On-Demand OFDMA: Control, Fairness and Non-Cooperation

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***Abstract*— Motivated by recent work on improving the efficiency of the IEEE 802.11 protocol at high speeds, we consider an OFDMA system in which the users make reservations requests over a collision channel. The controller schedules from only amongst the successful requests using an alpha-fair scheduler that balances the network throughput and fairness to nodes. We first analyze the performance of the alpha-fair scheduler when used with an Aloha reservation channel. We then assume that the network prescribes reservation rates to active nodes but that nodes may attempt reservations more aggressively so as to be scheduled more frequently (and unfairly). A simple game theoretic analysis of interaction between the Aloha reservation channel and the scheduler shows that in the presence of other cooperative users, a node attempting at a rate higher than that prescribed indeed obtains a larger (unfair) throughput. For such a network we propose a robust alpha-fair scheduler that penalizes aggressive users. This scheduler along with the prescribed reservation rates forms a Nash equilibrium.**

I. Introduction

Orthogonal frequency division multiple access (OFDMA) is being used for most of the current and emerging high speed networking technologies including IEEE 802.11 WLANs, LTE and WiMax. In OFDMA, the available spectrum is divided into a number of narrow subbands, or subchannels. The subbands can be dynamically allocated in different combinations to different users. This allows networks to use sophisticated opportunistic allocation mechanisms to provide high spectral efficiency. There is also significant interest in developing scheduling algorithms that achieve additional objectives like stabilizing queues (e.g., [2]), minimizing delays (e.g., [2], [3]), and providing fairness (e.g., [4]).

An OFDMA scheduling algorithm essentially allocates different combinations of resources—subchannels, transmission rate (by prescribing the modulation scheme) and transmission power to achieve an objective. In a typical scheme time is divided into frames. At the beginning of each frame, the active users provide ‘local state’ information e.g., channel state, queue length, battery state, etc.). The controller uses this information to allocate resources to achieve a network objective. The schemes in, among others, [2]–[4] have this structure. Clearly, such a system can lead to selfish behavior by the nodes and this has led to the design and analysis of OFDMA systems for non cooperative nodes, e.g., [4], [5].

An interesting recent use of OFDMA has been to reduce PHY inefficiency of 802.11 MAC. Use of CSMA/CA in 802.11 MAC makes it increasingly inefficient with increasing PHY data rates because the overheads remain fixed while data transmission times shrink. This has led to several schemes that improve the efficiency. Some of these reduce the overhead with clever changes to PHY e.g., [6], [7] while some amortize the fixed overheads by exploiting features of OFDMA, e.g., the fine grained channel (FICA) scheme of [8]. FICA works as follows. Time is divided into frames and OFDM subcarriers are divided into D groups called subchannels. R subcarriers of each group are allocated for sending RTS by the nodes to the access point (AP). Depending on the channel gains, each node can choose to transmit in any of the subchannels. For the chosen subchannel(s), the node randomly selects a subcarrier to signal an RTS. If more than one subcarrier is successful in a subchannel the AP performs a collision resolution and allocates the subchannel to one of the nodes using a collision resolution mechanism. Thus FICA improves efficiency by stretching the data transmission time (by simultaneously allocating smaller portions of the bandwidth to a larger number of users) while keeping the overheads constant.

The FICA scheme, omitting protocol details, can be seen to be an OFDMA channel in which the user-to-controller signaling is through a slotted-Aloha collision channel. The nodes send their requests for allocation of the OFDMA carriers and the controller allocates according to some criterion. Taking a cue from this scheme, we consider networks in which nodes are non cooperative in both the reporting of their local state and in using the signalling channel unfairly.

Systems in which nodes report better than actual channel conditions have been extensively studied under the assumption of an ideal signalling channel and fair scheduling algorithms have been proposed (e.g., [9]–[11]). These correspond to pure scheduling problems; this paper is not about pure scheduling. Note that the nodes could be non cooperative in the use of the Aloha signalling channel by attempting aggressively and drowning out the other requests. This results in fewer legitimate competing requests at the controller. The non-cooperative model of the slotted Aloha is also well investigated; but this work is also not about non cooperative Aloha. This work is about the use of Aloha in a non cooperative manner to signal to the controller about a resource (bandwidth) requirement in a possibly non cooperative manner. In the system of interest, the channel can lose signalling messages (or resource requests) via collisions on the Aloha signalling channel, and the probability of this loss can be influenced by other nodes.

In keeping with the terminology of OFDMA literature, the controller or the access point will be called the base station (BS) and the user node will be called mobile station (MS).

The rest of the paper is organized as follows. The notation, system details and some preliminary results are presented in the next section. In Section III we develop the game theoretic description of a system of non cooperative nodes interacting with an alpha-fair scheduler. In Section IV we describe and analyze the robust scheduler that will lead to a Nash equilibrium in which the nodes request reservations at the prescribed rate and the controller performs alpha-fair scheduling. We present extensive numerical results in Section V and conclude with a discussion in which we compare our system with related OFDMA systems and preview some future work. The proofs are omitted due to space constraints but are available in [1].

II. Notation and System Preliminaries

In this section, we first introduce the basic notations used in the paper. This is followed by a detailed description of the Aloha reservation channel. We then describe an alpha-fair scheduler that will be implemented at the base station which will allocate channel to the mobile stations based on some fairness criteria. We will also consider the packet arrival model describing the nature of data packet arrival at the mobiles.

Much like other OFDMA systems in the literature, time is divided into frames and each frame has two phases—reservation and data phases. In the reservation phase, the M mobiles in the system transmit RTS (request to send) to the BS (base station) according to a randomized algorithm over the Aloha reservation channel. The BS decodes the successful RTS's and applies its scheduling algorithm to grant channel access to a subset of the successful nodes. This schedule is conveyed via a common CTS (clear to send) signal. The data phase consists of D, $D \geq 1$, parallel data channels and the mobiles that are scheduled will transmit in these channels.

