Resource Allocation for the Multi-Cell OFDMA System and Its Capacity Bounds

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***Abstract*—Recently, joint subchannel allocation and transmission power control problems for multi-cell orthogonal frequency-division multiple access (OFDMA) systems have been actively studied. However, since the problems are notoriously difficult and complex, only heuristic approaches are mainly used to study the problems without trying to achieve the optimal resource allocation and the maximum system capacity. In this paper, we study the joint subchannel allocation and transmission power control problem for multi-cell OFDMA systems from the point of the optimal resource allocation and the maximum system capacity with a monotonic optimization approach. Even though we do not obtain the exact optimal resource allocation that achieves the maximum system capacity, we develop an algorithm for resource allocations that provide both upper and lower bounds on the maximum system capacity. Through numerical results, we evaluate our algorithm showing that it provides good approximations to the maximum system capacity in multi-cell OFDMA systems in most cases.**

***Index Terms*—Resource allocation; multi-cell OFDMA systems; subchannel allocation; transmission power control; capacity bounds.**

I. Introduction

Orthogonal frequency-division multiple access (OFDMA) has been one of the widely deployed multiple access schemes in many wireless systems, and efficient resource allocation for the OFDMA system is one of the most important research issues over the last decade. In the OFDMA system, subchannel allocation and transmission power control can be jointly considered to improve the system performance, and in the past years, this problem has been extensively studied with various system models and approaches.

In [1]–[7], joint subchannel allocation and transmission power control problems for the single-cell OFDMA system is studied. In the single-cell OFDMA system, since there is no intracell interference among users, we do not have to consider interference among users, which makes the resource allocation problem be relatively easy. Hence, the optimal resource allocation that achieves the maximum capacity of the system can be easily obtained. However, if we consider a multi-cell environment, which is a more realistic scenario, despite of no intracell interference by using OFDMA, we should deal with intercell interference among cells. Hence, the structure of the problem for the multi-cell system is totally different from that for the single-cell system, and the problem for the multi-cell system is much more difficult and complicated than that for the single-cell system. Hence, in general, it is difficult to directly apply the above proposed schemes for the single-cell OFDMA system to the multi-cell OFDMA system.

Hence, more recently, joint subchannel allocation and transmission power control problems in the multi-cell OFDMA system have been studied [8]–[21]. In [8]–[16], a joint subchannel allocation and transmission power control problem is divided into a subchannel allocation sub-problem and a transmission power control sub-problem, and heuristic algorithms to solve the two sub-problems are proposed. In [8], three heuristic iterative algorithms that iteratively compute subchannel allocation and transmission power are proposed. In [9], a graph-based two-phased heuristic algorithm is proposed, and, in [10], to reduce the high complexity of the graph-based algorithm, a heuristic subchannel allocation algorithm that requires a polynomial time is proposed. In [11], an algorithm that consists of the greedy search for subchannel allocation and the lagrange dual based transmission power control is proposed. In [12], the subchannel allocation is performed by using mixed integer linear programming and duality-based transmission power control algorithm is proposed. In [13], a heuristic algorithm considering the intercell interference with the cognitive radio functionality is proposed. In [14], to solve the joint subchannel allocation and transmission power control problem, an additional interference constraint is imposed, and an iterative algorithm that iteratively calculates subchannel allocation and transmission power is proposed. In [15], a joint subchannel allocation and transmission power control problem with a minimum number of subchannel requirement is decomposed by using Benders' Decomposition [22] into two sub-problems, and heuristic iterative algorithms are proposed to solve each of sub-problems. In [16], a two-stage heuristic resource allocation algorithm is proposed. In the first stage, subchannel allocation is performed in each cell without considering the interference from other cells, and in the second stage, transmission power control is performed by using geometric programming. The above proposed heuristic algorithms in [8]–[16] have the limitation that cannot achieve the optimal solution in the multi-cell OFDMA system.

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There are game-theoretic approaches to solve joint subchannel allocation and transmission power control problems in the multi-cell OFDMA system [17]–[19]. In [17], a virtual referee which monitors the non-cooperative resource competition is proposed to enhance the performance. In [18], integer data rates are used to formulate the resource allocation problem and a non-cooperative algorithm is proposed. In [19], a potential game is used to minimize the interference-sum for each mobile station. Note that in general, game-theoretic approaches do not guarantee achieving the optimal solution and the maximum system capacity.

In [20], [21], an algorithm that achieves the optimal solution for the joint subchannel allocation and transmission power control problem in a two-cell OFDMA system is proposed. However, the proposed approach in [20], [21], has the limitation that it can be applied only for the two-cell system.

