Optimal Delayed Wi-Fi Offloading

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***Abstract*—Wi-Fi offloading can help the mobile operator to obtain immediate capacity relief when facing the explosive growth of mobile data traffic. In this paper, we study the Wi-Fi offloading problem with delay tolerant applications. We formulate it as a finite-horizon sequential decision problem, where the objective is to minimize the total cellular usage plus the potential penalty for deadline violation. We solve the problem optimally using dynamic programming, and propose a general optimal delayed Wi-Fi offloading (ODWO) algorithm. For the special case with a convex penalty function and fixed location-independent data rates, we show that the optimal policy exhibits a threshold structure. A monotone ODWO algorithm with a lower complexity can be used in this case. Simulation results show that the ODWO scheme achieves both the minimal total cost and the highest file transfer efficiency as compared with two heuristic schemes.**

I. INTRODUCTION

With the proliferation of smartphones and mobile social networking, consumer demands for wireless data services are growing very rapidly, such that the capacity of the cellular network is pushed to its limit. According to Cisco's forecast, mobile data traffic will increase by 18-fold between 2011 and 2016 globally [1]. However, since the amount of wireless spectrum is limited, the mobile operators (MOs) worldwide are seeking ways to increase the network capacity in a cost-effective and timely manner. An efficient way to achieve this purpose is to use complementary technologies, such as Wi-Fi [2] or femtocells [3], to offload the traffic originally targeted towards the cellular network. There are two main approaches for the initiation of data offloading, namely mobile user (MU)-initiated and MO-initiated offloading. In the *MU-initiated* offloading, the MU is given the option to select the network technologies that it intends to use. In the *MO-initiated* offloading, however, the operator profile stored in the mobile device prompts the connection manager to initiate the offloading procedure. In this paper, we focus on the *MO-initiated Wi-Fi offloading*.

Traditionally, Wi-Fi is used for sharing broadband Internet connections at home or public hotspots. Recently, new Wi-Fi standards have been proposed to enable the use of *carrier-based Wi-Fi*, which promises the MUs a cellular-like experience [2]. For example, the IEEE 802.11u standard [4] automates the network discovery, selection, and authentication of Wi-Fi devices. Hotspot 2.0, which is built on the IEEE 802.11u standard, is a Wi-Fi Alliance (WFA) initiative that aims to provide seamless and secure Wi-Fi authentication at the hotspots [2]. Together with the increasing number of Wi-Fi access points (APs) that are deployed worldwide, these standardization efforts allow the huge amount of data traffic to be offloaded to the Wi-Fi networks efficiently.

Besides the effort from the industry, the academia is paying more attention to the theoretical study of Wi-Fi offloading. Recently, it was demonstrated in the measurement studies [5], [6] that the cellular network traffic can be reduced significantly by using Wi-Fi offloading. Dimatteo *et al.* in [7] evaluated the costs and benefits of Wi-Fi offloading in metropolitan area with real mobility traces. The number of Wi-Fi APs required for the support of a given quality of service (QoS) requirement was characterized. Joe-Wong *et al.* in [8] studied the user adoption of supplementary technology (e.g., Wi-Fi or femtocell) for cellular traffic offloading. The utility function of each user is related to its valuation of the technology, the congestion level, and the flat pricing of the service provider. The works in [9], [10] considered an offloading market, where the MOs pay the third-party deployed APs for data offloading. Gao *et al.* in [9] characterized the subgame perfect equilibrium in a data offloading game, where the base stations (BSs) propose the market prices, and the APs determine the volume of data traffic that they are willing to offload. Iosifidis *et al.* in [10] proposed an iterative and incentive compatible double auction that maximizes the social welfare.

For *delay-tolerant* applications, such as movie download, software update, and e-mail, which can tolerate some delays without sacrificing too much user satisfactions, the potential benefit of data offloading is even more significant [5], [6]. A number of recent research results have been devoted to the study of *delayed Wi-Fi offloading*. Zhuo *et al.* in [11] considered a 3G cellular network, where the MO motivates the MUs to use delayed data offloading by giving them discount coupons. The problem was formulated as a reverse auction with one buyer and multiple sellers, where the MO is the buyer, and the MUs are the sellers. Lee *et al.* in [12] studied the economic aspects of Wi-Fi offloading in a monopolistic market with multiple MUs and one MO. Each MU is characterized by its willingness to pay, traffic demand, delay profile, and Wi-Fi contact probability. Im *et al.* in [13] considered the cost-throughput-delay tradeoff in Wi-Fi offloading. Given the predicted future usage and the availability of Wi-Fi, the proposed system decides on the application that should offload its traffic to Wi-Fi at a given time, while taking into account the cellular budget constraint of the MU. In fact, there are only a few previous works in the literature related to the network control of Wi-Fi offloading, which includes [5], [13]. The work in [5] proposed a *heuristic* prediction-based offloading scheme, while we design and analyze an optimal delayed Wi-Fi offloading (ODWO) scheme. The work in [13] is related to the MU-initiated offloading, while this paper focuses on the

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MO-initiated offloading.

In this paper, we study the MO-initiated Wi-Fi offloading problem in a cellular network. We assume that the MU is moving under the coverage range of the cellular BS. However, the Wi-Fi connection is *location-dependent* and may not always be available to the user all the time. We consider that the MU is running a delay-tolerant file transfer application with a given deadline. To provide a satisfactory user experience while using as little cellular bandwidth as possible, the MO aims to minimize the total cellular usage, while taking into account the given deadline of the application. With information related to the mobile pattern of the MU [14], we formulate the delayed Wi-Fi offloading problem as a finite-horizon sequential decision problem, where each cellular time slot used leads to a unit cost, and the MU will incur a penalty if the file transfer cannot be completed by the deadline.

The main contributions of our work are as follows:

- We formulate the Wi-Fi offloading problem as a finite-horizon sequential decision problem, and propose a general ODWO algorithm that achieves the optimal performance for the general case.
- We show that the optimal policy has a threshold structure for the special case with a convex penalty function and fixed location-independent cellular and Wi-Fi data rates. A low-complexity monotone optimal ODWO algorithm is proposed for this special case.
- Simulation results show that ODWO algorithm results in the minimal total cost and the highest file transfer efficiency as compared with two heuristic schemes.

