, 299, 344, 382, 699, 767, 834, 1057, 1270, 1565, 1797, 1837, 3847*, 4328*, 4911*, 5093*, 6664*, 7313*, 7470*, 9789*	31	28
14	2, 3, 5, 7, 13, 22, 23, 52, 76, 82, 119, 125, 243, 281, 400, 437, 659, 994, 1377, 1426, 1940, 2303, 3853*, 6947*	24	26
15	2, 3, 5, 7, 13, 16, 31, 37, 38, 47, 48, 58, 66, 132, 167, 173, 269, 375, 542, 612, 911, 1073, 2821, 3237, 3656*, 3910*, 4424*, 4432*, 6978*	29	28
16	2, 3, 5, 7, 13, 26, 131, 145, 243, 821, 863, 1388, 1481, 4457*, 5309*, 8696*	16	27
17	2, 3, 5, 7, 13, 17, 59, 208, 325, 353, 926, 991, 1201, 1230, 1347, 1764, 8101*	17	28
18	2, 3, 5, 7, 13, 17, 23, 33, 45, 58, 107, 134, 217, 286, 552, 985, 1193, 2455, 4202*, 6176*, 8710*, 9831*	22	27
19	2, 3, 5, 7, 13, 17, 19, 91, 208, 219, 306, 312, 439, 494, 640, 656, 702, 872, 3346*, 3874*, 7769*, 7804*	22	28
20	2, 3, 5, 7, 13, 17, 19, 27, 35, 47, 53, 76, 88, 243, 281, 288, 355, 406, 436, 541, 761, 1055, 2343, 2585, 3705*, 4965*, 7033*	27	27
21	2, 3, 5, 7, 13, 17, 19, 28, 52, 61, 165, 263, 278, 284, 691, 1110, 1130, 1372, 1526, 2542, 2656, 3403*, 4162*, 5645*, 6156*, 6572*, 8496*	27	28
22	2, 3, 5, 7, 13, 17, 19, 34, 113, 191, 288, 451, 561, 646, 867, 1842, 2368, 5369*, 5827*, 5993*	20	27
23	2, 3, 5, 7, 13, 17, 19, 28, 37, 44, 86, 133, 142, 164, 307, 700, 2250, 3255, 9785*	19	28
24	2, 3, 5, 7, 13, 17, 19, 70, 79, 87, 99, 104, 140, 153, 214, 556, 743, 766, 905, 1824, 3983*, 4807*	22	27
25	2, 3, 5, 7, 13, 17, 19, 26, 39, 43, 46, 49, 65, 70, 79, 98, 188, 218, 337, 452, 539, 904, 3185, 5530*	24	28
26	2, 3, 5, 7, 13, 17, 19, 28, 35, 38, 61, 161, 263, 374, 475, 653, 668, 816, 914, 1073, 1841, 2353, 2794, 3176, 4278*, 4845*, 5305*, 6803*, 7748*, 8618*	30	27
27	2, 3, 5, 7, 13, 17, 19, 31, 52, 58, 101, 112, 209, 244, 281, 288, 326, 449, 468, 585, 718, 4814*, 5478*, 7622*	24	28
28	2, 3, 5, 7, 13, 17, 19, 35, 68, 86, 88, 89, 98, 102, 141, 160, 689, 785, 840, 1140, 1730, 1978, 2187, 3315	24	27
29	2, 3, 5, 7, 13, 17, 19, 44, 73, 85, 132, 135, 214, 221, 259, 358, 411, 426, 502, 740, 987, 1033, 1594, 1626, 2530, 2551, 2669, 5090*, 7568*, 9821*	30	28
30	2, 3, 5, 7, 13, 17, 19, 32, 41, 44, 45, 75, 82, 99, 107, 292, 338, 371, 1108, 2983, 4783*, 7932*	22	27
31	2, 3, 5, 7, 13, 17, 19, 31, 43, 45, 50, 52, 57, 81, 137, 172, 291, 472, 497, 502, 1003, 1746, 2007, 2115, 2226, 2318, 6116*, 7580*	28	28
32	2, 3, 5, 7, 13, 17, 19, 31, 35, 51, 57, 59, 65, 73, 85, 106, 124, 303, 318, 481, 503, 507, 681, 1624, 2633, 4186*, 4945*	27	27
33	2, 3, 5, 7, 13, 17, 19, 31, 44, 49, 52, 70, 71, 82, 85, 129, 138, 236, 249, 327, 704, 1373, 1406, 1477, 3370*, 3430*	26	28