Despite the above extensive studies, to our best knowledge, the optimal solution of the joint subchannel allocation and transmission power control problem in the multi-cell OFDMA system and its maximum system capacity have not been achieved yet. Hence, in the above works for the multi-cell OFDMA system, to evaluate the performance of their proposed algorithms, they are compared with relatively simple heuristic algorithms, e.g., random subchannel allocation, fixed subchannel allocation, equal transmission power control, etc. Hence, even though it has been shown that the proposed algorithms in the previous works [8]–[21] provide relatively higher performance than some other algorithms and provide some good properties, we still do not know how close to the optimal one their performance is.

In this paper, we study the joint subchannel allocation and transmission power control problem to maximize the sum capacity of the downlink in the multi-cell OFDMA system. Since the joint subchannel allocation and transmission power control problem is a coupled mixed integer non-convex problem, in general, it is highly difficult to obtain its optimal solution. Hence, instead of trying to achieve the optimal solution directly, in this paper, we will take an approach that provides the solutions for upper and lower bounds on the maximum sum capacity of the multi-cell OFDMA system. In addition, we will show that in practical scenarios such as the cases with interference power dominates noise power or high signal to interference and noise ratio (SINR), the gap between the two bounds is very small. To this end, we will use the monotonic optimization approach [23], [24]. This monotonic optimization has been successfully used to obtain the optimal transmission power control (and scheduling) in wireless networks with a single channel [25], [26]. However, the algorithms in [25], [26] cannot be directly used for our problem, since the OFDMA system is a multi-channel system in which we should deal with not only power control for subchannels that are coupled with each other but also subchannel allocation to users.

Compared with the previous works for the multi-cell OFDMA system, we can summarize our contributions as: 1) we have an algorithm that provides solutions for upper and lower bounds on the maximum sum capacity; 2) the solution that provides the lower bound could be a good approximation to the optimal subchannel allocation and transmission power control; 3) even though our algorithm might not be practically implementable due to some practical limitations, the two bounds could provide the approximated sum capacities that are very close to the maximum sum capacity of the multi-cell OFDMA system, which can be used as a benchmark to evaluate the performance of other heuristic algorithms.

The remainder of this paper is organized as follows. In Section II, we describe the system model and formulate the optimization problem. In Section III, joint subchannel allocation and transmission power control is studied. In Section IV, the numerical results are provided, and we finally conclude this paper in Section V.

II. System Model and Problem

In this paper, we consider a downlink multi-cell OFDMA system with K base stations (BSs) and M_k mobile stations (MSs) for each BS k. We denote the set of BSs as $\mathcal{K} = \{1, 2, ..., K\}$ and the set of MSs which communicate with BS k as $\mathcal{M}_k = \{1, 2, ..., M_k\}$. Each BS has L subchannels and we denote the set of subchannels as $\mathcal{L} = \{1, 2, ..., L\}$. We denote subchannel allocation indicator $a^l_{(k,m)}$ as

$$a^l_{(k,m)} = \begin{cases} 1, \text{ if BS } k \text{ allocates its subchannel } l \text{ to MS } m \\ 0, \text{ otherwise} \end{cases}, \tag{1}$$

and let $\boldsymbol{a} = \left(a^l_{(k,m)}\right)_{\forall k \in \mathcal{K}, \forall m \in \mathcal{M}_k, \forall l \in \mathcal{L}}$. We assume that each BS can assign a subchannel to only one MS in its cell, which is represented by

$$\sum_{m \in \mathcal{M}_k} a^l_{(k,m)} \leq 1, \quad \forall k \in \mathcal{K}, \quad \forall l \in \mathcal{L}. \tag{2}$$

We denote the transmission power of BS k at subchannel l as p^l_k. Each BS operates under the total transmission power constraint as

$$\sum_{l \in \mathcal{L}} p^l_k \leq p_{\text{max}}, \quad \forall k \in \mathcal{K}, \tag{3}$$

where p_{max} is the maximum total transmission power of the BS. Then, obviously, we also have the following transmission power constraint for each subchannel:

$$0 \leq p^l_k \leq p_{\text{max}}, \quad \forall l \in \mathcal{L}, \quad \forall k \in \mathcal{K}. \tag{4}$$

The SINR for MS m which communicates with BS k at subchannel l is obtained as

$$\gamma^l_{(k,m)}(\boldsymbol{p}) = \frac{g^l_{(k,m)} p^l_k}{\sum\limits_{n \in \mathcal{K}, n \neq k} g^l_{(n,m)} p^l_n + N_0}, \tag{5}$$

where $g^l_{(n,m)}$ is the channel gain between BS n and MS m at subchannel l, N_0 is the noise power, and $\boldsymbol{p} = \left(p^l_k\right)_{\forall k \in \mathcal{K}, \, \forall l \in \mathcal{L}}$.