The following notational convention is used. Lower case letters will represent flags or state indicators (some of which are random) while the corresponding bold letters (in lower case) represent the respective vectors. $\mathcal{X}_{\{\cdot\}}$ will be the indicator of the event in the subscript. The positive part of a variable is represented by $(\cdot)^+$, i.e., $(x)^+ = \max\{x, 0\}$. Mobiles are indexed by m and i, data channels by j and frames by t.

The state of channel j for MS (mobile station) m is denoted by $h_{m,j}$ with $\mathbf{h}_m := [h_{m,1}, \cdots, h_{m,D}]$ denoting the channel state vector for MS m. The channel states $\mathbf{h} := [\mathbf{h}_1, \cdots, \mathbf{h}_M]$ are assumed to be independent processes across mobiles and across time frames. Like in other such OFDMA systems, MS m will send $\mathbf{h}_m$ to the BS (base station) in the RTS.

The indicator that MS m has transmitted an RTS signal in the frame will be denoted by a_m with $\mathbf{a} := [a_1, a_2, \cdots, a_M]$ being the corresponding vector. Following conventional game theoretic notation, $\mathbf{a}_{-m} := [a_1, \cdots a_{m-1}, a_{m+1}, \cdots, a_M]$, i.e., $\mathbf{a}$ after excluding the m-th component.

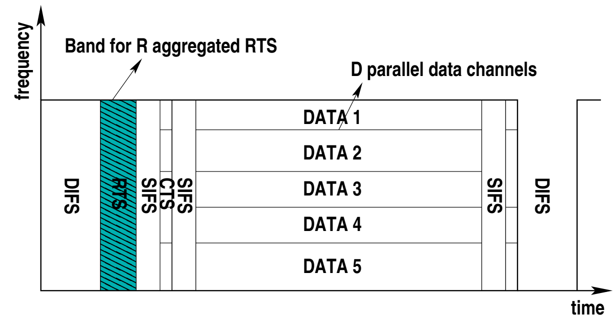

Fig. 1. Frequency-time diagram of a frame with aggregated Aloha reservation scheme. DIFS (DCF interframe space) and SIFS (Short interframe space) durations are specified by the protocol.

A. The Aloha Reservation Channel

For convenience, we will use an *aggregated Aloha reservation* channel that works as follows. Each frame has a single reservation phase and MS m transmits an RTS with probability p_m. If the total number of nodes transmitting an RTS is less than or equal to R, then all those that transmitted will be deemed successful[1]. If more than R mobiles transmit an RTS, then the BS cannot decode any of the RTS's and a collision is said to occur. In this case the corresponding data channel is wasted because it cannot be assigned to any MS. Recall that a_m is the indicator that MS m transmitted an RTS; thus $p_m = \mathsf{Pr}\left(a_m = 1\right)$. Let $\mathbf{p} := [p_m]$ be called the reservation rate vector. Let b_m be the indicator that MS m had a successful RTS. The probability of successful RTS, $p_m^{\text{succ}} := \mathsf{Pr}\left(b_m = 1\right)$, is easy to calculate. For example, with $p_m = p$ for all m, this will be the same for all m and is

$$p^{\text{succ}} = p \sum_{i=0}^{R-1} \binom{N-1}{i} p^i (1-p)^{N-1-i}.$$

p_m^{succ} can be easily obtained even for asymmetric $\mathbf{p}$ albeit with slightly messier expressions.

Refer Fig. 1 for detailed frequency v/s time diagram of a frame in the aggregated reservation scheme. A reservation scheme that resembles FICA more closely is given in [1]. Our analysis will also hold for that scheme.

B. Scheduler

In each frame, a set of mobiles will have submitted a reservation request via a successful RTS. The scheduler at the BS schedules a subset of these mobiles and sends CTS to the scheduled set. The scheduler could choose an MS with maximum utility, in which case network efficiency is maximized; the allocation may not be fair to the mobiles. It could also choose to allocate such that the minimum utility of the scheduled nodes is maximized, i.e., use a max-min fair schedule. The latter is of course not very efficient. An

[1]This model is appropriate when the reception depends on the SINR, which in turn depends on the number simultaneously attempting the RTS.

alpha-fair scheduler helps achieve a desired trade-off between fairness and network efficiency via a suitable choice of the tuning parameter α, $0 \le \alpha \le \infty$. Specifically, we define and use the following alpha-fair scheduler described in, among others, [9], [11].

Recall that $b_m := \mathcal{X}_{\{\text{successful RTS by } m\}}$; Define $\hat{h}_{m,j} := h_{m,j} b_m$, i.e., $\hat{h}_{m,j}$ is the effective state of channel j for MS m after accounting for RTS. Define $\hat{\mathbf{h}} := [[\hat{h}_{m,j}]]$. Let $f(h)$ be the utility from a channel with channel state h with $f(h) > 0$ if $h \neq 0$ and $f(0) = 0$. f could, for example, indicate the number of bits per frame per channel that the MS can transmit when the channel state is h. The utility that MS m achieves on being scheduled for transmission in data channel j is $f(\hat{h}_{m,j})$.

