

# Point and Area Sweep Coverage in Wireless Sensor Networks

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**Abstract**—Sweep coverage term is recently introduced for coverage in wireless sensor networks. The criteria for sweep coverage is different from the traditional coverage problem where a continuous monitoring with sensor nodes is required. But in sweep coverage, periodic monitoring is sufficient with a small number of mobile sensor nodes. Finding minimum number of mobile sensor nodes with a constant velocity to guarantee sweep coverage is NP-hard and it cannot be approximated within a factor of 2 [11]. In this paper we have proposed a 2-approximation algorithm for solving the sweep coverage for a given set of points of interest (PoI). The best known approximation factor is 3 for this problem [11]. When all PoI are static sensor nodes, we have proposed a distributed approximation algorithm with approximation ratio 2, where the sensor nodes compute the number of mobile nodes and their positions. We have introduced sweep coverage for a given area of interest (AoI) and proved that the problem is NP-complete. A  $2\sqrt{2}$ -approximation algorithm is proposed in order to solve the problem for a square region. A generalized version of the area sweep coverage problem for an arbitrary bounded region is also investigated in this paper.

**Key words:** Sweep Coverage, Approximation Algorithm, Mobile Sensor, MST, Euler tour, TSP, Wireless Networks.

## I. INTRODUCTION

Coverage in wireless sensor networks (WSNs) have been widely studied for different monitoring applications. In general coverage is defined as the measurement of the quality of surveillance of sensing function. In practice, sensor nodes are deployed over an area of interest (AoI) to get desire information from the area. Amount of information collected from the AoI depends on how well the AoI is covered by the deployed sensor nodes. Coverage problems are broadly categorized in three types depending on applications. First one is point coverage [5], [9], [12], where a set of discrete points in the AoI is continuously monitored, second one is area coverage [2], [13], [16], where all points in the AoI are continuously monitored, and third one is barrier coverage [6], [14], where specified path or boundary of the AoI is continuously monitored by sensor nodes. For example in forest monitoring [1] full coverage is required, where every location of the forest must be covered by at least one sensor node such that any unusual activities like forest fire, activities of poachers, etc. can be immediately detected. Similarly, if boundary of the forest is in the coverage range of sensor nodes then it might be possible for controlling and elimination of the poaching activities, and illegal entry through the boundary.

A continuous monitoring with static sensor nodes is required for the aforementioned types of coverage problems. But there are typical applications where only periodic patrol inspections are sufficient for a certain set of points of interest instead of continuous monitoring like traditional coverage.

For those applications periodic monitoring of a given set of PoI is sufficient and which is termed as *sweep coverage*. In the sweep coverage scenario deployment of static sensor nodes at the PoI may partially solve the purpose but it suffers from poor efficiency and unnecessary extra overhead.

In this paper we have considered point sweep coverage problem [4], [7], [11], [18] where a given set of discrete points are monitored by a set of mobile sensor nodes at least once within a given period of time. Finding minimum number of mobile sensor nodes for the sweep coverage of a given set of discrete points in a plane is NP-hard, proved in the paper [11]. An area sweep coverage problem is formulated in this paper with the basic concept of point sweep coverage.

## II. OUR CONTRIBUTION

In this paper, our contributions on sweep coverage problems are following:

- We have proposed a 2-approximation algorithm for point sweep coverage problem, where the best known approximation algorithm proposed by Li et al. in [11] is 3-factor.
- A distributed 2-approximation algorithm is proposed for the point sweep coverage problem when the PoIs are static sensor nodes.
- We have introduced the area sweep coverage problem and proved that the problem is NP-complete. A  $2\sqrt{2}$ -approximation sequential algorithm is proposed in order to solve the area sweep coverage problem for a square region. For arbitrary bounded region  $\mathcal{A}$  the approximation factor is  $2\left(\sqrt{2} + \frac{2r\mathcal{P}}{\text{Area}(\mathcal{A})}\right)$ , where  $\mathcal{P}$  is the perimeter of  $\mathcal{A}$ . To the best of our knowledge, there is no previous work which discussed about the complexity and approximation algorithm on area sweep coverage problem.