Table 6: Primes of the form $L_k^{(n)}$, continued

n	k for which $L_k^{(n)}$ is prime or prp(*)	number of primes for $k \leq 10000$	est. # of primes for $k \leq 10000$
34	2, 3, 5, 7, 13, 17, 19, 31, 73, 85, 104, 135, 244, 296, 317, 360, 366, 373, 397, 470, 1238, 2118, 2125, 2594, 5870*, 8742*	26	27
35	2, 3, 5, 7, 13, 17, 19, 31, 38, 50, 91, 97, 109, 132, 276, 361, 535, 1281, 1720, 2285, 2302, 3971*, 4322*, 5177*, 7336*	25	28
36	2, 3, 5, 7, 13, 17, 19, 31, 53, 117, 230, 298, 367, 701, 797, 846, 1008, 2122, 2210	19	27
37	2, 3, 5, 7, 13, 17, 19, 31, 69, 101, 341, 349, 577, 1147, 1963, 4542*, 8999*	17	28
38	2, 3, 5, 7, 13, 17, 19, 31, 44, 50, 58, 70, 77, 89, 145, 181, 197, 211, 247, 460, 475, 498, 579, 625, 687, 1288, 1736, 1740, 4172*, 7810*, 9629*	31	28
39	2, 3, 5, 7, 13, 17, 19, 31, 52, 85, 91, 243, 867, 1106, 1167, 1633, 2684, 2917, 3733*, 4593*, 5055*	21	28
40	2, 3, 5, 7, 13, 17, 19, 31, 47, 56, 64, 70, 154, 230, 714, 995, 2478, 3965*, 6837*, 8498*	20	28
41	2, 3, 5, 7, 13, 17, 19, 31, 45, 81, 91, 109, 131, 236, 405, 616, 803, 2886, 3875*, 5014*, 7930*, 9405*	22	28
42	2, 3, 5, 7, 13, 17, 19, 31, 53, 77, 95, 111, 119, 429, 463, 729, 879, 904, 1182, 3467*, 5562*, 6347*	22	28
43	2, 3, 5, 7, 13, 17, 19, 31, 50, 75, 81, 99, 192, 315, 338, 362, 369, 499, 1202, 1926, 2304, 6049*	22	28
44	2, 3, 5, 7, 13, 17, 19, 31, 93, 99, 179, 227, 337, 385, 550, 613, 795, 2506, 2627, 3910*, 6169*	21	28
45	2, 3, 5, 7, 13, 17, 19, 31, 87, 127, 176, 219, 295, 331, 420, 498, 697, 814, 2174, 2535, 6243*, 7158*, 8429*, 9065*	24	28
46	2, 3, 5, 7, 13, 17, 19, 31, 53, 65, 76, 93, 98, 154, 181, 194, 595, 644, 1829, 1994, 2410, 3242, 7321*	23	28
47	2, 3, 5, 7, 13, 17, 19, 31, 55, 56, 97, 128, 185, 192, 304, 348, 405, 543, 631, 750, 942, 1492, 1889, 2276, 4861*, 6206*, 6882*, 7001*, 7764*, 8146*	30	28
48	2, 3, 5, 7, 13, 17, 19, 31, 59, 69, 92, 123, 135, 283, 308, 387, 726, 897, 1325, 3561*, 4130*, 5153*, 9109*	23	28
49	2, 3, 5, 7, 13, 17, 19, 31, 50, 52, 87, 92, 129, 131, 185, 287, 494, 706, 728, 793, 1271, 1297, 1655, 1954, 2965, 3674*, 5129*, 5823*, 8847*, 9667*	30	28