To model the capacity of a link through which BS k communicates with MS m at subchannel l, we use the Shannon capacity as

$$r^l_{(k,m)}(\boldsymbol{a},\boldsymbol{p}) = a^l_{(k,m)} \log\left(1+\gamma^l_{(k,m)}\left(\boldsymbol{p}\right)\right). \tag{6}$$

Then, the achieved sum capacity of MS m which communicates with BS k is obtained as

$$\begin{aligned} R_{k,m}\left(\boldsymbol{a},\boldsymbol{p}\right) &= \sum_{l\in\mathcal{L}} r^l_{(k,m)}(\boldsymbol{a},\boldsymbol{p}) \\ &= \sum_{l\in\mathcal{L}} a^l_{(k,m)} \log\left(1+\gamma^l_{(k,m)}\left(\boldsymbol{p}\right)\right), \end{aligned} \tag{7}$$

and finally the achieved sum capacity of the downlink in the multi-cell OFDMA system is obtained as

$$\begin{aligned} \psi\left(\boldsymbol{a},\boldsymbol{p}\right) &= \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}_k} R_{k,m}\left(\boldsymbol{a},\boldsymbol{p}\right) \\ &= \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}_k} \sum_{l\in\mathcal{L}} a^l_{(k,m)}\ \log\left(1+\gamma^l_{(k,m)}\left(\boldsymbol{p}\right)\right). \end{aligned} \tag{8}$$

In this paper, we want to maximize the sum capacity for the downlink multi-cell OFDMA system by optimizing subchannel allocation for each user and transmission power for each subchannel considering the constraints for subchannel allocation in (2) and transmission power in (3) and (4). Hence, the optimization problem is formulated as

$$\begin{aligned} \text{(P1)} \quad & \underset{\boldsymbol{a},\,\boldsymbol{p}}{\text{maximize}} \quad \psi\left(\boldsymbol{a},\boldsymbol{p}\right) \\ & \text{subject to} \quad \sum_{m\in\mathcal{M}_k} a^l_{(k,m)} \le 1,\ \forall k\in\mathcal{K},\ \forall l\in\mathcal{L}, \\ & \qquad \sum_{l\in\mathcal{L}} p^l_k \le p_{\max},\ \forall k\in\mathcal{K}, \\ & \qquad \boldsymbol{a}\in\mathcal{A},\ \boldsymbol{p}\in\mathcal{P}, \end{aligned}$$

where

$$\mathcal{A} = \left\{\boldsymbol{a}\ \middle|\ a^l_{(k,m)} \in \{0,1\},\ \forall k\in\mathcal{K},\ \forall m\in\mathcal{M}_k,\ \forall l\in\mathcal{L}\right\}$$

and

$$\mathcal{P} = \left\{\boldsymbol{p}\ \middle|\ 0 \le p^l_k \le p_{\max},\ \forall k\in\mathcal{K},\ \forall l\in\mathcal{L}\right\}.$$

Note that problem (P1) is a coupled mixed integer non-linear non-convex problem, which is in general difficult to solve.

III. Subchannel Allocation and Transmission Power Control

To solve problem (P1), in this section, we propose an algorithm that is called a joint subchannel allocation and transmission power control based on polyblock outer approximation (JSPPA) algorithm, which is based on the algorithm for monotonic optimization [23]–[26]. In monotonic optimization, since the objective function is a monotonic increasing or decreasing function, we can find its optimal solution at the boundary of the feasible region. The proposed algorithm iteratively finds a feasible solution which is located at the boundary of the feasible region, and finally converges the optimal solution. We first introduce some mathematical preliminaries and then present the proposed algorithm.

A. Mathematical Preliminaries [23], [24]

To begin with, we first introduce some preliminary definitions and properties of monotonic optimization. In this paper, for any two vectors $\boldsymbol{x},\boldsymbol{x'}\in\mathcal{R}^n$, we write $\boldsymbol{x}\preceq\boldsymbol{x'}$ if $x_i\le x'_i, \quad \forall i=1,\ldots,n$, and say that $\boldsymbol{x'}$ dominates $\boldsymbol{x}$.

Definition 1 (Box): Given any two vectors $\boldsymbol{x},\boldsymbol{x'}\in\mathcal{R}^n_+$ (the n-dimensional nonnegative real domain), if $\boldsymbol{x}\preceq\boldsymbol{x'}$, the hyperrectangle $[\boldsymbol{x},\boldsymbol{x'}] = \left\{\boldsymbol{z}\in\mathcal{R}^n_+ \middle| \boldsymbol{x}\preceq\boldsymbol{z}\preceq\boldsymbol{x'}\right\}$ is referred to as a *box*.