## III. RELATED WORK

Several approaches have been found in the literature to overcome coverage problems in wireless sensor networks. As most of the coverage problems presented in the papers [6], [10], [11] are NP-complete, several heuristics [13], [16] and approximation algorithms [5], [6] are proposed to solve the

problems. The point coverage problems are studied in the works [5], [9], [12]. In [5], the authors consider the geometric version of the point coverage problem called unit disk cover problem. They discussed about the computational complexity of the problem which is NP-hard. An constant factor approximation algorithm is provided to solve the problem.

In paper [17], Wang et al. proposed three movement assisted algorithms for area coverage, which are VEC (vector based algorithm), VOR (voronoi based algorithm) and Minimax. In this paper the authors used voronoi diagram to identify coverage holes. Three different movement strategies give efficient improvement of the coverage with mobile sensor nodes. In the paper [13], Ma et al. proposed an distributed heuristic where sensor nodes move such a way that the overall topology becomes close to an equilateral triangulation of the plane which is the optimal layout for area coverage problem.

The concept of sweep coverage initially comes from the context of robotics [3]. Recently there are several works [4], [7], [11], [18] found in the literature in the area of point sweep coverage problem. In [7], Du et al. considered two different movement constraints on the mobile sensor nodes. In the first approach, mobile sensor nodes move in the same path in every time period and in the second approach the nodes move in different paths in different time periods.

The problem of point sweep coverage is rigorously studied in the paper [11] by Li et al. The authors proved that finding minimum number of mobile sensor nodes to sweep coverage a set of discrete points is NP-complete. They show that this problem is equivalent to solve the TSP and proved that the point sweep coverage problem cannot be approximated less than a factor of 2. One  $(2 + \epsilon)$ -approximation and a 3-approximation algorithm are proposed. The authors remarked on the impossibility of design distributed local algorithms, which can guarantee the required sweep coverage, *i.e.*, a mobile sensor node cannot locally determine whether all PoI are sweep covered without global information.

The rest of the paper is organized as follows. We have discussed about the point sweep coverage in section IV. In section V, we have proposed a distributed algorithm for point sweep coverage. In section VI we have introduced area sweep coverage problem and proposed an algorithm for solving the problem.

#### IV. POINT SWEEP COVERAGE

The definition of the point sweep coverage is given below as on the paper [11].

*Definition 1:* Let  $\mathcal{U} = \{u_1, u_2, \dots, u_n\}$  be a set of discrete PoI on a two dimensional plane and  $S = \{s_1, s_2, \dots, s_m\}$  be a set of mobile sensor nodes. A point  $u_i$  is said to be  $t_i$  point sweep covered if and only if at least one mobile sensor node visits  $u_i$  within every  $t_i$  time period. The set  $\mathcal{U}$  is said to be globally point sweep covered by the mobile sensor nodes of  $S$  if all  $u_i \in \mathcal{U}$  are  $t_i$  sweep covered. The time period  $t_i$  is said to be the *sweep period* of the point  $u_i$ .

Let us consider a set of PoI on a two dimensional plane containing  $n$  points. Let  $G = (V, E)$  be the complete graph

with each point as a vertex and the line joining two points  $i$  and  $j$  on the plane as edge. The Euclidean distance  $d(i, j)$  is the weight  $w(i, j)$  of edge  $(i, j)$ . Let  $v$  be the uniform velocity of all mobile sensor nodes and  $t$  be the common sweep period for the set of  $n$  points in  $V$ .

**Problem Definition (Point Sweep Coverage):** The point sweep coverage problem is to find optimal (minimum) number of mobile sensor nodes to guarantee sweep coverage for all points in  $V$ .

Li et al. in the paper [11] proved that the above point sweep coverage problem is equivalent to finding minimum TSP tour of  $G$ . Using ‘Metric TSP - factor 2’ approximation algorithm [15], one TSP tour  $L$  (say) can be found on  $G$ . Let  $L_{opt}$  be the optimal TSP tour of  $G$ . Clearly,  $L \leq 2L_{opt}$ . Partition  $L$  into  $\lceil \frac{L}{vt} \rceil$  parts of length  $vt$  each as shown in the figure 1.