A scheduler β maps every channel state $\hat{\mathbf{h}}$ to decisions $[[\beta_{m,j}]]$ where $\beta_{m,j} \in [0, 1]$ is the probability with which MS m is allocated data channel j. Let

$$u_{m,j} := \mathsf{E}\left[f(\hat{h}_{m,j})\beta_{m,j}\right]; \quad u_m := \sum_j u_{m,j}, \quad (1)$$

$$\Gamma_\alpha(u) = \frac{u^{1-\alpha}}{1-\alpha}\mathcal{X}_{\{\alpha\neq 1\}} + \log(u)\ \mathcal{X}_{\{\alpha=1\}}. \quad (2)$$

Define $\mathbf{u} = [u_1,\ u_2,\ \cdots, u_M]$. Assume that utilities are additive, i.e., if MS m is allocated more than one data channel in a frame, then the total utility is the sum of the utilities from each data channel. The alpha-fair scheduler will send CTS to a subset of MS selected according to (see [9], [11])

$$\arg\max_\beta \sum_m \Gamma_\alpha(u_m). \quad (3)$$

The efficient scheduler maximizes the sum throughput and is obtained by using $\alpha = 0$ in (3); for this case we have

$$\begin{aligned} \beta^0_{m,j} &= \mathcal{X}_{\{m=\arg\max_i f(\hat{h}_{m,j})\ a_i\}} \quad \text{for all } m, j \\ &= \mathcal{X}_{\{m=\arg\max_i f(h_{i,j})\ a_i\}} b_m. \end{aligned} \quad (4)$$

The max-min fair scheduler maximizes minimum of all utilities and is obtained with $\alpha = \infty$ in (3); intermediate levels of fairness-throughput trade-offs are obtained with $\alpha \in (0, \infty)$.

Unlike for $\alpha = 0$, for $\alpha > 0$ we do not have a closed form for the scheduler. However, using the concavity of $\Gamma_\alpha(u)$ and the linear dependence of $\mathbf{u}$ on the scheduler, we obtain a fixed point structure for the alpha-fair scheduler as described in the following theorem, with proof in Appendix.

***Theorem** 1:* Assume the channel states $h_{m,j}$ are independent across mobiles m, data channels j, and time frames.
(1) Consider , $\Lambda(\cdot) := [\Lambda_m(\cdot)]_m$, defined component wise by

$$\Lambda_m(\mathbf{u}) = \sum_j \mathsf{E}\left[\frac{f(h_{m,j})}{\left|\arg\max_i \frac{f(\hat{h}_{i,j})}{u_i^\alpha}\right|}\mathcal{X}_{\left\{m\in\arg\max_i \frac{f(\hat{h}_{i,j})}{u_i^\alpha}\right\}}\right], \quad (5)$$

with $|A|$ denoting the cardinality of set A. If there exists a fixed point $\mathbf{u}^*$ of the above function, then an alpha-fair scheduler maximizing (3) can be defined using this fixed point $\mathbf{u}^* := [u_1^*, \cdots, u_m^*]$ as

$$\beta^\alpha_{m,j} = \frac{\mathcal{X}_{\left\{m\in\arg\max_i \frac{f(\hat{h}_{i,j})}{(u_i^*)^\alpha}\right\}}}{\left|\arg\max_i \frac{f(\hat{h}_{i,j})}{(u_i^*)^\alpha}\right|} \text{ for all } m, j. \quad (6)$$

(2) Assume for each m and j that $h_{m,j}$ is a continuous random variable with continuous density $g_{m,j}(\cdot)$, and that f is integrable, i.e., that $E[f(h_{m,j})] < \infty$. Then there exists a fixed point $\mathbf{u}^*$ of the function (5). $\square$

The theorem says that if a fixed point for $\Lambda_m(\cdot)$ exists and if the scheduler given by (6) is used, then the corresponding average utility is indeed $\Lambda_m(\cdot)$. In (6), before making a scheduling decision the instantaneous utilities are scaled by weights that are inversely proportional to the time average of the utilities obtained by the MS. Thus users with lower utility obtain better allocations and hence increasing fairness. Also note that the weights $(u_i^*)^\alpha$ are such that the fairness increases with α. This structure will be used in further analysis as well as in constructing practical policies.

The case with $D = 1$ and with ideal reservation channel is considered in [9] where an iterative algorithm that asymptotically maximizes the alpha-fair utility (3) is proposed. This algorithm implements the fixed point equation of the preceding theorem (see [11] for more details). We now define the following iterative algorithm which obtains the fixed point of the above theorem for general D and for Aloha reservation channel. For any time frame $t+1$, any MS m and for the j-th data channel, the iteration in the t-th time step is

$$u_{m,j,t+1} = u_{m,j,t} + \mu\left(f(h_{m,j,t+1})\beta^\alpha_{m,j,t} - u_{m,j,t}\right) \quad (7)$$

$$\beta^\alpha_{m,j,t} := \mathcal{X}_{\{m=\arg\max_i f(\hat{h}_{i,j,t+1})(u_{i,t}+d_i)^{-\alpha}\}} \quad (8)$$

$$u_{m,t} = \sum_{j=1}^D u_{m,j,t}.$$

Here the constant $\mu > 0$ is the step size and $[d_i]$ are small stabilizing constants.

C. Packet Arrival Model

The assumption of saturated nodes where every departing packet is replaced by a new packet at the source has been extensively used to provide tractability to the performance models. We also take recourse to this model and all our analyses will be for this saturated model. This is also a good model when the sources have only elastic traffic and the active nodes will want to send data at the best available rate. Of course, this is not very realistic and systems are operated in non saturated (or stable regime,) with arrival rates less than departure rates with queues becoming empty infinitely often. The non-saturated users with load factor (ratio of the arrival to the departure rate) close to one, have empty buffers with a very small probability almost as in a saturated case. In this paper we analyze the saturated case; using numerical analysis it is demonstrated in [1], that the non-saturated case with a reasonable load factor also behaves similarly.

III. Selfish Mobiles: Towards a Game

We begin by showing that MS has incentive to be selfish—it can improve its utility by choosing a higher RTS reservation rate. This motivates a game theoretic problem formulation.