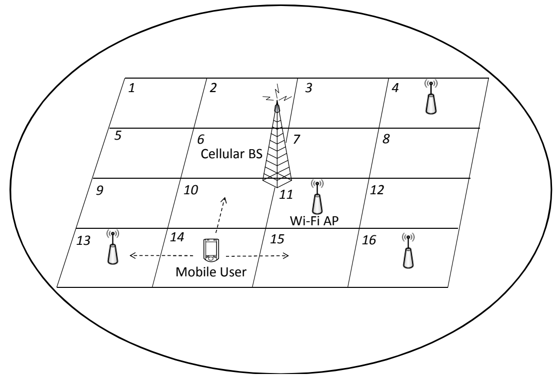


Fig. 1. An example of the network setting, where the MU is moving within a set of $\mathcal{L} = \{1, \ldots, 16\}$ locations. The MU is always under the coverage of a cellular BS, but Wi-Fi is only available at four locations, where $\mathcal{L}^{(1)} = \{4, 11, 13, 16\}$ and $\mathcal{L}^{(0)} = \mathcal{L} \backslash \mathcal{L}^{(1)}$. We assume that the MU is launching a file transfer of size K bits that should be completed by deadline T. Given the mobility pattern of the MU, the MO aims to decide whether the MU should remain idle ($a = 0$), use the cellular network ($a = 1$), or use the Wi-Fi network ($a = 2$) if it is available in each time slot to offload as much traffic as possible, while taking into account the deadline of the application.

II. System Model

As shown in Fig. 1, we consider a MU moving within the coverage of the cellular network, such that the cellular connection is always available to the MU. Occasionally, the MU may meet some Wi-Fi APs available for use at some locations (e.g., in a coffee shop or in a shopping mall). As a result, the Wi-Fi connection is *location-dependent* and may not be available to the MU all the time. We consider that the MU is running a file transfer application, which requires transferring of K bits within T time slots. In other words, the file transfer application is *delay-tolerant* with a deadline T. We consider that the MU moves in a set $\mathcal{L} = \{1, \ldots, L\}$ of possible locations. We consider a known mobility model of the MU based on the past mobility pattern of the MU [14] using the global positioning system (GPS) information. We assume that the MU has a unlimited data plan in the cellular network, and the Wi-Fi hotspots are offered free to the MU, so pricing is not an issue to the MU.

As mentioned before, we consider the MO-initiated Wi-Fi data offloading, where a MO can use the Wi-Fi networks to offload the data traffic originally targeted towards the cellular network. Specifically, we assume that the Wi-Fi offloading is initiated by the network server of the MO remotely by prompting the connection manager in the mobile device of the MU. Under this MO-initiated offloading, both the cellular usage and the QoS requirement of the MU should be taken into account. First, the MO has the incentive to offload as much data traffic to the Wi-Fi network as possible, and reduce the congestion in the cellular network. In this way, the MO may prefer not to start the data transmission using the cellular connection immediately, but defer the transmission until a Wi-Fi hotspot is available. On the other hand, the MO should also consider the satisfaction of the MU in its offloading decision. For example, if the deadline T is short, then the deferred transmission may violate the deadline if the MU does not encounter enough Wi-Fi APs in the near future. Instead, the MO should start the file transfer using the cellular connection as soon as possible to reduce the latency. Thus, a MO-initiated Wi-Fi offloading needs to achieve a good *tradeoff* between the total cellular usage and the QoS requirement of the MU.

Due to the dynamic nature of the delayed Wi-Fi offloading problem, we formulate it as a finite-horizon sequential decision problem in the following section. We consider that the MO aims to find the optimal transmission policy that minimizes the total cellular usage, while taking into account the deadline of the file transfer application. By defining the total *cost* as the total cellular usage and a *penalty* for not able to finish the file transfer by the deadline, we can derive the optimal transmission policy through dynamic programming (DP).

III. Problem Formulation

In this section, we formulate the delayed Wi-Fi offloading problem of the MO involving a *single* MU as a *finite-horizon sequential* decision problem [15]. Without loss of generality, we normalize the length of a time slot to be one. The MU needs to choose an action (to be explained later) at each *decision epoch*

$$t \in \mathcal{T} = \{1, \ldots, T\}. \quad (1)$$

The system *state* is defined as $\boldsymbol{s} = (k, l)$. The state element $k \in \mathcal{K} \subseteq [0, K]$ represents the *remaining* size (in bits) of a file to be transferred. The state element $l \in \mathcal{L} = \{1, \ldots, L\}$ is the location index, where L is the total number of possible locations that the MU may reach within the T time slots. We define the function $w(l)$ as the availability of Wi-Fi at location l. Specifically, we define $w(l) = 1$ if Wi-Fi is available at location $l \in \mathcal{L}$, and $w(l) = 0$ otherwise. We let $\mathcal{L}^{(0)} = \{l \in \mathcal{L} : w(l) = 0\}$ and $\mathcal{L}^{(1)} = \{l \in \mathcal{L} : w(l) = 1\}$ be the sets of locations where Wi-Fi is not and is available, respectively.

The *action* a specifies the transmission decision of the MU at each decision epoch. Specifically, we have $a \in \mathcal{A} = \{0, 1, 2\}$, where $a = 0$ means that the MU chooses to remain idle, $a = 1$ means that the MU uses the cellular connection, and $a = 2$ represents that the MU uses Wi-Fi in a time slot. Notice that actions $a = 0$ and $a = 1$ are always available to the MU. Action $a = 2$, however, is only available at location $l \in \mathcal{L}^{(1)}$. Thus, the available choice of action a depends on the state element l, so $a \in \mathcal{A}^{(l)} \subseteq \mathcal{A}$, where $\mathcal{A}^{(l)}$ is the set of available transmission actions at location l:

$$\mathcal{A}^{(l)} = \begin{cases} \{0,1,2\}, & \text{if } l \in \mathcal{L}^{(1)}, \\ \{0,1\}, & \text{if } l \in \mathcal{L}^{(0)}. \end{cases} \tag{2}$$

Let $\mu(l, a)$ be the data rate at location l with action $a \in \mathcal{A}^{(l)}$, respectively, where $\mu(l, 0) = 0, \forall\, l \in \mathcal{L}$ when the MU remains idle (i.e., when $a = 0$).

We define the *cost* at state $\boldsymbol{s}$ with action a at time slot t as

$$c_t(\boldsymbol{s}, a) = c_t(k, l, a) = I(a = 1) = \begin{cases} 1, & \text{if } a = 1, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $a \in \mathcal{A}^{(l)}$ and $I(\cdot)$ is the indicator function. Whenever the cellular connection is used in a time slot (i.e., when $a = 1$), the network incurs a unit cost, so that the sum of the cost over all the time slots is equal to the *total cellular usage*.

After the deadline has passed at $T+1$, we define the *penalty* for not being able to finish the file transfer at state $\boldsymbol{s}$ as

$$\hat{c}_{T+1}(\boldsymbol{s}) = \hat{c}_{T+1}(k, l) = h(k), \tag{4}$$

where $h(k) \geq 0$ is a nondecreasing function of k with $h(0) = 0$. It is chosen according to the latency requirement of the application.