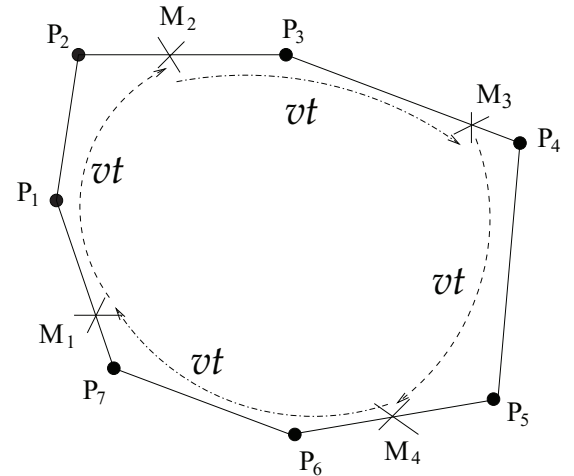


Fig. 1: The figure showing TSP tour for the PoI,  $P_1, P_2, \dots, P_7$ , and initial positions of the mobile sensor nodes  $M_1, M_2, M_3, M_4$  for sweep coverage of the PoI.

Then deploy  $\lceil \frac{L}{vt} \rceil$  mobile sensor nodes at all partition points, one for each. To cover each PoI periodically, each mobile sensor node starts moving at the same time along  $L$  in the same direction (either clockwise or anti clockwise). Based on the above discussion we have proposed POINTSWEEP COVERAGE algorithm.

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#### Algorithm 1 POINTSWEEP COVERAGE

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- 1: Find a TSP tour  $L$  of  $G$  by the method of finding Euler tour from a MST of  $G$  by ‘Metric TSP - factor 2’ approximation algorithm [15].
  - 2: Partition  $L$  into  $\lceil \frac{L}{vt} \rceil$  parts and deploy  $\lceil \frac{L}{vt} \rceil$  mobile sensor nodes at all partition points, one for each.
  - 3: Each mobile sensor node then starts moving at the same time along  $L$  in same direction.
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Following lemma and theorem prove correctness of the above algorithm.

*Lemma 1:* Each point on  $L$  can be visited by at least one mobile sensor node in every time period  $t$ .

*Proof:* Let us consider any point  $p$  on  $L$  and let  $t_0$  be the time when a mobile sensor node visited  $p$  last time. Now we have to prove that the point  $p$  must be visited by at least one mobile sensor node in  $t_0 + t$  time. According the deployment strategy of mobile sensor nodes any two consecutive mobile sensor nodes are within the distance of  $vt$  at any time. So, when a mobile sensor node visited  $p$  at  $t_0$  another mobile sensor node is on the way to  $p$  and within the distance of  $vt$  along  $L$ . Hence  $p$  will be again visited by another mobile sensor node within next  $t$  time. ■

*Theorem 1:* The number of mobile sensor nodes required for the algorithm: POINTSWEEP\_COVERAGE is no more than twice the number needed for the optimal solution.

*Proof:* Let  $L_{opt}$  be the optimal TSP tour for the graph  $G$ . Let  $L$  be the TSP tour calculated by our algorithm: POINTSWEEP\_COVERAGE. Then by the ‘Metric TSP - factor 2’ approximation algorithm [15],  $L \leq 2L_{opt}$ . Let  $N_{opt}$  be the number of mobile sensor nodes required for optimal solution. Then  $N_{opt} \times vt \geq L_{opt}$ , i.e.,  $N_{opt} \geq \left\lceil \frac{L_{opt}}{vt} \right\rceil$ . The number of mobile sensor nodes calculated by algorithm: POINTSWEEP\_COVERAGE is  $\left\lceil \frac{L}{vt} \right\rceil$  ( $=N$ , say). Therefore, the approximation ratio of the algorithm: POINTSWEEP\_COVERAGE is equal to  $\frac{N}{N_{opt}} \leq \left\lceil \frac{2L_{opt}}{vt} \right\rceil / \left\lceil \frac{L_{opt}}{vt} \right\rceil \leq 2$ . ■

## V. DISTRIBUTED POINT SWEEP COVERAGE

In this section we have proposed distributed point sweep coverage problem, where static sensor nodes are placed at every point of interest. The static sensor nodes collect coverage information from the surrounding environment. Mobile sensor nodes are scheduled for their visits to the respective static sensor nodes. The mobile sensor nodes are moving on the plane according to their schedule and designated path for collecting data from all static sensor nodes. In the distributed algorithm each static sensor node communicates with its immediate neighbors and collectively computes the number of mobile sensor nodes required for sweep coverage.