The system that we consider has two parts—the Aloha reservation channel and the data channel scheduled by an alpha-fair scheduler. While the literature on Aloha is extensive, a game theoretic understanding is emerging, e.g. [12] for an early survey, and [11], [13] for some recent work. Also, there is significant literature on OFDMA scheduling some of which were pointed out in Section I. Our interest in this paper is to analyze the interaction between the success probability in the Aloha channel and the long term average scheduled throughput obtained on the data channel. More specifically, recall that MS m obtains a utility

$$\begin{aligned} u_m(\mathbf{p},\beta) &= \sum_j \mathsf{E}\left[f(h_{m,j})\beta_{m,j}\right] \\ &= \sum_j \sum_{\mathbf{a}} \mathsf{E}\left[f(h_{m,j})\beta_{m,j}|\mathbf{a}\right] \mathsf{Pr}\left(\mathbf{a}\right). \end{aligned}$$

Note that this utility is a function of $\mathbf{p}$. The MS attempt RTS independent of one another and hence the joint probability of attempting is $\mathsf{Pr}\left(\mathbf{a}\right) = \Pi_m \mathsf{Pr}\left(a_m\right)$. Hence,

$$\begin{aligned} u_m(\mathbf{p},\beta) &= \sum_j \sum_{\mathbf{a}} \mathsf{E}\left[f(h_{m,j})\beta_{m,j}|\mathbf{a}\right] \prod_i \mathsf{Pr}\left(a_i\right) \\ &= p_m \sum_j \sum_{\mathbf{a}_{-m}} \mathsf{E}\left[f(h_{m,j})\beta_{m,j}|\mathbf{a}_{-m}, a_m=1\right] \prod_{i\neq m} \mathsf{Pr}\left(a_i\right). \quad (9) \end{aligned}$$

The BS derives the scheduling decision in each frame by maximizing the utility according to (3). Thus following [11], we can define a natural utility function for the BS to be

$$u_{BS}(\mathbf{p},\beta) = \sum_m \Gamma_\alpha(u_m(\mathbf{p})). \quad (10)$$

We call u_{BS} as the *network utility.*

The utility of MS m can be split as $u_m = u_m^r + u_m^b$ where

$$\begin{aligned} u_m^r &:= p_m \Pi_{i\neq m}(1-p_i) \sum_j \mathsf{E}\left[f(h_{m,j})\right] \\ u_m^b &:= p_m \sum_{j,\mathbf{a}_{-m}\neq(0,\cdots,0)} \mathsf{E}\left[f(h_{m,j})\beta_{m,j}|\mathbf{a}_{-m}, a_m=1\right] \mathsf{Pr}\left(a_{-m}\right). \end{aligned}$$

In the above, u_m^r is the utility obtained by MS m, when no other MS attempts RTS. In such a situation, there is only one MS contending and hence the MS is always scheduled. Thus u_m^r is the utility which cannot be changed or controlled by the scheduler and we call this the *private utility* of MS m. u_m^b is a function of the parameter of the scheduler and we call this the *public utility* of mobile m. Note that private utility can also influence total utility of other mobiles, e.g when an MS attempts RTS more aggressively than the prescribed rate.

A. The Game

A game theoretic setting is clear from the preceding discussion—the mobiles can choose p_m to maximize individual utility while the BS can choose a scheduling scheme to desirably trade-off the network utility and fairness.

Consider a system where the mobiles are assigned reservation attempt rates, say p_m^{ref} for MS m; denote $\mathbf{p}^{\text{ref}} := \left[p_m^{\text{ref}}\right]$. Clearly, $\mathbf{p}^{\text{ref}}$ determines the long term average utility that each of MS can obtain. The data channel is only granted to an MS with a successful RTS. Thus the MS have an incentive to attempt at a rate $p_m > p_m^{\text{ref}}$ to improve their utility. We thus have the following non cooperative game that arises naturally.

$$\begin{array}{lclllll} \text{Players} &=& \{1, & 2, & \cdots, & M, & BS\}, \\ \text{Utilities} &=& \{u_1, & u_2, & \cdots, & u_M, & u_{BS}\}, \\ \text{Actions} &=& \{p_1, & p_2, & \cdots, & p_M, & \beta\}. \end{array} \quad (11)$$

Observe that (9) suggests that u_m increases linearly with the reservation rate p_m. Indeed this is true for $\alpha = 0$ i.e., for the efficient scheduler of (4) u_m increases linearly with p_m. One can use continuity arguments to show that this behavior is true even for small values of $\alpha > 0$. Using an example with finite channel states, we will now show that, a mobile can cheat even at high values of α.

B. An Example

We illustrate the preceding discussions, with the help of a simple example. Assume $D = 1$, $M = 3$. Two mobiles are far from the BS and have similar random variations in their channel states. Both can communicate with the BS at one of two rates 5 and 3 units with probability 0.2 and 0.8 respectively. A third MS is close to the BS and can communicate with the BS at one of two rates 12 and 10 units with probability 0.8 and 0.2 respectively. Assume that $p_i^{\text{ref}} = p$ for all i.

When an efficient scheduler (4) is used at BS, MS 3 obtains a higher utility than the other two. However as α increases, higher priority is given to fairness. Maximum fairness is obtained when MS 3 is scheduled only when there are no other MS with successful RTS. If such a fair scheduler exists, then the three mobiles would obtain the following utilities:

$$\begin{aligned} u_3 &= p(1-p)^2(0.8*12+0.2*10) = 11.6p(1-p)^2, \\ u_1 &= u_2 = (p(1-p)^2 + p^2(1-p))(0.8*3+0.2*5) \\ &+p^2(1-p)(0.8*0.2*5+0.2*0.2*\frac{5}{2}+0.8*0.8*\frac{3}{2}) \\ &= 3.4p(1-p)^2 + 5.86p^2(1-p). \end{aligned}$$

Using part 1 of Theorem 1 (6), such a scheduling is implemented by an alpha-fair scheduler maximizing (3) for all those values of α which satisfy $12/u_3^\alpha < 1/u_1^\alpha$, i.e., when $12^{1/\alpha}\left(3.4p(1-p)^2 + 5.86p^2(1-p)\right) < 11.6p(1-p)^2$. The above inequality is satisfied for all $\alpha > 2.63$ when the common RTS reservation rate $p = .45$. Fig. 2 plots the utility for the three mobiles as a function of α for different combinations of $\mathbf{p}$. We see that MS 3 indeed has a nearly constant utility, close to minimum, for all $\alpha > 2.6$. Also observe that the utility of the MS 3 when it increases it's $p = 0.75$. Clearly, MS 3 has a higher utility when it uses a higher p. The remaining two have reduced, unfair utilities.