The *state transition probability* $p\big(\boldsymbol{s}' \,|\, \boldsymbol{s}, a\big) = p\big((k', l') \,|\, (k, l), a\big)$ is the probability that the system will go into state $\boldsymbol{s}' = (k', l')$ if action a is taken at state $\boldsymbol{s} = (k, l)$. Since the transition of the Wi-Fi availability from l to l' is independent of the value of k and action a, we have

$$p\big(\boldsymbol{s}' \,|\, \boldsymbol{s}, a\big) = p\big((k', l') \,|\, (k, l), a\big) = p(l' \,|\, l)\, p\big(k' \,|\, (k, l), a\big), \tag{5}$$

where

$$p\big(k' \,|\, (k, l), a\big) = \begin{cases} 1, & \text{if } k' = [k - \mu(l, a)]^+ \text{ and } a \in \mathcal{A}^{(l)}, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

and $[x]^+ = \max\{0, x\}$. Here, we assume that $p(l' \,|\, l)$ is defined according to the Markov chain estimated based on the past mobility pattern of the MU [14].

Let $\delta_t : \mathcal{K} \times \mathcal{L} \to \mathcal{A}$ be a function that specifies the transmission decision of the MU at state $\boldsymbol{s} = (k, l)$ and time slot t. We define a *policy* $\boldsymbol{\pi} = (\delta_t(k, l), \forall\, k \in \mathcal{K}, l \in \mathcal{L}, t \in \mathcal{T})$ as the set of decision rules for states and time slots. We denote $\boldsymbol{s}_t^{\boldsymbol{\pi}} = (k_t^{\boldsymbol{\pi}}, l_t^{\boldsymbol{\pi}})$ as the state at time slot t if policy $\boldsymbol{\pi}$ is used, and we let Π be the feasible set of $\boldsymbol{\pi}$. We consider that the MO aims to find an optimal policy $\boldsymbol{\pi}^*$ that minimizes the sum of the expected total cost from $t = 1$ to $t = T$ and the penalty at $t = T + 1$ as

$$\underset{\boldsymbol{\pi} \in \Pi}{\text{minimize}} \quad E_{\boldsymbol{s}}^{\boldsymbol{\pi}} \left[\sum_{t=1}^{T} c_t\big(\boldsymbol{s}_t^{\boldsymbol{\pi}}, \delta_t(\boldsymbol{s}_t^{\boldsymbol{\pi}})\big) + \hat{c}_{T+1}(\boldsymbol{s}_{T+1}^{\boldsymbol{\pi}}) \right]. \tag{7}$$

$E_{\boldsymbol{s}}^{\boldsymbol{\pi}}$ denotes the expectation with respect to the probability distribution by the mobility model of the MU and policy $\boldsymbol{\pi}$ with an initial state $\boldsymbol{s} = (K, l_1)$, where l_1 is the location of the MU at time slot $t = 1$.

IV. General Optimal Delayed Wi-Fi Offloading

In this section, we solve problem (7) *optimally* using the *finite-horizon DP* for the general case with general penalty function and cellular/Wi-Fi data rates. We propose a general ODWO algorithm that computes the optimal policy.

Let $v_t(\boldsymbol{s})$ be the minimal expected total cost of the MU from time slot t to $T + 1$, given that the system is in state $\boldsymbol{s}$ immediately before the decision at time slot t. The *optimality equation* [15, pp. 83] relating the minimal expected total cost at different states for $t \in \mathcal{T}$ is given by

$$v_t(\boldsymbol{s}) = v_t(k, l) = \min_{a \in \mathcal{A}^{(l)}} \{\psi_t(k, l, a)\}, \tag{8}$$

where for $k \in \mathcal{K}$, $l \in \mathcal{L}$, and $a \in \mathcal{A}^{(l)}$, we have

$$\psi_t(k, l, a) = c_t(k, l, a) + \sum_{l' \in \mathcal{L}} \sum_{k' \in \mathcal{K}} p\big((k', l') \,|\, (k, l), a\big)\, v_{t+1}(k', l') \tag{9}$$

$$= I(a = 1) + \sum_{l' \in \mathcal{L}} p(l' \,|\, l)\, v_{t+1}\big([k - \mu(l, a)]^+, l'\big). \tag{10}$$

The first and second terms on the right hand side of (9) are the *immediate cost* and the *expected future cost* in the remaining time slots for choosing action a, respectively. The derivation of (10) from (9) follows directly from (5) and (6). Moreover, for $t = T + 1$, we set the boundary condition as

$$v_{T+1}(\boldsymbol{s}) = \hat{c}_{T+1}(\boldsymbol{s}) = h(k), \quad \forall\, k \in \mathcal{K}, l \in \mathcal{L}. \tag{11}$$

A. Properties of the Optimal Policy

In this subsection, we discuss some analytical results related to the properties of the optimal policy. They are useful in establishing the threshold policy in Section V.

Lemma 1: $v_t(k, l)$ is a nondecreasing function in k, $\forall\, l \in \mathcal{L}, t \in \mathcal{T}$.

The proof of Lemma 1 is given in Appendix A. Intuitively, a larger remaining file size k leads to a higher expected cost (given a fixed location $l \in \mathcal{L}$). Next, we show the result related to location $l \in \mathcal{L}^{(1)}$, where Wi-Fi is available. Lemma 2(a) states that action $a = 2$ (i.e., use Wi-Fi) is always preferred to action $a = 0$ (i.e., remain idle). Lemma 2(b) states that if the Wi-Fi data rate is higher than the cellular data rate, then Wi-Fi will always be used.

Lemma 2: For all location $l \in \mathcal{L}^{(1)}$ (hence Wi-Fi is available), we have: (a) $\psi_t(k, l, 0) \geq \psi_t(k, l, 2)$, $\forall\, k \in \mathcal{K}, t \in \mathcal{T}$. (b) If $\mu(l, 1) \leq \mu(l, 2)$, then $\delta_t^*(k, l) = 2$, $\forall\, k \in \mathcal{K}, t \in \mathcal{T}$.

The proof of Lemma 2 is given in Appendix B. Notice that at $l \in \mathcal{L}^{(1)}$, although $\mathcal{A}^{(l)} = \{0, 1, 2\}$ from (2), Lemma 2(a) implies that we do not need to consider action $a = 0$ in (8). Specifically, let

$$\tilde{\mathcal{A}}^{(l)} = \begin{cases} \{1, 2\}, & \text{if } l \in \mathcal{L}^{(1)}, \\ \{0, 1\}, & \text{if } l \in \mathcal{L}^{(0)}. \end{cases} \tag{12}$$

We can simplify the optimality equation in (8) as

$$v_t(k, l) = \min_{a \in \mathcal{A}^{(l)}} \{\psi_t(k, l, a)\} = \min_{a \in \tilde{\mathcal{A}}^{(l)}} \{\psi_t(k, l, a)\}. \tag{13}$$

B. General ODWO Algorithm

With the simplified optimality equation, we are ready to propose the general ODWO algorithm in Algorithm 1. The algorithm consists of two phases, namely the planning phase and the transmission and Wi-Fi offloading phase. Let $\sigma > 0$ be the granularity of the discrete state element k in the algorithm (such as 1 Kbyte). First, in the planning phase, based on the simplified optimality equation in (13) and the boundary condition in (11), we obtain the *optimal policy* $\boldsymbol{\pi}^*$ that solves problem (7) using *backward induction* [15, pp. 92]. Specifically, we first set $v_{T+1}(k, l)$ based on the boundary condition (line 2). Then, we obtain the values of $\delta_t^*(k, l)$ and $v_t(k, l)$ by updating them recursively backward from time slot $t = T$ to time slot $t = 1$ (lines 3 to 16). The complexity of the algorithm is directly proportional to $|\mathcal{K}| \times L \times T$ [16].