### A. System Model

Let  $n$  static sensor nodes with communication range  $r_c$  are placed at all points of interest with unique  $id \in \{0, 1, 2, \dots, n-1\}$ . Let  $G_c$  be the connected communication graph, where the sensor node  $i$  is a vertex and  $(i, j)$  is the edge between nodes  $i$  and  $j$ . Edge weight  $w(i, j)$  for the edge  $(i, j)$  is the euclidian distance ( $dist(i, j)$ ) between nodes  $i$  and  $j$ . Let  $G$  be the corresponding complete graph of  $G_c$ . Let  $t$  be the sweep period of every static sensor node. All mobile sensor nodes can move with constant speed  $v$ . Every static node uses *control message* for finding Euler tour by marking the edges and the positions of the mobile sensor nodes by marking the point on the edges. There is an initiator node  $s$  for initiating the control message. Every static sensor node maintains a variable *status* for each edge incident to it, which can be either *used* or *unused*. An edge *status* := *used* means one control message for finding *Euler tour* has passed through that edge, otherwise *status* := *unused*.

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### Algorithm 2 DISTRIBUTEDSWEEP\_COVERAGE

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- 1: Find a MST  $T_c$  of  $G_c$  by the distributed MST algorithm presented in [8].
  - 2: Doubling every edge  $(i, j) \in T_c$  with same weight, one edge is  $(i, j)$  and other is  $(j, i)$ .
  - 3: Initial status of each edge  $(i, j)$  is *unused*
  - 4: Initiator  $s$  selects an *unused* edge  $(s, j)$ , marks the edge for *Euler tour* and assigns  $d = w(s, j)$
  - 5: If  $\lfloor \frac{d}{vt} \rfloor \geq 1$  then  $s$  calculates  $\lfloor \frac{d}{vt} \rfloor$  number of points on  $(s, j)$  and sends  $\langle m, d - vt \times \lfloor \frac{d}{vt} \rfloor \rangle$  to  $j$ . Mark the points for mobile sensor nodes. Otherwise  $s$  sends a message  $\langle m, d \rangle$  to the node  $j$ . The status of the edge  $(s, j)$  become *used*.
  - 6: A node  $j$  executes following after receiving a message  $\langle m, d \rangle$  from a node  $i$ , Choose an *unused* edge  $(j, k)$  other than  $(j, i)$  and marks the edge for *Euler tour*. Now  $d = d + w(j, k)$ . If  $\lfloor \frac{d}{vt} \rfloor \geq 1$ , calculates  $\lfloor \frac{d}{vt} \rfloor$  number of points on and sends  $\langle m, d - vt \times \lfloor \frac{d}{vt} \rfloor \rangle$  to  $k$  and mark the points for mobile sensor nodes. Otherwise  $j$  sends a message  $\langle m, d \rangle$  to the node  $k$ . The status of the edge  $(j, k)$  become *used*. If no such *unused* edge  $(j, k)$  other than  $(j, i)$  exists then  $j$  executes step 6 for the edge  $(j, i)$ .
  - 7: When the message  $\langle m, d \rangle$  returns back to the initiator  $s$  and every edge incident to  $s$  is *used*, and if  $d > 0$ , it marks the position of  $s$  for a mobile sensor node.
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The DISTRIBUTEDSWEEP\_COVERAGE algorithm returns marked edges for Euler tour and corresponding marked points for placement of mobile sensor nodes. The mobile sensor nodes are start moving along the edges of the Euler tour at the same time once they are placed. The movement of all mobile sensor nodes guarantee sweep coverage for all the static sensor nodes. Following theorems give correctness and complexity measure of the algorithm.

Based on the following theorem 2 the cost of the MST on  $G$  constructed according to the algorithm 1 is same as the cost of the MST on  $G_c$  constructed according to the algorithm 2.

*Theorem 2:* Let  $G_c$  be the connected communication graph with communication range  $r_c$  of  $n$  static sensor nodes and  $G$  be the corresponding complete graph. Then cost of the respective MST of  $G_c$  and  $G$  are same.