Definition 2 (Normal): A set $\mathcal{G}\subset\mathcal{R}^n_+$ is said to be *normal*, if for any two vectors, $\boldsymbol{x},\boldsymbol{x'}\in\mathcal{R}^n_+$ such that $\boldsymbol{x}\preceq\boldsymbol{x'}$, if $\boldsymbol{x'}\in\mathcal{G}$, then $\boldsymbol{x}\in\mathcal{G}$.

Proposition 1: The intersection and the union of normal sets are still normal sets.

Definition 3 (Reverse Normal): A set $\mathcal{H}\subset\mathcal{R}^n_+$ is said to be *reverse normal*, if for any two vectors, $\boldsymbol{x},\boldsymbol{x'}\in\mathcal{R}^n_+$ such that $\boldsymbol{x}\preceq\boldsymbol{x'}$, if $\boldsymbol{x}\in\mathcal{H}$, then $\boldsymbol{x'}\in\mathcal{H}$.

Definition 4 (Increasing Function): A function $f:\mathcal{R}^n\longrightarrow\mathcal{R}$ is said to be *increasing function* on $\mathcal{R}^n_+$, if for any two vectors, $\boldsymbol{x},\boldsymbol{x'}\in\mathcal{R}^n_+$ such that $\boldsymbol{x}\preceq\boldsymbol{x'}$, $f\left(\boldsymbol{x}\right)\le f\left(\boldsymbol{x'}\right)$.

Definition 5 (Monotonic Optimization): A monotonic optimization problem is a class of optimization problems which have the following formulation:

$$\begin{aligned} & \text{maximize} \quad f\left(\boldsymbol{x}\right) \\ & \text{subject to} \quad x\in\mathcal{G}\cap\mathcal{H}, \end{aligned}$$

where the domain $\mathcal{G}$ is a nonempty normal set, the domain $\mathcal{H}$ is a closed reverse normal set, and the function $f\left(\boldsymbol{x}\right)$ is an increasing function.

Definition 6 (Polyblock): A set $\mathcal{P}\subset\mathcal{R}^n_+$ is referred to as a *polyblock* with the set $\mathcal{T}$ in $[\boldsymbol{x},\boldsymbol{x'}]\subset\mathcal{R}^n_+$, if the set $\mathcal{P}$ is the union of a finite number of boxes $[\boldsymbol{x},\boldsymbol{z}]$, $\boldsymbol{z}\in\mathcal{T}\subset[\boldsymbol{x},\boldsymbol{x'}]$. The set $\mathcal{T}$ is referred to as the *vertex set* of the polyblock $\mathcal{P}$.

B. JSPPA Algorithm

To deal with problem (P1), in this subsection, we first modify problem (P1) by relaxing the total transmission power constraint (3), which results in the following problem:

$$\begin{aligned} \text{(P2)} \quad & \underset{\boldsymbol{a},\,\boldsymbol{p}}{\text{maximize}} \quad \psi\left(\boldsymbol{a},\boldsymbol{p}\right) \\ & \text{subject to} \quad \sum_{m\in\mathcal{M}_k} a^l_{(k,m)} \le 1,\ \forall k\in\mathcal{K},\ \forall l\in\mathcal{L}, \\ & \qquad \boldsymbol{a}\in\mathcal{A},\ \boldsymbol{p}\in\mathcal{P}. \end{aligned}$$

In this subsection, we first focus on solving problem (P2), and then return to problem (P1) and issue on the total transmission power constraint (3).

We now show that problem (P2) can be represented as a monotonic optimization problem. The link capacity (6) can be reformulated as

$$\begin{aligned} r^l_{(k,m)}(\boldsymbol{a},\boldsymbol{p}) &= a^l_{(k,m)} \log\left(1+\gamma^l_{(k,m)}\left(\boldsymbol{p}\right)\right) \\ &= \log\left(1+a^l_{(k,m)}\gamma^l_{(k,m)}\left(\boldsymbol{p}\right)\right), \\ & \quad \forall k\in\mathcal{K},\ \forall m\in\mathcal{M}_k,\ \forall l\in\mathcal{L}. \end{aligned} \tag{9}$$

Algorithm 1 Polyblock outer approximation algorithm [25]