We now make the following observations.

- Utility of MS 3 is highest when $\alpha = 0$ and decreases monotonically as $\alpha \to \infty$; see Fig. 2.
- For $\alpha > 2.6$, u_3 does not change with α; this is the best that the scheduler can do towards fairness. For $\alpha > 2.6$, MS 1 or 2 are scheduled whenever their RTS succeeds.
- Private utility of MS 1, 2 is $3.4p(1-p)^2$ and is $11.6p(1-p)^2$ for MS 3. This cannot be changed by a scheduler.
- MS 3 can increase its utility even at high alpha through its private utility by using $p_3 > p$. Also, the cooperating MS receive lower utilities than their fair share. And this is true even for higher α.

IV. A Robust Scheduling Policy

We now focus on constructing robust scheduling policies that use additional information to penalize aggressive mobiles. Robustness is shown by arguing that the scheduling policy and $\mathbf{p}^{\text{ref}}$ form a Nash equilibrium of an equivalent game.

A. Construction of the Robust Policy

The BS can observe $\mathbf{b}_t := \{b_{m,t}\}$, the time sequence of the RTS success flags and estimate their time average. Since time average equals the ensemble average, the BS can estimate the reservation rates $\{\hat{p}_m\}$ using $\hat{p}_m = \hat{b}_m/c_m$ where $\hat{b}_m$ is the time average of the rate of successful RTS from MS m and c_m is obtained below. It can then estimate the extent of any non-cooperation by observing and penalize MS m when $(\hat{p}_m - p_m^{\text{ref}}) > 0$. This should force the MS to be cooperative. For the case of saturated mobiles and aggregated reservations, we have $b_m = a_m \mathcal{X}_{\left\{\sum_{i\neq m} a_i < R\right\}}$. By independence, $\Pr(b_m = 1) = p_m c_m$ where $c_m = \Pr\left(\sum_{i\neq m} a_i < R\right)$. We can thus estimate p_m. The estimates, henceforth, are represented without ˆ to avoid messy equations.

Robust Scheduler: The robust modification of the iterative scheduling algorithms (7) is:

$$p_{m,t+1} = p_{m,t} + \mu_t \left(b_{m,t}/c_m - p_{m,t}\right) \tag{12}$$

$$u_{m,j,t+1} = u_{m,j,t} + \mu_t \left(\frac{f(h_{m,j,t})}{1+\rho_{m,t}}\beta_{m,j,t} - u_{m,j,t}\right) \tag{13}$$

$$\omega_{m,t} = \rho_{m,t} + u_{m,t}, \quad \rho_{m,t} = \Delta\left(p_{m,t} - p_m^{\text{ref}}\right)^+$$

$$\beta_{m,j,t} = \cap_{i\neq m} \mathcal{X}_{\left\{\frac{f(\hat{h}_{m,j,t})}{(1+\rho_{m,t})(\omega_{m,t})^\alpha} \geq \frac{f(\hat{h}_{i,j,t})}{(1+\rho_{m,t})(\omega_{i,t})^\alpha}\right\}}. \tag{14}$$

In the above, the BS estimates the actual reservation rates used by the MS iteratively using (12). It then identifies the selfish MS as the ones with $p_{m,t} > p_m^{\text{ref}}$. It makes robust scheduling decisions $\{\beta_{m,j,t}\}$ by weighing down u_m for the selfish MS using a larger punitive term $(1+\rho_{m,t})(\omega_{m,t})^\alpha$ instead of the original $(u_{m,t})^\alpha$ as in (8). It further reduces the instantaneous utilities $f(\hat{h}_{m,j,t})$ by an extra factor $1+\rho$, to ensure that the private utilities are also punished (whenever the MS is non-cooperative).

B. Analysis

We analyze the proposed robust scheduler, using ODE analysis for the case with saturated packet arrivals. Let $\Theta := [\{p_m\}_m, \{u_{m,j}\}_{m,j}]$ represent a vector of all the components related to the robust algorithm. We will show that the trajectory of the robust scheduler (12)–(14) can be approximated by the solution $\Theta(t)$ of the following ODE for all $m \leq M$ and j:

$$\dot{p}_m(t) = \frac{\Pr(b_m = 1)}{c_m} - p_m(t) \tag{15}$$

$$\dot{u}_{m,j}(t) = E\left[\frac{f(h_{m,j})}{1+\rho_m(t)}\beta_{m,j}(t)\right] - u_{m,j}(t) \tag{16}$$

$$\omega_m(t) := u_m(t) + \rho_m(t), \quad \rho_m(t) = \Delta(p_m(t) - p_m^{\text{ref}})^+$$

$$\beta_{m,j}(t) = \Pi_{i\neq m}\mathcal{X}_{\left\{\frac{f(\hat{h}_{m,j})}{(1+\rho_{m,t})(\omega_m(t))^\alpha} \geq \frac{f(\hat{h}_{i,j})}{(1+\rho_{m,t})(\omega_i(t))^\alpha}\right\}}.$$

We consider a slight modification of the non-cooperative game (11) to show the robustness of the proposed algorithm. We replace the utilities defined in the game (11) by the asymptotic time limits of the robust algorithm (12)–(14) and show that the robust algorithm and the assigned reservation rate vector form a Nash equilibrium.