Theorem 1: The policy $\boldsymbol{\pi}^* = (\delta_t^*(k, l), \forall\, k \in \mathcal{K}, l \in \mathcal{L}, t \in \mathcal{T})$, where

$$\delta_t^*(k, l) = \arg\min_{a \in \mathcal{A}^{(l)}} \{\psi_t(k, l, a)\} = \arg\min_{a \in \tilde{\mathcal{A}}^{(l)}} \{\psi_t(k, l, a)\}, \tag{14}$$

is the optimal solution of problem (7).

Proof: Using the principle of optimality [17, pp. 18], we can show that $\boldsymbol{\pi}^*$ is the optimal solution of problem (7). ■

Notice that the optimal policy $\boldsymbol{\pi}^*$ is a *contingency plan* that contains information about the optimal transmission decision at *all* the possible states (k, l) in any time slots $t \in \mathcal{T}$, and it is computed *offline* before the file transfer begins in the second phase. In the second phase, the location index l in each time slot is first determined based on the location information obtained by GPS (line 20). Then, the transmission decisions are carried out based on the optimal policy $\boldsymbol{\pi}^*$ through checking a table (lines 21 to 25), and the state element k is updated accordingly (line 24).

V. Threshold Policy and Monotone ODWO

In this section, we consider a special case, where the penalty function $h(k)$ is convex, and the cellular and Wi-Fi data rates are location-independent (but these two rates are different). We show that the optimal policy has a *threshold* structure in the remaining file size k. As a result, we propose a monotone ODWO algorithm with a lower computational complexity to achieve the optimal performance. Notice that the convex penalty function is commonly used for resource allocation [16], and the assumption of fixed data rates is valid for a free space uniformly distributed with MUs.

Algorithm 1 *General Optimal Delayed Wi-Fi Offloading (ODWO) Algorithm.*

1: Planning Phase:
2: Set $v_{T+1}(k, l)$, $\forall\, k \in \mathcal{K}, l \in \mathcal{L}$ using (11)
3: Set $t := T$
4: **while** $t \geq 1$
5:    **for** $l \in \mathcal{L}$
6:       Set $k := 0$
7:       **while** $k \leq K$
8:          Calculate $\psi_t(k, l, a)$, $\forall\, a \in \tilde{\mathcal{A}}^{(l)}$ using (10)
9:          Set $\delta_t^*(k, l) := \min_{a \in \tilde{\mathcal{A}}^{(l)}} \{\psi_t(k, l, a)\}$
10:          Set $v_t(k, l) := \psi_t\big(k, l, \delta_t^*(k, l)\big)$
11:          Set $k := k + \sigma$
12:       **end while**
13:    **end for**
14:    Set $t := t - 1$
15: **end while**
16: Output the optimal policy $\boldsymbol{\pi}^*$ for use in the transmission and Wi-Fi offloading phase
17: Transmission and Wi-Fi Offloading Phase:
18: Set $t := 1$ and $k := K$
19: **while** $t \leq T$ **and** $k > 0$
20:    Determine the location index l from GPS
21:    Set action $a := \delta_t^*(k, l)$ based on the optimal policy $\boldsymbol{\pi}^*$
22:    **If** $a > 0$
23:       Send $\mu(l, 1)$ bits to the cellular network if $a = 1$ or offload $\mu(l, 2)$ bits to the Wi-Fi network if $a = 2$
24:       Set $k := [k - \mu(l, a)]^+$
25:    **end if**
26:    Set $t := t + 1$
27: **end while**

To show the threshold policy, we need to leverage on the concepts of *superadditivity* and *subadditivity* [15, pp. 103]. Specifically, with the assumptions we made on $h(k)$ and cellular/Wi-Fi data rates, we show in Appendix C that $\psi_t(k, l, a)$ is superadditive or subadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ under different conditions.

Definition 1: Given $l \in \mathcal{L}$, the function $\psi_t(k, l, a)$ is *superadditive* on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ if for $\forall\, \hat{k}, \check{k} \in \mathcal{K}$ and $\forall\, \hat{a}, \check{a} \in \mathcal{A}$, where $\hat{k} \geq \check{k}$ and $\hat{a} \geq \check{a}$, we have

$$\psi_t(\hat{k}, l, \hat{a}) + \psi_t(\check{k}, l, \check{a}) \geq \psi_t(\hat{k}, l, \check{a}) + \psi_t(\check{k}, l, \hat{a}). \tag{15}$$

Moreover, $\psi_t(k, l, a)$ is *subadditive* on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ if the reverse inequality holds.

Then, with $\delta_t^*(k, l)$ defined in (14), we can establish the threshold structure of the optimal policy [15, pp. 104, 115].

Theorem 2: If $h(k)$ is a convex and nondecreasing function in k, and the cellular and Wi-Fi data rates are location independent such that $\mu_1 = \mu(l, 1), \forall\, l \in \mathcal{L}$ and $\mu_2 = \mu(l, 2), \forall\, l \in \mathcal{L}^{(1)}$, then the optimal policy $\boldsymbol{\pi}^* = (\delta_t^*(k, l), \forall\, k \in \mathcal{K}, l \in \mathcal{L}, t \in \mathcal{T})$ has a *threshold* structure in k as follows: First, for $l \in \mathcal{L}^{(0)}$, we have

$$\delta_t^*(k, l) = \begin{cases} 1, & \text{if } k \geq k_t^*(l), \\ 0, & \text{otherwise}, \end{cases} \quad \forall\, t \in \mathcal{T}, \tag{16}$$

where $k_t^*(l)$ is the threshold that depends on both l and t. Second, for $l \in \mathcal{L}^{(1)}$, if the Wi-Fi data rate is lower than the cellular data rate (i.e., $\mu_2 \leq \mu_1$), we have

$$\delta_t^*(k,l) = \begin{cases} 1, & \text{if } k \geq k_t^*(l), \\ 2, & \text{otherwise,} \end{cases} \quad \forall\, t \in \mathcal{T}, \tag{17}$$

otherwise (hence $\mu_1 < \mu_2$), we have

$$\delta_t^*(k,l) = 2, \forall\, k \in \mathcal{K}, t \in \mathcal{T}. \tag{18}$$