*Proof:* Let  $T$  be a minimum spanning tree of  $G$  and  $T_c$  be the minimum spanning tree of  $G_c$ . Clearly,  $w(T) \leq w(T_c)$ . For all the edges  $e \in E(G_c)$  implies  $w(e) \leq r_c$ , since  $r_c$  is the communication range. Therefore,  $e \in E(T_c)$  implies  $w(e) \leq r_c$ . Now let us consider the following two cases:

**Case 1:** If all the edges of  $T$  are with weight  $\leq r_c$ . Then  $T$  is a spanning tree of  $G_c$  and therefore  $w(T_c) = w(T)$ .

**Case 2:** If case 1 is not true then there exist an edge  $e$  of  $T$  such that  $w(e) > r_c$ . Let us consider the graph  $T_c \cup \{e\}$ . This graph must contains a cycle containing the edge  $e$  and  $w(e)$  is greater than the weights of all other edges of the cycle. So, by the cycle property of minimum spanning tree,  $e$  can not be a part of any MST of  $G$ , contradicting the fact that  $e$  is

a part of  $T$ . Which implies  $w(e) \leq r_c$ . Therefore, by case 1,  $w(T_c) = w(T)$ . ■

*Theorem 3:* The DISTRIBUTEDSWEEP\_COVERAGE algorithm produces approximation ratio 2 for finding minimum number of mobile sensor nodes.

*Proof:* Let  $L_{opt}$  be the optimal TSP tour on the corresponding complete graph  $G$ . If  $T$  be an MST on  $G$  with cost  $w(T)$ , then one can easily prove  $w(T) \leq w(L_{opt})$ . The algorithm 2 computes an MST  $T_c$  on the graph  $G_c$  and by theorem 2,  $w(T_c) = w(T)$ . Therefore,  $w(T_c) \leq w(L_{opt})$ . Let  $G_e$  be the Eulerian graph formed after doubling each edge of  $T_c$ . Then  $w(G_e) \leq 2w(L_{opt})$ . By theorem 1, the approximation ratio of the algorithm DISTRIBUTEDSWEEP\_COVERAGE is 2. ■

*Theorem 4:* Total number of messages required for computing the number of mobile sensor nodes by the algorithm DISTRIBUTEDSWEEP\_COVERAGE is  $O(n \log n + |E|)$ .

*Proof:* Total number of messages required for finding the number and the initial positions of the mobile sensor nodes depend on the distributed construction of MST and finding Euler tour by doubling each edge of the MST. Total number of messages required for construction of the MST by the distributed algorithm presented in [8] is  $O(n \log n + |E|)$ . Total  $2(n - 1)$  messages are required for finding Euler tour by the algorithm DISTRIBUTEDSWEEP\_COVERAGE, since in the steps 4 and 5 each node sends one message through all the tree edges which are incident on it. Therefore the total number of messages required for the algorithm is  $O(n \log n + |E|)$ . ■

## VI. AREA SWEEP COVERAGE

In area sweep coverage all the points of a bounded region must be covered at least once in a certain time period. Same strategy like point sweep coverage cannot be applied to solve the area sweep coverage since there are uncountable number of points in any bounded region. Here we are considering homogeneous mobile sensor nodes with sensing range  $r$ . The sensor nodes can be considered as sensing disks with radius  $r$ , centering at the nodes.

*Definition 2:* Let  $\mathcal{A}$  be a given bounded area of interest (AoI) and  $S = \{s_1, s_2, \dots, s_m\}$  be a set of mobile sensor nodes.  $\mathcal{A}$  is said to be  $t$ -area sweep covered if and only if each point of  $\mathcal{A}$  is in the sensing disk of at least one mobile sensor node in every  $t$  time period. The time period  $t$  is said to be *sweep period* of the area  $\mathcal{A}$ .

**Problem Definition (Area Sweep Coverage):** The area sweep coverage problem is to find optimal (minimum) number of mobile sensor nodes such that for a given time period  $t$ , each point of a given area  $\mathcal{A}$  is  $t$ -sweep covered.

### A. Complexity result

The decision version of the area sweep coverage problem: Whether every point of the area  $\mathcal{A}$  is covered by at least once in  $t$  time period by given number of mobile sensor nodes and their corresponding paths of travel?

*Theorem 5:* The area sweep coverage problem is NP-complete.

*Proof:* Let a number of mobile sensor nodes with their corresponding paths be given. Then it is possible to verify whether or not all points of the AoI are covered at least once by a mobile sensor node in polynomial time. Hence the decision version of the area sweep coverage problem is in NP.