ODE Approximation: We begin by discussing the existence of solutions of the above ODEs. The ODE (15) and (16) have a unique solution as will be established below:

***Lemma** 1:* The ODE (16) has unique bounded solution for any initial condition in $\mathcal{A}$ with $\mathcal{A} := [0,1]^M \times \mathcal{R}_+^{MD}$ and it is bounded as below where $\eta := \sup_{m,j} E[|f(h_{m,j})|]$.

$$|u_{m,j}(t)| \leq \eta - (\eta - |u_{m,j,0}|)e^{-t} \text{ for all } t.$$

Proof: The proof is along lines of Lemma 1 in [10]. See [1] for details. □

We now define the limit set of the ODEs (15)–(16) and its δ-neighborhood:

$$\mathbb{L}^{ODE} := \lim_{t\to\infty} \cup_{\Theta\in\mathcal{A}}\{\Theta(s) : s \geq t \text{ and } \Theta(0) = \Theta\}.$$

$$\mathbb{B}_\delta(\mathbb{L}^{ODE}) := \left\{\Theta : |\Theta - \tilde{\Theta}| \leq \delta \text{ for some } \tilde{\Theta} \in \mathbb{L}^{ODE}\right\}.$$

The following theorem now establishes that the trajectory $\{\Theta_t; t\}$ with $\Theta_t := \{\{p_{m,t}\}_m, \ \{u_{m,j,t}\}_{m,j}\}$, ultimately spends time in the limit set defined above (Proof in [1]).

***Theorem** 2:* Assume the following:

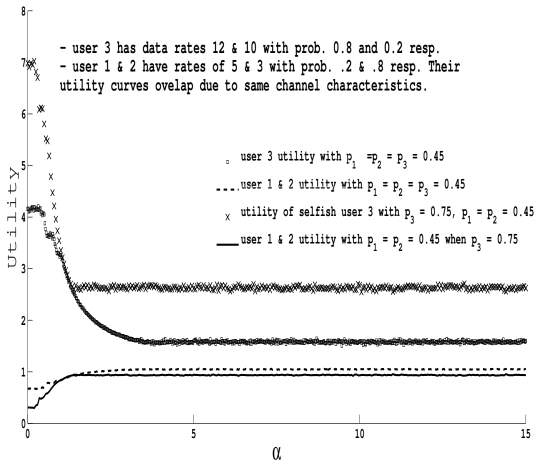

Fig. 2. Discrete channel states: A non-cooperative MS gains at the cost of cooperative ones

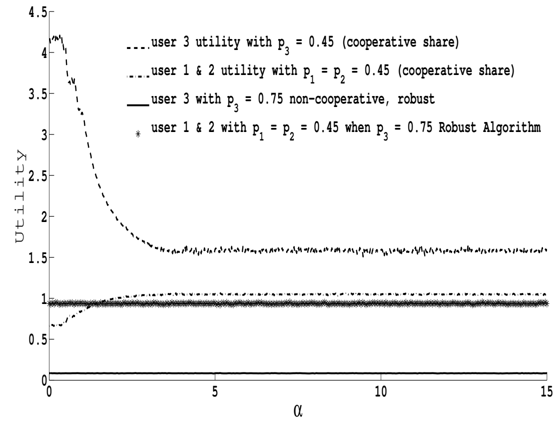

Fig. 3. Discrete channel states: Robust scheduler punishes the non-cooperative MS and aids the cooperative ones

- There exists a sequence $\epsilon_k \to \infty$ with $\lim_k \sup_{0 \le l \le \epsilon_k} \mu_{k+l}/\mu_k = 0$.
- The channel state $\{\mathbf{h}_k\}$ is an independent and identically distributed (IID) sequence with finite mean and variance.
- The average rates are bounded by the same constant, i.e., $|f(h_{m,j})| \le B$ for all m, j.
- $\{\mathbf{h}_k\}$ has continuous and bounded density.

Then for every $\delta > 0$, the fraction of time the tail of the algorithm $\{\Theta_\tau\}_{\tau \ge t}$ for any initial condition $\Theta_0 \in \mathcal{A}$ spends in the δ-neighborhood of the limit set $\mathbb{B}_\delta(\mathbb{L}^{ODE})$ tends to one as $t \to \infty$. □

Further analysis is obtained by studying the limit set of the above ODEs. The first ODE (15) has a unique solution and a unique attractor:

$$p_m(t) \;=\; p_m^* - (p_m^* - p_{m,0})\, e^{-t}, \qquad p_m^* = \frac{\mathsf{E}\,[b_m]}{c_m}. \quad (17)$$

That means, the BS via iteration (12) estimates the RTS reservation rates $[p_m]$ used by all the MS's. And as we will see below for any MS m, it uses only the excess (w.r.t. the assigned reservation rate) given by $(p_m^* - p_m^{ref})^+$ to punish it. When one MS becomes non cooperative, the c_m of the other MS's actually should decrease, however the BS uses the larger one corresponding to all cooperative case. As seen from (17) this results in a reduced estimate for the reservation rates of the other MS's. Since one uses only the positive part of the excess this would not alter the analysis.

We now study the limit set of the second ODE (16). In particular we study its equilibrium points. Any equilibrium point of the ODE (16) satisfies the following fixed point equation:

$$u_m^* \;=\; \sum_j E\left[\frac{f(h_{m,j})}{1+\rho_m^*}\beta_{m,j}\right], \quad (18)$$

$$\omega_m \;:=\; u_m^* + \rho_m^*, \qquad \rho_m^* = \Delta(p_m^* - p_m^{\rm ref})^+,$$
$$\beta_{m,j} \;=\; \cap_{i \ne m} \mathcal{X}_{\left\{\frac{f(\hat h_{m,j})}{(1+\rho_m^*)(\omega_m)^\alpha} \ge \frac{f(\hat h_{i,j})}{(1+\rho_i^*)(\omega_i)^\alpha}\right\}}. \quad (19)$$

Existence of a fixed point for the above equation can be established as in Theorem 1. We further have the following:

Lemma 2: (19) has a unique fixed point i.e., the ODE (16) has an unique equilibrium point.

Proof: The proof is in [1]. □

One still needs to show that the above unique equilibrium is indeed a limit point. This is given by [9, Theorem 2.2] for the case with $\alpha \le 1$ and without aloha reservation. The proof of [9, Theorem 2.2] can easily be imitated for the cooperative case, i.e., when $p_{m,0} = p_m^{\rm ref}$ for all m. Note that, with $p_{m,0} = p_m^{\rm ref}$ for all m, from (17) that $p_m(t) = p_m^* = p_m^{\rm ref}$ for all t and m and hence one can treat ω, ρ like constants and then [9, Theorem 2.2] can be extended. One need to show that the unique equilibrium point of Lemma 2 is a limit point for general case and this is work in progress.