50	2, 3, 5, 7, 13, 17, 19, 31, 57, 99, 107, 116, 161, 290, 518, 685, 688, 1044, 7182*	19	28
51	2, 3, 5, 7, 13, 17, 19, 31, 74, 97, 112, 184, 380, 382, 397, 411, 646, 3092, 6313*, 7128*, 7917*	21	28
52	2, 3, 5, 7, 13, 17, 19, 31, 57, 62, 76, 95, 192, 223, 333, 752, 1518, 2479, 3980*, 6056*, 7620*	21	28
53	2, 3, 5, 7, 13, 17, 19, 31, 79, 115, 116, 128, 129, 141, 243, 348, 380, 503, 664, 1135, 1212, 1250, 2221, 2417, 3141, 3841*, 4322*, 5007*, 6038*, 6161*, 6521*, 7344*, 7385*	33	28
54	2, 3, 5, 7, 13, 17, 19, 31, 59, 76, 82, 223, 285, 485, 791, 941, 1215, 1224, 1339, 3649*, 3737*, 5594*, 5758*, 5907*	24	28
55	2, 3, 5, 7, 13, 17, 19, 31, 56, 112, 142, 309, 499, 607, 1109, 1648, 2866, 3170, 4006*, 4345*, 5494*, 7059*	22	28
56	2, 3, 5, 7, 13, 17, 19, 31, 116, 178, 204, 303, 390, 475, 725, 1190, 2031, 5112*, 5234*, 8130*, 9776*	21	28
57	2, 3, 5, 7, 13, 17, 19, 31, 62, 69, 70, 187, 201, 425, 596, 928, 1125, 1951, 6164*, 7609*	20	28
58	2, 3, 5, 7, 13, 17, 19, 31, 82, 83, 92, 94, 112, 515, 793, 1117, 1385, 1471, 3503*, 6102*, 8559*	21	28
59	2, 3, 5, 7, 13, 17, 19, 31, 64, 68, 2260, 8086*, 9189*	13	28
60	2, 3, 5, 7, 13, 17, 19, 31, 99, 113, 172, 238, 576, 1877, 5904*, 8860*	16	28
61	2, 3, 5, 7, 13, 17, 19, 31, 61, 64, 68, 165, 291, 480, 1140, 1214, 1371, 2384, 2717, 3773*, 4203*, 5534*, 6924*	23	28
62	2, 3, 5, 7, 13, 17, 19, 31, 61, 68, 74, 131, 287, 593, 798, 4705*, 4827*, 5384*	18	28
63	2, 3, 5, 7, 13, 17, 19, 31, 61, 70, 92, 116, 135, 145, 152, 377, 387, 437, 603, 690, 695, 803, 1215, 1217, 1367, 1781, 2347, 2406, 7764*, 9297*	30	28
64	2, 3, 5, 7, 13, 17, 19, 31, 61, 77, 113, 163, 265, 351, 369, 824, 906, 2589, 2731, 3516*, 4820*, 9498*	22	28
65	2, 3, 5, 7, 13, 17, 19, 31, 61, 81, 93, 175, 276, 307, 382, 804, 1279, 1668, 2067, 2189, 3235, 5452*	22	28
66	2, 3, 5, 7, 13, 17, 19, 31, 61, 76, 196, 340, 382, 391, 655, 1276, 1289, 1419, 1488, 2314, 2795, 7465*, 8961*	23	28
67	2, 3, 5, 7, 13, 17, 19, 31, 61, 87, 116, 117, 207, 221, 226, 391, 402, 578, 733, 961, 984, 1078, 1107, 2019, 3232, 6755*, 6799*, 7198*, 9892*	29	28