The proof of Theorem 2 is given in Appendix D. With this threshold structure, we propose Algorithm 2 with a lower computational complexity than Algorithm 1 for the special case with a convex penalty function and fixed data rates. In the first phase, we aim to obtain the set of thresholds ($k_t^*(l)$, $\forall\, l \in \mathcal{L}, t \in \mathcal{T}$) (line 21), which completely characterizes the optimal policy $\boldsymbol{\pi}^*$. Basically, the procedure THRESHOLD(j, $flag$) is used to obtain the thresholds, where we set $j = 0$ if $l \in \mathcal{L}^{(0)}$ (line 7) and $j = 2$ if $l \in \mathcal{L}^{(1)}$ (line 10) as mentioned in Appendix C. Let $\breve{\mathcal{A}} \subseteq \mathcal{A}$ be the set of actions that should be considered in line 5 of the procedure. The variable $flag$ indicates whether the threshold has been obtained in an iteration. Specifically, for $l \in \mathcal{L}^{(0)}$ or $l \in \mathcal{L}^{(1)}$ with $\mu_2 \leq \mu_1$, we set $flag = 0$ (lines 7 and 11) to consider $\breve{\mathcal{A}} = \{j, 1\}$ (procedure line 2) and search for the threshold. However, for $l \in \mathcal{L}^{(1)}$ with $\mu_1 < \mu_2$, we know from (18) that $\delta_t^*(k,l) = 2$, so we set $flag = 1$ (line 11) to consider $\breve{\mathcal{A}} = \{j\} = \{2\}$ (procedure line 2). In THRESHOLD(j, $flag$), when the threshold is reached, where $\delta_t^*(k,l) = 1$ and $flag = 0$ (procedure line 7), the threshold $k_t^*(l)$ is recorded, and $\breve{\mathcal{A}}$ becomes a singleton (procedure line 8). In this way, the minimization in line 5 of the procedure is readily known, so the computational complexity is reduced. In the second phase, the action a is determined based on the threshold optimal policy stated in Theorem 2. Specifically, the decisions in lines 23, 26, and 28 are due to (16), (17), and (18), respectively.

Algorithm 2 *Monotone ODWO Algorithm for the special case with convex penalty function $h(k)$ and fixed location-independent cellular and Wi-Fi data rates μ_1 and μ_2.*

1: Planning Phase:
2: Set $v_{T+1}(k,l)$, $\forall\, k \in \mathcal{K}, l \in \mathcal{L}$ using (11)
3: Set $t := T$
4: **while** $t \geq 1$
5: **for** $l \in \mathcal{L}$
6: **if** $l \in \mathcal{L}^{(0)}$
7: Set $j := 0$ and $flag := 0$
8: Call THRESHOLD(j, $flag$)
9: **else if** $l \in \mathcal{L}^{(1)}$
10: Set $j := 2$
11: **if** $\mu_2 \leq \mu_1$, Set $flag := 0$, **else**, Set $flag := 1$, **end if**
12: Call THRESHOLD(j, $flag$)
13: **end if**
14: **end for**
15: Set $t := t - 1$
16: **end while**
17: Output the thresholds ($k_t^*(l)$, $\forall\, l \in \mathcal{L}, t \in \mathcal{T}$)
18: Transmission and Wi-Fi Offloading Phase:
19: Set $t := 1$ and $k := K$
20: **while** $t \leq T$ **and** $k > 0$
21: Determine the location index l from GPS
22: **If** $l \in \mathcal{L}^{(0)}$
23: **If** $k \geq k_t^*(l)$, Set $a := 1$, **else**, Set $a := 0$, **end if**
24: **else if** $l \in \mathcal{L}^{(1)}$
25: **If** $\mu_2 \leq \mu_1$
26: **If** $k \geq k_t^*(l)$, Set $a := 1$, **else**, Set $a := 2$, **end if**
27: **else**
28: Set $a := 2$
29: **end if**
30: **end if**
31: **If** $a > 0$
32: Send μ_1 bits to the cellular network if $a = 1$ or offload μ_2 bits to the Wi-Fi network if $a = 2$
33: Set $k := [k - \mu_a]^+$
34: **end if**
35: Set $t := t + 1$
36: **end while**

procedure THRESHOLD(j, $flag$)

1: Set $k := 0$
2: **If** $flag = 0$, Set $\breve{\mathcal{A}} := \{j, 1\}$, **else**, Set $\breve{\mathcal{A}} := \{j\}$, **end if**
3: **while** $k \leq K$
4: Calculate $\psi_t(k,l,a)$, $\forall\, a \in \breve{\mathcal{A}}$ using (10)
5: Set $\delta_t^*(k,l) := \min_{a \in \breve{\mathcal{A}}} \{\psi_t(k,l,a)\}$
6: Set $v_t(k,l) := \psi_t\big(k,l,\delta_t^*(k,l)\big)$
7: **if** $\delta_t^*(k,l) = 1$ and $flag = 0$
8: Set $\breve{\mathcal{A}} := \{1\}$, $k_t^*(l) := k$, and $flag := 1$
9: **end if**
10: Set $k := k + \sigma$
11: **end while**

VI. Performance Evaluations

In this section, we evaluate the performance of the ODWO scheme in Algorithm 1 by comparing it with the on-the-spot offloading [6] and Wiffler [5] schemes. The threshold policy stated in Theorem 2, of which Algorithm 2 is based on, is illustrated. For the *on-the-spot offloading* scheme, the data traffic is offloaded to the Wi-Fi network whenever Wi-Fi is available. Cellular connection will be used immediately when Wi-Fi is not available. Furthermore, we consider a prediction-based offloading scheme that was proposed in the *Wiffler* system [5]. Let ζ be the estimated amount of data that can be transferred using Wi-Fi by the deadline. Under the Wiffler scheme, a history-based predictor is used, which estimates ζ based on the inter-meeting time and throughput of the last m Wi-Fi AP encounters. If Wi-Fi is available in the current location, then Wi-Fi will be used immediately. Otherwise, if Wi-Fi is not available, then we need to check whether the condition $\zeta \geq ck$ is satisfied, where k is the remaining size of the file to be transferred, and $c > 0$ is the conservative coefficient that tradeoffs the amount of data offloaded with the completion time of the file transfer. If this condition is satisfied, meaning that the estimated data transfer using Wi-Fi is large enough, then it will stay idle and wait for the Wi-Fi connection. Otherwise, it will use the cellular connection. We adopt $c = 1$ and $m = 4$ as suggested in [5].