Nash Equilibrium: By Theorem 2, the robust algorithm converges weakly to the limit set of the ODEs (15)–(16) as discussed earlier. The analysis of the robust algorithm (12)–14 can be obtained by further studying these equilibrium points and it is easy to verify the following result:

Lemma 3: At any rate vector $\mathbf{p} = [p_1, \cdots, p_M]$, the unique equilibrium point is upper bounded for all m by:

$$u_m^* \le \frac{\sum_j E[f(h_{m,j})}{1 + \Delta(p_m^* - p_m^{\rm ref})^+}.$$

Proof: The proof is immediate from the fixed point equation (18). □

So, one can chose Δ large enough such that

$$u_m^* < u_m^{\text{ref}} \text{ for all } m \text{ with } p_m^* > p_m^{\text{ref}},$$

where u_m^{ref} is the limit when all the MS are cooperative, i.e., the fixed point of Lemma 2 with $\mathbf{p}^{\text{ref}}$. With this Δ, the algorithm (12)–14 converges weakly (for any MS m with w representing weak convergence) to:

$$\lim_{t\to\infty} u_{m,t} \stackrel{w}{=} u_m^* < u_m^{\text{ref}} \text{ when } p_m^* > p_m^{\text{ref}}.$$

Thus any MS m obtains a smaller u_m when it deviates unilaterally from its designated reservation rate $p_m^* > p_m^{\text{ref}}$. In other words, any MS obtains a smaller utility if it attempts more aggressively. Hence the reservation rates $\mathbf{p}^{\text{ref}}$ and the robust policy β given by (12)–(14) form a Nash Equilibrium for a game with utilities defined by the weak time limits:

Utilities

$$= \left\{ \lim_{t\to\infty} u_{1,t}, \cdots, \lim_{t\to\infty} u_{M,t}, \sum_m \Gamma_\alpha \left(\lim_{t\to\infty} u_{m,t} \right) \right\}.$$

V. Numerical Examples

We now consider numerical examples illustrating different scheduling algorithms considered in this work. The objective is to demonstrate how quickly convergence occurs and to get a sense of the penalty incurred by non cooperative nodes.

Example 1 We begin with the discrete channel example of Section III-B when the common assigned reservation rate equals 0.45. We consider the case when MS 3, is selfish and uses an increased RTS reservation rate $p_3 = 0.75$. We plot the asymptotic utilities $\lim_k u_{m,k}$ as a function of α. These limits are plotted for the non robust, (7), as well as the robust scheduler, (14), respectively in figures 2 and 3. Both the figures also show the limits corresponding to the cooperative case, referred to as cooperative shares.

While the utility of MS 3 increases with increased reservation rate, it however reduces the utility of the other two MS's resulting in unfair allocations (see Fig. 2). When the scheduler is replaced with the proposed robust algorithm, MS 3 is penalized while the utilities of other MS's are improved. Note that the utility curves for MS 1 and 2 overlap (due to identical channel statistics) while MS 3 has almost 0 utility in Fig. 3 with robust scheduler.

Example 2: We now consider continuous channel states. Fig. 4 shows the convergence of the scheduling algorithm for the case with $\alpha = 0$, $M = 3$ and with 2 data channels. The MS have asymmetric channel states and the $h_{m,j}$'s are truncated Rayleigh distributed with parameter σ_m (as in [10], [11]). Here $\sigma_1 = 60, \sigma_2 = 30$ and $\sigma_3 = 10$. The MS's are again assigned equal reservation rates, i.e., $p_m^{\text{ref}} = 0.45$ for all m and $\mu = 4 \times 10^{-5}$. MS 1 is a non-cooperative node and uses an increased reservation rate $p_1 = 0.65 > p_1^{ref}$.

The first plot in Fig. 4 shows the convergence of estimates $p_{m,k}$ for the 3 users; $p_{1,k}$ converges to 0.65 while $p_{2,k}$ and

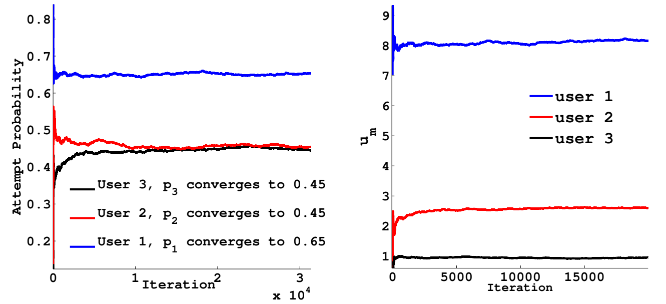

Fig. 4. Estimated transmission probabilities p_m and estimated utility u_m

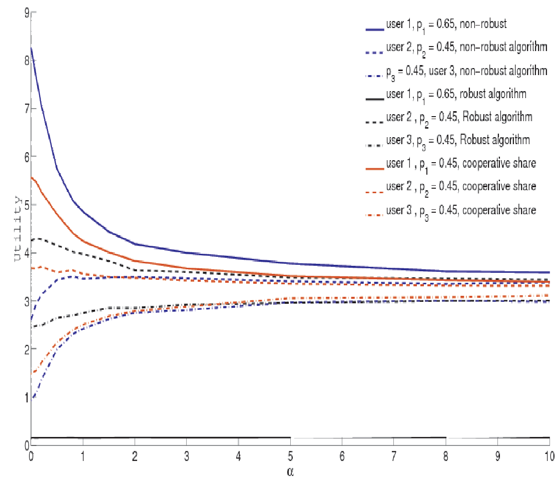

Fig. 5. Continuous channel states: comparison of non robust and robust schedulers

$p_{3,k}$ converge to 0.45. The second part of Fig. 4 shows the convergence of estimated utilities $u_{m,k}$. In summary, the trajectories converge to their corresponding limits, even when started from an unknown initial point different from the limit, reaffirming the stochastic approximation Theorem 2.