Li et al. in the article [10] have shown that covering a bounded region with minimum number of static sensor node is NP-hard. Covering a bounded region is a special instance of the area sweep coverage problem when the sweep period  $t = 0$ . Hence the decision version of the area sweep coverage problem is also NP-hard. Therefore the area sweep coverage problem is NP-complete. ■

### B. Solution of area sweep coverage

Although the area sweep coverage is different from point sweep coverage, but it can be solved using a solution of the point sweep coverage. Let us consider a point set  $P = \{p_1, p_2, \dots, p_k\}$ , where each  $p_i \in \mathcal{A}$  such that a placement of static sensor nodes at each point  $p_i$  covers  $\mathcal{A}$ .

*Lemma 2:* The point sweep coverage of the set of points  $P$  on  $\mathcal{A}$  guarantees the area sweep coverage of  $\mathcal{A}$ .

*Proof:* Suppose we have a solution for point sweep coverage for the set of points  $P$ . Therefore at least one mobile sensor node visits each point  $p_i$  at least once in every  $t$  time period. Since one static sensor node at each  $p_i$  gives complete coverage of  $\mathcal{A}$  (according to above assumption), each point of  $\mathcal{A}$  is inside at least one sensing disk in every  $t$  time period. Hence the solution of point sweep coverage problem for  $P$  is also a solution of area sweep coverage problem for  $\mathcal{A}$ . ■

Based on the above lemma 2 we have proposed AREASWEEP\_COVERAGE algorithm for area sweep coverage problem, where  $\mathcal{A}$  is a square region. Assume side of the square is divisible by  $\sqrt{2}r$ . Divide the area into square grid of side  $\sqrt{2}r$ . Let  $P$  be the set of center points of each grid cell. Clearly, putting one static sensor node with sensing radius  $r$  at each point of  $P$  covers  $\mathcal{A}$ . Therefore by lemma 2, the solution of point sweep coverage problem for  $P$  on  $\mathcal{A}$  is also a solution of the area sweep coverage problem for  $\mathcal{A}$ .

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### Algorithm 3 AREASWEEP\_COVERAGE

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- 1: Divide the area into square grid of side  $\sqrt{2}r$ .
  - 2: Let  $P$  is the set of center points of each square grid.
  - 3: Apply algorithm 1 to compute number of mobile sensor nodes required for sweep coverage for  $P$
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### C. Analysis and complexity measure

*Lemma 3:* For a mobile sensor node with sensing range  $r$ , the area covered by the node due to its movement is maximum when the node moves in straight line.

*Proof:* Let us consider a straight line  $l$  and an arbitrary curve  $c$  (non straight line) of same length  $\epsilon$ , where  $\epsilon < 2r$ . Place one sensing disk at the starting point and other sensing disk at the ending point of  $l$  and  $c$  respectively. Clearly, the overlap between the sensing disks of  $c$  is more than the

overlap between the sensing disks of  $l$ , as the distance between two end points of  $c$  is less than  $\epsilon$ . Hence the proof. ■

*Lemma 4:* If  $N_{opt}$  is the number of mobile sensor nodes for the optimal solution of the area sweep coverage problem then  $N_{opt} \geq \frac{Area(\mathcal{A})}{2rvt}$ .

*Proof:* The total path moved by a node in  $t$  time is  $vt$  where  $v$  has been considered as velocity of the nodes. According to the Lemma 3, the coverage area is maximum when the node moves in straight line. Let at time  $t_0$  the node is at  $I$  and at time  $t_0+t$  the node is at  $F$  on the straight line  $IF$  as shown in Fig. 2. At time  $t_0$ , the node covers the area  $\pi r^2$ ,

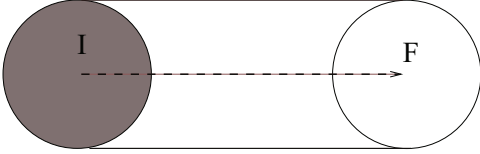


Fig. 2: The area covered by a mobile sensor node in time  $t$  shown in the white area. The colored disk is the initial position of the sensing disk

which is shown in colored disk in the figure 2. The remaining area  $2rvt$  is covered due to movement of the node along the line  $IF$ , shown in white area in the same figure. So, the total area covered by the node with radius  $r$  in time  $t$  is  $2rvt$ . If  $N_{opt}$  is the number of mobile sensor nodes in the optimal solution to sweep cover  $\mathcal{A}$ , then  $2rvt \times N_{opt} \geq Area(\mathcal{A})$ , implies  $N_{opt} \geq \frac{Area(\mathcal{A})}{2rvt}$ . ■

*Theorem 6:* The approximation ratio of the algorithm AREASWEEPCOVERAGE is  $2\sqrt{2}$ .