Table 7: Primes of the form $L_k^{(n)}$, continued

n	k for which $L_k^{(n)}$ is prime or prp(*)	number of primes for $k \le 10000$	est. # of primes for $k \le 10000$
68	2, 3, 5, 7, 13, 17, 19, 31, 61, 107, 131, 139, 241, 369, 388, 435, 577, 782, 813, 1605, 1730, 1770, 5429*, 5482*, 7001*, 7277*, 7635*, 8019*	28	28
69	2, 3, 5, 7, 13, 17, 19, 31, 61, 70, 81, 94, 116, 146, 167, 176, 185, 237, 463, 563, 722, 1501, 1953, 3049, 5437*	25	28
70	2, 3, 5, 7, 13, 17, 19, 31, 61, 98, 123, 124, 187, 285, 582, 613, 761, 826, 1127, 1199, 2723, 3966*, 5867*, 6575*, 7888*	25	28
71	2, 3, 5, 7, 13, 17, 19, 31, 61, 103, 115, 129, 187, 238, 425, 1349, 1688, 2354, 2554, 5197*, 8616*	21	28
72	2, 3, 5, 7, 13, 17, 19, 31, 61, 113, 386, 393, 443, 811, 1045, 1229, 2032, 3721*, 5478*, 5935*, 6610*, 7391*, 7406*, 7664*, 7769*, 9063*	26	28
73	2, 3, 5, 7, 13, 17, 19, 31, 61, 87, 93, 242, 401, 409, 495, 604, 772, 893, 1682, 4864*, 5852*	21	28
74	2, 3, 5, 7, 13, 17, 19, 31, 61, 93, 99, 101, 199, 452, 823, 1934, 2994	17	28
75	2, 3, 5, 7, 13, 17, 19, 31, 61, 81, 123, 166, 213, 370, 515, 763, 2210, 2512, 2840, 3177, 8727*, 9893*	22	28
76	2, 3, 5, 7, 13, 17, 19, 31, 61, 128, 146, 261, 304, 491, 1005, 1032, 1229, 1671, 1887, 2835, 3723*, 3836*, 9200*	23	28
77	2, 3, 5, 7, 13, 17, 19, 31, 61, 133, 142, 154, 182, 187, 206, 444, 604, 796, 839, 916, 1152, 1528, 1541, 2222, 3002, 3662*, 3768*, 5792*, 7384*, 9444*, 9495*	31	28
78	2, 3, 5, 7, 13, 17, 19, 31, 61, 88, 368, 555, 719, 1101, 1418, 1904, 4475*, 4676*, 5287*, 5585*	20	28
79	2, 3, 5, 7, 13, 17, 19, 31, 61, 81, 91, 161, 251, 476, 535, 590, 662, 814, 841, 971, 1097, 1205, 2967, 3052, 5403*, 6036*, 7671*	27	28
80	2, 3, 5, 7, 13, 17, 19, 31, 61, 87, 93, 249, 366, 461, 528, 535, 632, 675, 844, 1157, 1894, 2068, 2241, 2849, 3617*, 6028*, 7381*, 8614*	28	28
81	2, 3, 5, 7, 13, 17, 19, 31, 61, 99, 141, 233, 235, 290, 425, 535, 615, 649, 1317, 1722, 5129*, 5831*, 8044*, 8595*, 8681*	25	28
82	2, 3, 5, 7, 13, 17, 19, 31, 61, 161, 286, 287, 788, 2402, 6383*, 7236*, 8738*	17	28
83	2, 3, 5, 7, 13, 17, 19, 31, 61, 249, 422, 423, 1031, 1062, 1180, 2637, 2836, 4919*, 5641*, 8043*, 8197*	21	28
84	2, 3, 5, 7, 13, 17, 19, 31, 61, 95, 219, 303, 736, 1120, 1447, 1590, 2076, 2788, 4163*, 4490*	20	28
85	2, 3, 5, 7, 13, 17, 19, 31, 61, 127, 221, 740, 1607, 3139, 3800*, 5533*, 8133*, 9747*	18	28
86	2, 3, 5, 7, 13, 17, 19, 31, 61, 93, 299, 379, 416, 470, 606, 662, 1791, 2908, 3999*	19	28
87	2, 3, 5, 7, 13, 17, 19, 31, 61, 109, 121, 269, 326, 367, 551, 593, 883, 1083, 3036, 3294, 5433*, 8169*, 9523*	23	28
88	2, 3, 5, 7, 13, 17, 19, 31, 61, 107, 112, 129, 493, 1173, 1549, 3407*, 3424*, 5895*	18	28
89	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 164, 169, 1829, 1880, 2888, 3408*, 3694*, 5173*, 5567*, 7612*, 7983*	21	28
90	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 202, 256, 262, 297, 373, 383, 485, 844, 3390*, 5261*, 5533*, 6743*, 7572*	23	28
91	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 169, 201, 250, 526, 570, 731, 807, 964, 1062, 2515, 2908, 3139	22	28
92	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 164, 170, 196, 215, 225, 284, 353, 375, 379, 491, 628, 668, 1148, 1888, 2135, 4069*, 6331*, 6483*, 7904*	29	28
93	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 112, 146, 225, 470, 568, 599, 654, 941, 993, 3012, 7306*	21	28
94	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 159, 325, 581, 708, 743, 1260, 1527, 1983, 4575*, 6507*, 7747*	21	28
95	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 176, 181, 193, 219, 283, 348, 387, 392, 527, 549, 761, 766, 848, 852, 911, 1041, 1067, 3721*, 3914*, 4304*, 5552*, 5905*, 7409*	33	28
96	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 125, 166, 184, 287, 316, 392, 822, 1252, 2095, 2185, 3056, 3235, 3663*, 7381*, 8634*	25	28
97	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 104, 213, 274, 697, 1982, 2908, 3678*, 4236*	18	28
98	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 161, 221, 478, 591, 748, 825, 860, 943, 1488, 1567, 4615*, 5292*, 8338*	23	28
99	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 129, 261, 348, 639, 1301, 1531, 4151*, 6775*	18	28
100	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 124, 179, 211, 243, 531, 599, 814, 934, 1529, 3803*, 4768*, 5496*, 6554*, 7858*, 8764*, 9083*, 9976*	27	28

2000 *Mathematics Subject Classification*: Primary 11A41; Secondary 11B39.
Keywords: prime numbers, generalized Fibonacci and Lucas numbers.

(Concerned with sequences A000032, A000045, A000073, A000078, A001591, A001592, A001605, A001606, A001644, A005478, A005479, A066178, A073817, A074048, A074584, A079262, A092835, A092836, A104534, A104535, A104576, A104577, A104621, A104622, A105753, A105754, A105755, A105756, A105757, A105758, A105759, A105760, A105761, A105762, A105763, A105764, A105765, A105766, A105767, and A105768.)

Received April 19 2005; revised version received September 5 2005. Published in *Journal of Integer Sequences*, September 20 2005.

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