1: **Initialization:** Construct the vertex set $\mathcal{T}_0$ as

$$\mathcal{T}_0 = \left\{ \boldsymbol{z} \,\middle|\, \boldsymbol{z} = \boldsymbol{1} + \boldsymbol{a} \circ \boldsymbol{\sigma}, \quad \forall \boldsymbol{a} \in \mathcal{A}^\dagger \right\},$$

where

$$\sigma^l_{(k,m)} = \frac{g^l_{(k,m)} p_{\max}}{N_0}, \quad \forall k \in \mathcal{K},\ \forall m \in \mathcal{M}_k,\ \forall l \in \mathcal{L},$$

and set $n = 0$. Furthermore, we define $\boldsymbol{e}^l_{(k,m)}$ as the vector whose every element is equal to zero except that (k, m, l)th element is equal to one.

2: **repeat**

3: In the set $\mathcal{T}_n$, select the vertex $\boldsymbol{z}_n$ that maximize the objective of the problem when the selected vertex is applied as

$$\boldsymbol{z}_n = \arg\max_{\boldsymbol{z} \in \mathcal{T}_n} \left\{ f(\boldsymbol{z}) = \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}_k} \sum_{l \in \mathcal{L}} \log\left(z^l_{(k,m)} \right) \right\}.$$

4: For $\boldsymbol{z}_n$, calculate the point $\pi_{\mathcal{Z}}(\boldsymbol{z}_n)$ as

$$\pi_{\mathcal{Z}}(\boldsymbol{z}_n) = \lambda_n (\boldsymbol{z}_n - \boldsymbol{1}) + \boldsymbol{1},$$

where λ_n can be achieved by using the projection algorithm, i.e., Algorithm 2.

5: Update the vertex set as

$$\mathcal{T}_{n+1} = (\mathcal{T}_n - \{\boldsymbol{z}_n\}) \cup \mathcal{T}^\dagger_n,$$

where

$$\mathcal{T}^\dagger_n = \left\{ \boldsymbol{y}^l_{(k,m)} \,\middle|\, \begin{array}{l} \boldsymbol{y}^l_{(k,m)} = \boldsymbol{z}_n - (\boldsymbol{z}_n - \pi_{\mathcal{Z}}(\boldsymbol{z}_n)) \circ \boldsymbol{e}^l_{(k,m)}, \\ \forall k \in \mathcal{K},\ \forall m \in \mathcal{M}_k,\ \forall l \in \mathcal{L} \end{array} \right\}.$$

6: $n \leftarrow n + 1$.

7: **until** $\dfrac{\|\boldsymbol{z}_n - \pi_{\mathcal{Z}}(\boldsymbol{z}_n)\|}{\|\boldsymbol{z}_n\|} \leq \epsilon$.

8: We achieve the optimal vertex $\boldsymbol{z}^*$, i.e., the optimal vector of SINR plus one, as $\boldsymbol{z}^* = \pi_{\mathcal{Z}}(\boldsymbol{z}_n)$ and the optimal transmission power $\boldsymbol{p}^*$ is achieved from results of the projection algorithm, Algorithm 2.

Furthermore, we define new variable $z^l_{(k,m)}$ that denotes the SINR plus one for MS m which communicates with BS k at subchannel l and we let $\boldsymbol{z} = \left(z^l_{(k,m)} \right)_{\forall k \in \mathcal{K},\, \forall m \in \mathcal{M}_k,\, \forall l \in \mathcal{L}}$ that is called a vertex. By using this variable, problem (P2) can be reformulated as the following equivalent optimization problem:

$$\text{(P3)} \quad \underset{\boldsymbol{z}}{\text{maximize}} \quad f(\boldsymbol{z}) = \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}_k} \sum_{l \in \mathcal{L}} \log\left(z^l_{(k,m)} \right)$$

$$\text{subject to} \quad \boldsymbol{z} \in \mathcal{Z},$$

where

$$\mathcal{Z} = \left\{ \boldsymbol{z} \,\middle|\, \begin{array}{l} 1 \leq z^l_{(k,m)} \leq 1 + a^l_{(k,m)} \gamma^l_{(k,m)}(\boldsymbol{p}), \\ \forall k \in \mathcal{K},\ \forall m \in \mathcal{M}_k,\ \forall l \in \mathcal{L},\ \forall \boldsymbol{a} \in \mathcal{A}^\dagger,\ \forall \boldsymbol{p} \in \mathcal{P} \end{array} \right\}$$

and

$$\mathcal{A}^\dagger = \left\{ \boldsymbol{a} \,\middle|\, \sum_{m \in \mathcal{M}_k} a^l_{(k,m)} \leq 1,\ \forall k \in \mathcal{K},\ \forall l \in \mathcal{L},\ \forall \boldsymbol{a} \in \mathcal{A} \right\}.$$

We now show that problem (P3) is a monotonic optimization problem.