*Proof:* The total number of points in the set  $P$  is  $\frac{Area(\mathcal{A})}{2r^2}$ . Now let us consider the graph with the set of points in  $P$  as vertex and the edges are the line joining the centers of the adjacent cells. Clearly each edge of this graph is of length  $\sqrt{2}r$ . Therefore by the theorem 2, MST of this graph and the corresponding complete graph is same. As each edge of this graph is  $\sqrt{2}r$ , the cost of the minimum spanning tree is  $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r$ . By the algorithm ‘Metric TSP - factor 2’ [15], the cost of the TSP computed by the algorithm 1 is  $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times 2$ . Therefore the number of mobile sensor nodes computed by the algorithm 3 is  $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times \frac{2}{vt}$ . Therefore the approximation ratio of our algorithm is  $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times \frac{2}{N_{opt} \times vt} \leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times \frac{2}{\frac{Area(\mathcal{A})}{2rvt} \times vt} = 2\sqrt{2}$ . ■

#### D. Solution for arbitrary bounded region

In the previous section, we discuss the area sweep coverage problem for a bounded square region. In this section we consider the same problem for an arbitrary bounded region.

Let  $\mathcal{A}$  be an arbitrary bounded region. We divide the region  $\mathcal{A}$  into square grids of length  $\sqrt{2}r$ . Since the shape of the region is arbitrary, the grid cells which are adjacent to the perimeter  $\mathcal{P}$  (say) of the region may not be complete or non-square. The maximum number of non-square grid cell is  $\leq$

$\frac{\mathcal{P}}{\sqrt{2}r}$ , whereas  $\frac{Area(\mathcal{A})}{2r^2}$  is the maximum number of square grid cells. Now we will apply the same algorithm 3 on the set of points  $P$ .

*Theorem 7:* The approximation ratio of the algorithm 3 for an arbitrary bounded region is  $2 \left( \sqrt{2} + \frac{2r\mathcal{P}}{Area(\mathcal{A})} \right)$ .

*Proof:* The number of grid points in  $\mathcal{A}$  is  $\leq \frac{Area(\mathcal{A})}{2r^2} + \frac{\mathcal{P}}{\sqrt{2}r}$ . Therefore the number of nodes required for sweep coverage of  $P$  is  $\leq \left( \frac{Area(\mathcal{A})}{2r^2} + \frac{\mathcal{P}}{\sqrt{2}r} \right) \times \sqrt{2}r \times \frac{2}{vt}$ . Hence the approximation ratio of our algorithm is  $\leq \left( \left( \frac{Area(\mathcal{A})}{2r^2} + \frac{\mathcal{P}}{\sqrt{2}r} \right) \times \sqrt{2}r \times \frac{2}{vt} \right) / \left( \frac{Area(\mathcal{A})}{2rvt} \right) = 2 \left( \sqrt{2} + \frac{2r\mathcal{P}}{Area(\mathcal{A})} \right)$ . ■

## VII. CONCLUSION AND FUTURE WORKS

Sweep coverage has introduced recently for sensor network monitoring. In this paper we have proposed two kinds of sweep coverage problems, point sweep coverage and area sweep coverage. In point sweep coverage our proposed algorithm achieved the best possible approximation factor 2, since according the result in [11] it cannot be approximated within a factor of 2.

If the PoI’s are static sensor nodes, a distributed version of the same algorithm is proposed where the static sensor nodes can coordinately find the numbers and locations of the mobile sensor nodes through message passing. Area sweep coverage problem has introduced in this paper with the idea of point sweep coverage. We have proved that the area sweep coverage problem is NP-complete.  $2\sqrt{2}$ -approximation algorithm is proposed for a square region to solve the problem. For arbitrary bounded region the approximation factor is a function of perimeters and area of the bounded region. In future, we try to improve the approximation factor for the area sweep coverage for arbitrary bounded regions.