Example 3 : We continue with Example 2. As in Example 1, we compare the two scheduling algorithms (7) with (14). The asymptotic limits ($\{\lim_k u_{m,k}\}_m$) are plotted for the non robust (blue curves) as well as the robust scheduler (black curves) in Fig. 5. Orange curves represent the cooperative shares. From Fig. 5 near $\alpha = 0$, the selfish behavior of MS 1 increases $\lim_k u_{1,k}$ corresponding to non robust scheduler (continuous blue curve), which is at the cost of the same for other users (other two blue curves). There is a significant improvement in MS 1 utility for all the values of α. Such a behavior is not seen in the corresponding black curves, wherein the robust algorithm actually penalizes the selfish user. Note actually that the utility of MS 1 is almost equal to 0. Further, it improves the utility of the cooperative users.

In all the above examples, we observe that the robust algorithm not only punishes non-cooperative MS but also improves the utilities of the cooperative MS (see Fig. 3 and 5). Thus it exhibits anti jamming property, whenever the noncooperation is within manageable limits.

VI. Discussion

We have considered and analyzed a combined system with collision based reservation requests and a scheduler that trades-off the system efficiency with desired level of fairness. That the system is prone to noncooperation is established. We have proposed a scheduling mechanism in which users violating a prescribed reservation rates are penalized. However we have not addressed the choice of reservation rates. Two possible criteria to determine this choice are immediate – (a) choosing a reservation rate vector that optimizes a network utility and (b) a reservation rate vector to simultaneously achieve the assigned utility for each MS. This is part of the future work. While the initial numerical analysis of the non-saturated packet arrivals is established in [1], an extensive analysis is being carried out.

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Appendix A

Proof of Theorem 1: We first prove the second part. We fix the reservation rates $\mathbf{p}$ and neglect them in the rest of the proof. Consider the function defined component wise by:

$$\Theta_m(\mathbf{u}) := \sum_j E\left[f(h_{m,j}) \Pi_{i\neq m} \mathcal{X}_{\left\{ \frac{f(\hat{h}_{m,j})}{u_m^\alpha} > \frac{f(\hat{h}_{i,j})}{u_i^\alpha} \right\}} \right] \quad (20)$$

Under the theorem hypothesis, Θ_m is continuous in $\mathbf{u}$ because:

a) for any sequence $\mathbf{u}^{(n)} \to \mathbf{u}$, the term inside the expectation:

$$\Pi_{i\neq m} \mathcal{X}_{\left\{ \frac{f(\hat{h}_{m,j})}{\left(u_m^{(n)}\right)^\alpha} > \frac{f(\hat{h}_{i,j})}{\left(u_i^{(n)}\right)^\alpha} \right\}} \to \Pi_{i\neq m} \mathcal{X}_{\left\{ \frac{f(\hat{h}_{m,j})}{(u_m)^\alpha} > \frac{f(\hat{h}_{i,j})}{(u_i)^\alpha} \right\}}$$

at all $\mathbf{h}$ which are the points of continuity and hence for almost all $\mathbf{h}$ and this true for all j.

b) It is easy to see that,

$$|\Theta_m(\mathbf{u})| \leq \sum_j E[f(h_{m,j})] < \infty \text{ for any } \mathbf{u}. \quad (21)$$

Thus the expected value $\Theta_m(\mathbf{u}^{(n)}) \to \Theta_m(\mathbf{u})$ by Dominated Convergence theorem.

c) Sequential continuity implies continuity in a finite dimension space, hence $\Theta := [\Theta_1, \cdots, \Theta_M]$ is continuous in $\mathbf{u}$.

From (21) Θ is bounded and hence by Brouwer's fixed point theorem, there exist a fixed point $\mathbf{u}^* = [u_1^*, \cdots, u_M^*]$ for the map Θ and hence for Λ proving the second part of the theorem.

Fair scheduler using the fixed point: If there exists a fixed point of Λ, then define β^α by (6) and define,

$$G^\alpha(\beta) = \sum_m \Gamma_\alpha(u_m(\beta)).$$

Since Γ_α is a concave function of u and hence for any β,

$$G^\alpha(\beta) - G^\alpha(\beta^\alpha) \leq \sum_m \left[u_m(\beta) - u_m(\beta^\alpha)\right] d\Gamma_\alpha(u_m(\beta^\alpha)). \quad (22)$$

Consider the following function:

$$\begin{aligned} \beta \quad \mapsto \quad & \sum_m u_m(\beta) d\Gamma_\alpha(u_m(\beta^\alpha)) \\ = \quad & \mathbb{E}_{\mathbf{h}} \left[\sum_m \sum_j f(\hat{h}_m, j) d\Gamma_\alpha(u_m(\beta^\alpha)) \beta(m,j) \right]. \quad (23) \end{aligned}$$

From (6) for every m, j and channel state $\mathbf{h}$ and for any scheduler β with $\beta(m,j) \in [0,1]$ and $d\Gamma_\alpha(u) = 1/u^\alpha$,

$$f(\hat{h}_m, j) d\Gamma_\alpha(u_m(\beta^\alpha))\beta(m,j) \leq f(\hat{h}_m, j) d\Gamma_\alpha(u_m(\beta^\alpha))\beta^\alpha(m,j).$$

Hence β^α maximizes the function (23) and so

$$\sum_m \left[u_m(\beta) - u_m(\beta^\alpha)\right] d\Gamma_\alpha(u_m(\beta^\alpha)). \leq 0$$

Hence from (22) $G^\alpha(\beta) - G^\alpha(\beta^\alpha) \leq 0$. Hence β^α maximizes G^α among all the schedulers and hence is an alpha-fair scheduler. □