Proposition 2: Problem (P3) is a monotonic optimization problem.

Proof: According to the definition of *Box*, the feasible set $\mathcal{Z}$ of problem (P3) can be represented as a union of infinite number of boxes. If we define the hyperrectangle for each subchannel allocation $\boldsymbol{a} \in \mathcal{A}^\dagger$ and each transmission power $\boldsymbol{p} \in \mathcal{P}$ as

$$\mathcal{Z}_{(\boldsymbol{a},\boldsymbol{p})} = \{ \boldsymbol{z} \mid \boldsymbol{1} \preceq \boldsymbol{z} \preceq \boldsymbol{1} + \boldsymbol{a} \circ \boldsymbol{\gamma}(\boldsymbol{p}) \}^{1},$$

where $\boldsymbol{\gamma}(\boldsymbol{p}) = \left(\gamma^l_{(k,m)}(\boldsymbol{p}) \right)_{\forall k \in \mathcal{K},\, \forall m \in \mathcal{M}_k,\, \forall l \in \mathcal{L}}$, then feasible set $\mathcal{Z}$ can be represented as

$$\mathcal{Z} = \bigcup_{\boldsymbol{a} \in \mathcal{A}^\dagger,\ \boldsymbol{p} \in \mathcal{P}} \mathcal{Z}_{(\boldsymbol{a},\boldsymbol{p})}.$$

Since $\mathcal{Z}_{(\boldsymbol{a},\boldsymbol{p})}$ for any $\boldsymbol{a}$ and $\boldsymbol{p}$ is normal, by Proposition 1, feasible set $\mathcal{Z}$ is also normal. Furthermore, since the objective function of problem (P3) is an increasing function, problem (P3) is a monotonic optimization problem. ■

We can solve problem (P3), which is monotonic optimization, by using Algorithm 1, which is called a polyblock outer approximation algorithm for monotonic optimization [23]–[26]. For the brevity of the presentation, in this paper, we do not provide the details for the derivation of Algorithm 1, but we refer the readers to [23]–[26] for the details. Note that, in problem (P3), since the objective function is a monotonic increasing function, the optimal solution of problem (P3) can be achieved at the boundary of feasible region $\mathcal{Z}$. Hence, the polyblock outer approximation algorithm, which solves monotonic optimization, iteratively finds the optimal vertex $\boldsymbol{z}^*$, i.e., SINR plus one, at the boundary of feasible region $\mathcal{Z}$.

To find the optimal vertex $\boldsymbol{z}^*$, Algorithm 1 starts with an infeasible vertex that is located in the outside of the feasible region of the problem. In Algorithm 1, we first define the maximum achievable SINR for MS m which communicates with BS k at subchannel l as $\sigma^l_{(k,m)} = \frac{g^l_{(k,m)} p_{\max}}{N_0}$, and let $\boldsymbol{\sigma} = \left(\sigma^l_{(k,m)} \right)_{\forall k \in \mathcal{K},\, \forall m \in \mathcal{M}_k,\, \forall l \in \mathcal{L}}$. By using $\boldsymbol{\sigma}$, we set the initial vertex set $\mathcal{T}_0$ as

$$\mathcal{T}_0 = \left\{ \boldsymbol{z} \,\middle|\, \boldsymbol{z} = \boldsymbol{1} + \boldsymbol{a} \circ \boldsymbol{\sigma}, \quad \forall \boldsymbol{a} \in \mathcal{A}^\dagger \right\}.$$

Since a vertex in the initial vertex set consists of the maximum achievable SINR for each MS at each subchannel assuming that there is no interference, it could be an infeasible vertex that is located in the outside of the feasible region $\mathcal{Z}$. Hence,

[1]For two vectors with the same dimension, $\mathcal{A}, \mathcal{B} \in \mathcal{R}^n$, the Hadamard product $\mathcal{A} \circ \mathcal{B}$ is a vector of the same dimension, $\mathcal{A} \circ \mathcal{B} \in \mathcal{R}^n$, whose (i, j) element $(\mathcal{A} \circ \mathcal{B})_{(i,j)}$ is given by $(\mathcal{A})_{(i,j)} \times (\mathcal{B})_{(i,j)}$.