## REFERENCES

- [1] Fadi M. Al-Turjman, Hossam S. Hassanein, and Mohamed A. Ibnkahla. Connectivity optimization for wireless sensor networks applied to forest monitoring. In *Proceedings of the 2009 IEEE international conference on Communications, ICC’09*, pages 285–290, Piscataway, NJ, USA, 2009. IEEE Press.
- [2] Novella Bartolini, Tiziana Calamoneri, Emanuele G. Fusco, Annalisa Massini, and Simone Silvestri. Push & pull: autonomous deployment of mobile sensors for a complete coverage. *Wireless Networks*, 16(3):607–625, 2010.
- [3] Maxim A. Batalin and Gaurav S. Sukhatme. Multi-robot dynamic coverage of a planar bounded environment. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (Submitted)*, 2002.
- [4] Hung-Chi Chu, Wei-Kai Wang, and Yong-Hsun Lai. Sweep coverage mechanism for wireless sensor networks with approximate patrol times. *Ubiquitous, Autonomic and Trusted Computing, Symposia and Workshops on*, 0:82–87, 2010.
- [5] Gautam K. Das, Robert Fraser, Alejandro López-Ortiz, and Bradford G. Nickerson. On the discrete unit disk cover problem. In *WALCOM*, pages 146–157, 2011.
- [6] Dinesh Dash, Arijit Bishnu, Arobinda Gupta, and Subhas C. Nandy. Approximation algorithms for deployment of sensors for line segment coverage in wireless sensor networks. In *COMSNETS*, pages 1–10, 2012.
- [7] Junzhao Du, Yawei Li, Hui Liu, and Kewei Sha. On sweep coverage with minimum mobile sensors. In *Proceedings of the 2010 IEEE 16th International Conference on Parallel and Distributed Systems, ICPADS ’10*, pages 283–290, Washington, DC, USA, 2010. IEEE Computer Society.

- [8] R. G. Gallager, P. A. Humblet, and P. M. Spira. A distributed algorithm for minimum-weight spanning trees. *ACM Trans. Program. Lang. Syst.*, 5(1):66–77, January 1983.
- [9] Yu Gu, Yusheng Ji, Jie Li, and Baohua Zhao. Fundamental results on target coverage problem in wireless sensor networks. In *GLOBECOM'09*, pages 1–6, 2009.
- [10] Jing Li, Ruchuan Wang, Haiping Huang, and Lijuan Sun. Voronoi-based coverage optimization for directional sensor networks. *Wireless Sensor Network*, 1(5):417–424, 2009.
- [11] Mo Li, Wei-Fang Cheng, Kebin Liu, Yunhao Liu, Xiang-Yang Li, and Xiangke Liao. Sweep coverage with mobile sensors. *IEEE Trans. Mob. Comput.*, 10(11):1534–1545, 2011.
- [12] Mingming Lu, Jie Wu, Mihaela Cardei, and Minglu Li. Energy-efficient connected coverage of discrete targets in wireless sensor networks. In *ICCNMC'05*, pages 43–52, 2005.
- [13] Ming Ma and Yuanyuan Yang. Adaptive triangular deployment algorithm for unattended mobile sensor networks. *IEEE Trans. Computers*, 56(7), 2007.
- [14] Seapahn Meguerdichian, Farinaz Koushanfar, Miodrag Potkonjak, and Mani B. Srivastava. Coverage problems in wireless ad-hoc sensor networks. In *INFOCOM'01*, pages 1380–1387, 2001.
- [15] Vijay V. Vazirani. *Approximation algorithms*. Springer-Verlag New York, Inc., New York, NY, USA, 2001.
- [16] Guiling Wang, Guohong Cao, and T. LaPorta. A bidding protocol for deploying mobile sensors. In *Proceedings. 11th IEEE International Conference on Network Protocols (ICNP 2003)*, pages 315 – 324, Nov. 2003.
- [17] Guiling Wang, Guohong Cao, and Thomas F. LaPorta. Movement-assisted sensor deployment. *IEEE Trans. Mob. Comput.*, 5(6):640–652, 2006.
- [18] Min Xi, Kui Wu, Yong Qi, Jizhong Zhao, Yunhao Liu, and Mo Li. Run to potential: Sweep coverage in wireless sensor networks. In *ICPP*, pages 50–57, 2009.