For the performance evaluations, we run each experiment in 1000 different network scenarios and show the average value. Unless specified otherwise, we assume that the cellular data rate $\mu(l,1), \forall\, l \in \mathcal{L}$ and the Wi-Fi data rate $\mu(l,2), \forall\, l \in \mathcal{L}^{(1)}$ are obtained by rounding off a set of normally distributed random variables, with mean equal to 3 Mbps and standard deviation equal to 1 Mbps, to the nearest non-negative numbers. The probability that a Wi-Fi connection is available

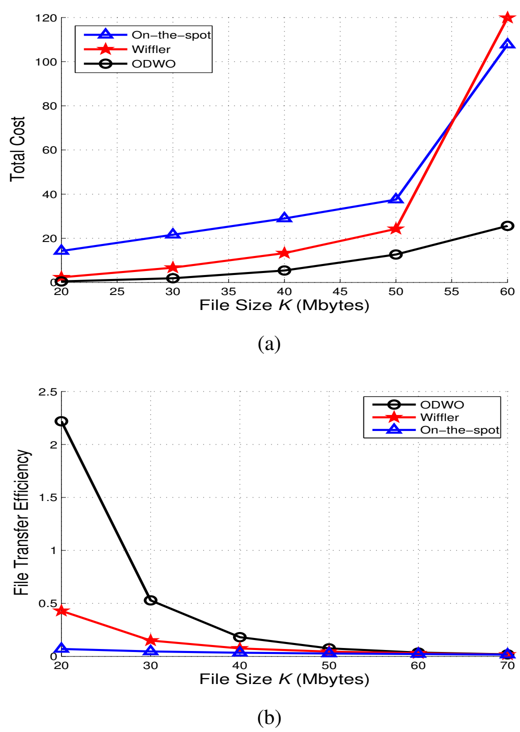


Fig. 2. Total cost and file transfer efficiency (i.e., probability of completing file transfer / average number of cellular time slots used) versus file size K for $D = 3$ min and $b = 1$. Our proposed ODWO scheme achieves both the minimal total cost and the highest file transfer efficiency.

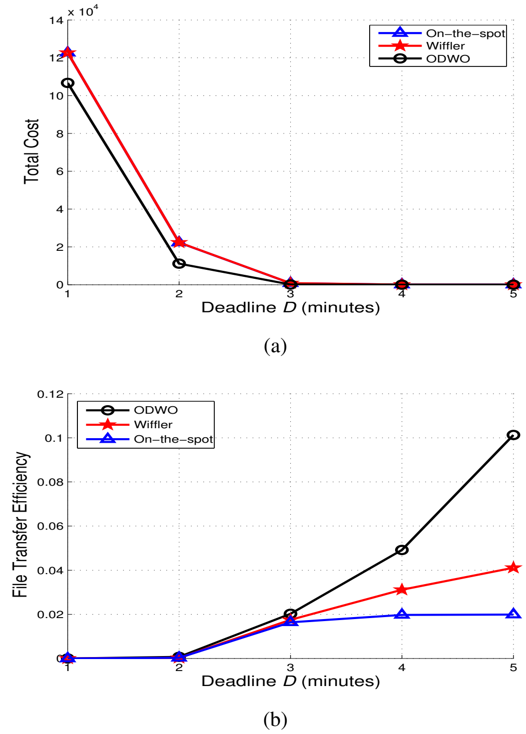


Fig. 3. Total cost and file transfer efficiency versus deadline D for $K = 70$ Mbytes and $b = 1$. Our proposed ODWO scheme achieves both the minimal total cost and the highest file transfer efficiency.

at a particular location is equal to 0.7. We set the length of a time slot to be equal to one second. We consider that the MU is downloading a file (e.g., a movie), where the deadline of the file transfer is D minutes (so $T = 60D$). Moreover, we consider that the MU is moving around $L = 6$ possible locations in a linear network with the state transition probabilities $p(l' \,|\, l)$ given as follows: $p(l \,|\, l) = 0.6, \forall\, l \in \mathcal{L}$, $p(l+1 \,|\, l) = 0.2$, $p(l-1 \,|\, l) = 0.2$ for $l = 2, \ldots, L-1$, $p(2 \,|\, 1) = 0.4$, $p(L-1 \,|\, L) = 0.4$, and zero for other state transitions. For the penalty, we use the convex function

$$h(k) = b\,k^2, \quad \forall\, k \in \mathcal{K}, \tag{19}$$

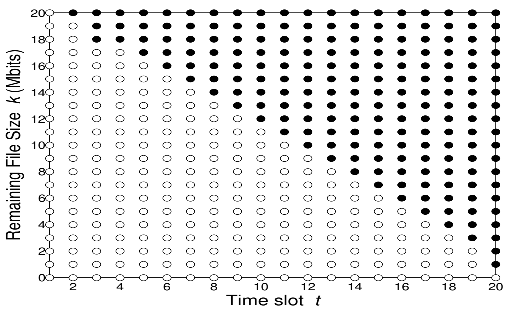
where $b \geq 0$ is a constant.

First, we plot the total cost (i.e., the objective function in problem (7)) against the file size K for $D = 3$ min and $b = 1$ in Fig. 2(a). Since ODWO computes and obtains the optimal policy, it achieves the minimal total cost as stated in Theorem 1. Moreover, we observe that the total cost increases with K. It is because a larger number of cellular time slots (i.e., the time slots with chosen action $a = 1$) is often used for a larger K. In addition, the chance of completing the file transfer is smaller for a large K, which results in a larger penalty.

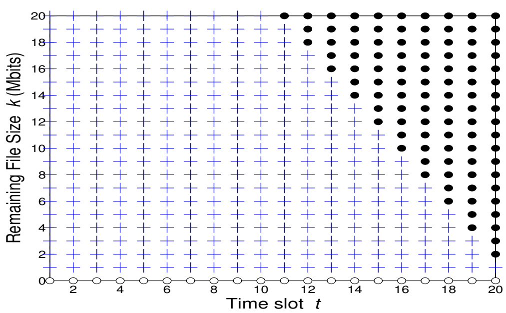
To measure the efficiency of the file transfer with respect to the cellular usage, we define a metric called the *file transfer efficiency*, which is defined as the probability of completing file transfer divided by the average number of cellular time slots used. As shown in Fig. 2(b), ODWO achieves the highest file transfer efficiency as compared with the two other heuristic schemes. The reason is that by setting the penalty parameter $b = 1$, ODWO achieves a high probability of completing file transfer while using a small number of cellular time slots.

Then, we study the effects on the total cost and file transfer efficiency by varying the deadline D for $K = 70$ Mbytes and $b = 1$ in Figures 3(a) and 3(b), respectively. Similarly, we observe that our proposed ODWO scheme achieves the minimal total cost and the highest file transfer efficiency as compared with the two other heuristic schemes. Moreover, as D increases, the MU has more time to wait for the availability of Wi-Fi, and thus reduces the cellular usage. Besides, for a larger D, the chance of completing the file transfer is higher, and the penalty is thus smaller. As a result, the total cost decreases with D, while the file transfer efficiency increases with D, as shown in Figures 3(a) and 3(b), respectively.