Algorithm 2 Projection algorithm [25], [27]

Initialization: For given $\boldsymbol{p}$, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}(k)$, $\forall l \in \mathcal{L}$, we define

$$\nu^l_{(k,m)}(\boldsymbol{p}) = g^l_{(k,m)} p^l_k,$$

and

$$\mu^l_{(k,m)}(\boldsymbol{p}) = \sum_{n \in \mathcal{K}, n \neq k} g^l_{(n,m)} p^l_n + N_0.$$

We let $\boldsymbol{\nu}(\boldsymbol{p}) = \left(\nu^l_{(k,m)}(\boldsymbol{p})\right)_{\forall k \in \mathcal{K}, \forall m \in \mathcal{M}(k), \forall l \in \mathcal{L}}$, and $\boldsymbol{\mu}(\boldsymbol{p}) = \left(\mu^l_{(k,m)}(\boldsymbol{p})\right)_{\forall k \in \mathcal{K}, \forall m \in \mathcal{M}(k), \forall l \in \mathcal{L}}$. Furthermore, for given $\boldsymbol{z}_n$, we define

$$\delta(\lambda, \boldsymbol{p}) = \min\left(\boldsymbol{\nu}(\boldsymbol{p}) - \lambda\left(\boldsymbol{\mu}(\boldsymbol{p}) \circ (\boldsymbol{z}_n - \boldsymbol{1})\right)\right),$$

where $\min(\boldsymbol{a})$ for a vector $\boldsymbol{a}$ represents the smallest element of the vector $\boldsymbol{a}$. Select arbitrary $\boldsymbol{p}_0 \in \mathcal{P}$, and set $t = 0$.

repeat

For given $\boldsymbol{p}_t$,

$$\lambda_t = \min\left(\boldsymbol{\nu}(\boldsymbol{p}_t) \circ (\boldsymbol{\mu}(\boldsymbol{p}_t) \circ (\boldsymbol{z}_n - \boldsymbol{1}))^{-1}\right).$$

For given λ_t,

$$\boldsymbol{p}_{t+1} = \arg\max_{\boldsymbol{p} \in \mathcal{P}} \min\ \delta(\lambda_t, \boldsymbol{p}).$$

$t \leftarrow t + 1$.

until $\delta(\lambda_t, \boldsymbol{p}_t) \leq 0$.

We achieve λ_t and $\boldsymbol{p}_t$.

to find the optimal (feasible) vertex which is at the boundary of the feasible region, we will iteratively find feasible vertices starting from the initial vertex set.

The operation in each iteration can be summarized as the following three steps. First, in the current vertex set, we select a vertex that maximizes the objective function of problem (P3). Note that the selected vertex may not be feasible. Second, we find a feasible vertex by projecting the selected vertex in the first step into the feasible region using Algorithm 2. Third, the vertex set is updated by using the selected vertex which is achieved in the first step and the feasible vertex which is achieved in the second step. A new vertex can be made by replacing the element of the selected vertex to the element of the feasible vertex. The above three steps are operated iteratively, and finally, we can achieve the optimal vertex $\boldsymbol{z}^*$ of problem (P3). Since each element of the optimal vertex $\boldsymbol{z}^*$ represents the SINR plus one, if an element of the optimal vertex is one, then it represents that the corresponding subchannel is not allocated to the corresponding MS. Hence, from optimal vertex $\boldsymbol{z}^*$, we can achieve subchannel allocation $\boldsymbol{a}^*$ as

$$a^{l*}_{(k,m)} = \begin{cases} 1, & z^{l*}_{(k,m)} > 1, \\ 0, & z^{l*}_{(k,m)} = 1 \end{cases}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}_k, \forall l \in \mathcal{L}. \tag{10}$$

When we achieve the optimal vertex by using Algorithm 2, from the result of Algorithm 2, we can obtain transmission power $\boldsymbol{p}^*$ that corresponds the optimal vertex $\boldsymbol{z}^*$. Hence, subchannel allocation $\boldsymbol{a}^*$ and transmission power $\boldsymbol{p}^*$ consist the optimal solution of problem (P2).

We now go back to problem (P1). Since in problem (P2), we relaxed the total transmission power constraint (3) from problem (P1), transmission power $\boldsymbol{p}^*$ may not satisfy the total transmission power constraint of problem (P1). In other words, the solution $(\boldsymbol{a}^*, \boldsymbol{p}^*)$ may not be a feasible solution of problem (P1). To obtain a feasible solution that satisfies all the constraints in problem (P1), we first define the largest total transmission power $\rho(\boldsymbol{p}^*)$ among those of BSs when $(\boldsymbol{a}^*, \boldsymbol{p}^*)$ is applied as

$$\rho(\boldsymbol{p}^*) = \max\left\{ p^*_k \,\middle|\, p^*_k = \sum_{l \in \mathcal{L}} p^{l*}_k,\ \forall k \in \mathcal{K} \right\}. \tag{11}$$