Finally, we illustrate the actions of the optimal policy for different system states. We first look at the special case with convex penalty function $h(k)$ and location-independent data rates μ_1 and μ_2 for $K = 20$ Mbits, $T = 20$, and $b = 10$. In Figures 4(a) and 4(b), we can observe the threshold structure in dimension k as stated in (16) for $l \in \mathcal{L}^{(0)}$ and in (17) for $l \in \mathcal{L}^{(1)}$ with $\mu_2 \leq \mu_1$ in Theorem 2, respectively. We also observe that the threshold structure exists in dimension t. We conjecture that this is not limited to this numerical example, but we have not established its proof analytically. Furthermore, we show an example of the optimal policy for the general case with non-convex penalty function $h(k)$ and location-dependent cellular/Wi-Fi data rates. We consider a step penalty function $h(k) = Z$ for $k > 0$ and $h(0) = 0$, where $Z >> 1$ is a large positive constant. With this penalty function, we place more importance on completing the file transfer than reducing the

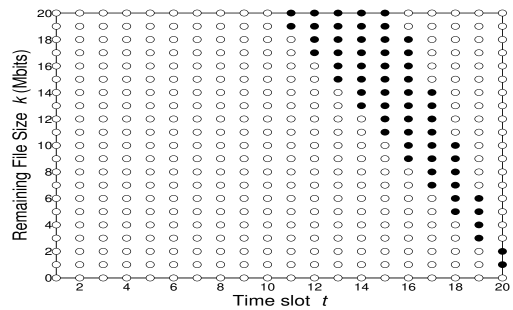


(a) $l \in \mathcal{L}^{(0)}$ for $\mu_1 = 2$ Mbps.

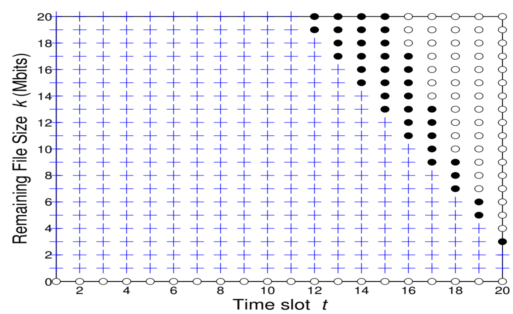


(b) $l \in \mathcal{L}^{(1)}$ for $\mu_1 = 2$ Mbps and $\mu_2 = 1$ Mbps.

Fig. 4. An example of the optimal policy at location $l \in \mathcal{L}$ for the case with convex penalty and location-independent data rates, where $K = 20$ Mbits, $T = 20$, and $b = 10$. The white dots, black dots, and blue crosses represent the transmission decisions of $a = 0$ (idle), 1 (use cellular), and 2 (use Wi-Fi), respectively. We can observe the threshold optimal policy as stated in Theorem 2.



(a) $l \in \mathcal{L}^{(0)}$ for $\mu(l, 1) = 2.1$ Mbps.



(b) $l \in \mathcal{L}^{(1)}$ for $\mu(l, 1) = 3.1$ Mbps and $\mu(l, 2) = 2.1$ Mbps.

Fig. 5. An example of the optimal policy at location $l \in \mathcal{L}$ for the case with step penalty and location-dependent data rates, where $K = 20$ Mbits, $T = 20$, and $Z = 100000$. The white dots, black dots, and blue crosses represent the transmission decisions of $a = 0$ (idle), 1 (use cellular), and 2 (use Wi-Fi), respectively.

cellular usage. We adopt $Z = 100000$. As shown in Fig. 5, we can see that multiple thresholds exist along dimension k, instead of a single threshold in the special case. For example, in Fig. 5(a), for $t \geq 16$, when k is increased from zero, the decision first changes from idle to using cellular, because a complete file transfer is still possible. However, when k is increased further that a complete file transfer is impossible, the idle action is chosen. Notice that it is very different from the policy in the special case as stated in Theorem 2.

VII. Conclusion

In this paper, we studied the delayed Wi-Fi offloading problem initiated by the mobile operator. The user is running a file transfer application while moving around different locations with different data rates and Wi-Fi availabilities. We considered that the mobile operator initiates the Wi-Fi offloading to minimize the cellular usage, while taking into account the completion of file transfer within the user-specified deadline using a penalty function. We solved this finite-horizon sequential decision problem using dynamic programming and proposed the general ODWO algorithm for the general setting. Then, for the special case with a convex penalty function and location-independent data rates, we proposed a low-complexity optimal monotone ODWO algorithm based on the threshold structure of the optimal policy in dimension k. Simulation results showed that our proposed ODWO achieves both the minimal total cost and the highest file transfer efficiency as compared with two heuristic schemes.

Appendix

A. Proof of Lemma 1

We prove it by induction. First, from (11), $v_{T+1}(k, l) = h(k)$ is a nondecreasing function in k, $\forall\, l \in \mathcal{L}$. Assume that $v_{t+1}(k, l)$ is a nondecreasing function in k, $\forall\, l \in \mathcal{L}$. From (10), since $p(l' \,|\, l) \geq 0$, $\forall\, l, l' \in \mathcal{L}$ and the function $I(a = 1)$ is independent of k, $\psi_t(k, l, a)$ is a nondecreasing function in k, $\forall\, l \in \mathcal{L}, a \in \mathcal{A}$. Thus, $v_t(k, l)$ in (8) is a nondecreasing function in k, $\forall\, l \in \mathcal{L}$. ■

B. Proof of Lemma 2

Let $k \in \mathcal{K}$ and $l \in \mathcal{L}$ be given.

(a) We have

$$\begin{aligned}\psi_t(k, l, 0) &= \sum_{l' \in \mathcal{L}} p(l' \,|\, l)\, v_{t+1}(k, l') \\ &\geq \sum_{l' \in \mathcal{L}} p(l' \,|\, l)\, v_{t+1}\big([k - \mu(l, 2)]^+, l'\big) = \psi_t(k, l, 2),\end{aligned} \tag{20}$$

where the two equalities are due to (10) and the inequality is due to Lemma 1.

(b) First, since $\mu(l, 1) \leq \mu(l, 2)$, we have

$$\begin{aligned}\psi_t(k, l, 1) &= 1 + \sum_{l' \in \mathcal{L}} p(l' \,|\, l)\, v_{t+1}\big([k - \mu(l, 1)]^+, l'\big) \\ &\geq \sum_{l' \in \mathcal{L}} p(l' \,|\, l)\, v_{t+1}\big([k - \mu(l, 2)]^+, l'\big) = \psi_t(k, l, 2),\end{aligned} \tag{21}$$

where the two equalities are due to (10) and the inequality is due to Lemma 1. Combining the results from (20) and (21), from (14), we have $\delta_t^*(k,l) = 2, \forall\, k \in \mathcal{K}, t \in \mathcal{T}$. ∎

C. Superadditivity and subadditivity of $\psi_t(k,l,a)$

The proof of Theorem 2 is based on the results in Lemmas 3 and 4. Let $l \in \mathcal{L}$ be given. Let $\tilde{\mathcal{A}}^{(l)} = \{j,1\}$, where $j=0$ if $l \in \mathcal{L}^{(0)}$ and $j=2$ if $l \in \mathcal{L}^{(1)}$ as in (12), and $\mu_0 = 0$. With only two possible actions in $\tilde{\mathcal{A}}^{(l)}$, we can rewrite (10) as

$$\psi_t(k,l,a) = I(a=1) + \sum_{l' \in \mathcal{L}} p(l' \,|\, l) \Big[I(a=1) \times v_{t+1}\big([k-\mu_1]^+, l'\big) + \big(1 - I(a=1)\big) v_{t+1}\big([k-\mu_j]^+, l'\big) \Big]. \tag{22}$$

Lemma 3: If $\mu_j \le \mu_1$ and $h(k)$ is a convex and nondecreasing function in k, then

$$v_t([k-\mu_j]^+, l) - v_t\big([k-\mu_1]^+, l\big) \ge v_t\big([k-\sigma-\mu_j]^+, l\big) - v_t\big([k-\sigma-\mu_1]^+, l\big), \forall\, k \in \mathcal{K}, l \in \mathcal{L}, t \in \mathcal{T} \cup \{T+1\}. \tag{23}$$

The proof of Lemma 3 is given in [18].

Lemma 4: If $\mu_j \le \mu_1$ and $\forall\, \hat{k}, \check{k} \in \mathcal{K}$, $l \in \mathcal{L}$, $t \in \mathcal{T}$ with $\hat{k} \ge \check{k}$, where

$$\begin{aligned} & v_{t+1}([\hat{k}-\mu_j]^+, l) - v_{t+1}\big([\hat{k}-\mu_1]^+, l\big) \\ & \quad \ge v_{t+1}([\check{k}-\mu_j]^+, l) - v_{t+1}\big([\check{k}-\mu_1]^+, l\big), \end{aligned} \tag{24}$$

then $\psi_t(k,l,a)$ is subadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ for $j=0$, and superadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ for $j=2$, $\forall\, t \in \mathcal{T}$, respectively.

Proof: Let $\hat{k}, \check{k} \in \mathcal{K}$, $\hat{a}, \check{a} \in \tilde{\mathcal{A}}^{(l)}$, $l \in \mathcal{L}$, and $t \in \mathcal{T}$ be given, where $\hat{k} \ge \check{k}$ and $\hat{a} \ge \check{a}$. Then

$$\begin{aligned} & \psi_t(\hat{k},l,\hat{a}) + \psi_t(\check{k},l,\check{a}) - \psi_t(\hat{k},l,\check{a}) - \psi_t(\check{k},l,\hat{a}) \\ & = \sum_{l' \in \mathcal{L}} p(l' \,|\, l) \Big(I(\check{a}=1) - I(\hat{a}=1) \Big) \Big[v_{t+1}([\hat{k}-\mu_j]^+, l) - \\ & v_{t+1}\big([\hat{k}-\mu_1]^+, l\big) - v_{t+1}([\check{k}-\mu_j]^+, l) + v_{t+1}\big([\check{k}-\mu_1]^+, l\big) \Big], \end{aligned} \tag{25}$$

where the equality is derived using (22). Notice that $p(l' \,|\, l) \ge 0, \forall\, l, l' \in \mathcal{L}$. First, for $j=0$, we have $\hat{a}, \check{a} \in \{0,1\}$, so $I(\check{a}=1) \le I(\hat{a}=1)$. From the given condition in Lemma 4 and Definition 1, we conclude that $\psi_t(k,l,a)$ is subadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$. On the other hand, for $j=2$, we have $\hat{a}, \check{a} \in \{1,2\}$, so $I(\check{a}=1) \ge I(\hat{a}=1)$. We can then conclude that $\psi_t(k,l,a)$ is superadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$. ∎

D. Proof of Theorem 2

First, for $l \in \mathcal{L}^{(1)}$ and $\mu_1 < \mu_2$, from Lemma 2(b), we have $\delta_t^*(k,l) = 2, \forall\, k \in \mathcal{K}, t \in \mathcal{T}$ as stated in (18).

Next, we consider that case $0 \le \mu_j \le \mu_1$. Let $\hat{k}, \check{k} \in \mathcal{K}$, $l \in \mathcal{L}$, and $t \in \mathcal{T}$ be given. Let $\check{k} = [\hat{k} - z\sigma]^+$, where $z > 0$. If the condition of Theorem 2 is satisfied, by repetitively applying Lemma 3, we have

$$\begin{aligned} & v_t([\hat{k}-\mu_j]^+, l) - v_t\big([\hat{k}-\mu_1]^+, l\big) \\ & \ge v_t\big([\hat{k}-\sigma-\mu_j]^+, l\big) - v_t\big([\hat{k}-\sigma-\mu_1]^+, l\big) \ge \cdots \\ & \ge v_t\big([\hat{k}-z\sigma-\mu_j]^+, l\big) - v_t\big([\hat{k}-z\sigma-\mu_1]^+, l\big) \\ & = v_t\big([\check{k}-\mu_j]^+, l\big) - v_t\big([\check{k}-\mu_1]^+, l\big). \end{aligned} \tag{26}$$

For $l \in \mathcal{L}^{(0)}$, we consider $j=0$ as mentioned in Appendix C. Since $0 = \mu_0 < \mu_1$, $\psi_t(k,l,a)$ is subadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ from Lemma 4. From [15, pp. 104, 115], $\delta_t^*(k,l)$ is a monotone nondecreasing function in k. From (12) and (14), since $\delta_t^*(k,l) \in \tilde{\mathcal{A}}^{(l)} = \{0,1\}$, $\delta_t^*(k,l)$ is in the form of (16).

Then, we consider $l \in \mathcal{L}^{(1)}$ for $\mu_2 \le \mu_1$. Since we consider $j=2$ as mentioned in Appendix C, $\psi_t(k,l,a)$ is superadditive on $\mathcal{K} \times \tilde{\mathcal{A}}^{(l)}$ from Lemma 4. From [15, pp. 104, 115], $\delta_t^*(k,l)$ is a monotone nonincreasing function in k. From (12) and (14), as $\delta_t^*(k,l) \in \tilde{\mathcal{A}}^{(l)} = \{1,2\}$, $\delta_t^*(k,l)$ is in the form of (17). ∎

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