We then obtain $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ as

$$\tilde{\boldsymbol{p}}^* = \left(\frac{p_{\max}}{\rho(\boldsymbol{p}^*)}\right)\boldsymbol{p}^* \text{ and } \tilde{\boldsymbol{a}}^* = \boldsymbol{a}^*. \tag{12}$$

Hence, $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ is obtained by transmission power that is scaled down from power allocation $\boldsymbol{p}^*$ by a scaling factor $\frac{p_{\max}}{\rho(\boldsymbol{p}^*)}$, while maintaining the same subchannel allocation as $\boldsymbol{a}^*$. Then, we can easily show that $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ is a feasible solution of problem (P1) that satisfies all the constraints in problem (P1). The overall description of our algorithm is provided in Algorithm 3.

Note that even though $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ is a feasible solution, it may not be an optimal solution of problem (P1). However, we can show that two solutions $(\boldsymbol{a}^*, \boldsymbol{p}^*)$ and $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ that can be obtained from our algorithm provide upper and lower bounds on the maximum sum capacity of the multi-cell OFDMA system. We first let $(\boldsymbol{a}^{\boldsymbol{o}}, \boldsymbol{p}^{\boldsymbol{o}})$ be the optimal solution of problem (P1) that maximizes its objective function $\psi(\boldsymbol{a}, \boldsymbol{p})$, while satisfying all of its constraints. Then, we can show the following relationship, which indicates that $(\boldsymbol{a}^*, \boldsymbol{p}^*)$ provides the upper bound on the maximum sum capacity of the multi-cell OFDMA system while $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ provides its lower bound.

Theorem 1:

$$\psi\left(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*\right) \leq \psi\left(\boldsymbol{a}^{\boldsymbol{o}}, \boldsymbol{p}^{\boldsymbol{o}}\right) \leq \psi\left(\boldsymbol{a}^*, \boldsymbol{p}^*\right).$$

Proof: From the fact that the constraint set of problem (P1) is a subset of the constraint set of problem (P2), we have $\psi\left(\boldsymbol{a}^{\boldsymbol{o}}, \boldsymbol{p}^{\boldsymbol{o}}\right) \leq \psi\left(\boldsymbol{a}^*, \boldsymbol{p}^*\right)$. In addition, since $(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*)$ is a feasible solution of problem (P1), we also have $\psi\left(\tilde{\boldsymbol{a}}^*, \tilde{\boldsymbol{p}}^*\right) \leq \psi\left(\boldsymbol{a}^{\boldsymbol{o}}, \boldsymbol{p}^{\boldsymbol{o}}\right)$, which completes the proof. ■

IV. NUMERICAL RESULTS

In this section, we provide numerical results to evaluate the performance of our algorithm and also characterize upper and lower bounds of the maximum sum capacity that are achieved by our algorithm. We consider a one-tier seven-cell system in which each base station is located in the center of each cell and four mobile stations are located in each cell, as depicted in Fig. 1. We set the inter-site distance (ISD) to be

Algorithm 3 Joint Subchannel allocation and transmission Power control based on Polyblock outer Approximation

1: By relaxing the total transmission power constraint (3), problem (P1) can be modified to problem (P2).

2: Reformulate the link capacity (6) as

$$\begin{aligned} r^{l}_{(k,m)}(\boldsymbol{a},\boldsymbol{p}) &= a^{l}_{(k,m)} \log\left(1+\gamma^{l}_{(k,m)}(\boldsymbol{p})\right) \\ &= \log\left(1+a^{l}_{(k,m)}\gamma^{l}_{(k,m)}(\boldsymbol{p})\right), \\ &\qquad \forall k\in\mathcal{K},\ \forall m\in\mathcal{M}_k,\ \forall l\in\mathcal{L}. \end{aligned}$$

3: Define $z^{l}_{(k,m)}$ that denotes the SINR plus one for MS m which communicates with BS k at subchannel l and we let $\boldsymbol{z}=\left(z^{l}_{(k,m)}\right)_{\forall k\in\mathcal{K},\ \forall m\in\mathcal{M}_k,\ \forall l\in\mathcal{L}}$.

4: By using this variable, problem (P2) can be reformulated as problem (P3) that is a monotonic optimization problem as

$$\begin{aligned} \text{(P3)}\quad & \underset{\boldsymbol{z}}{\text{maximize}} && f(\boldsymbol{z})=\sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}_k}\sum_{l\in\mathcal{L}}\log\left(z^{l}_{(k,m)}\right) \\ & \text{subject to} && \boldsymbol{z}\in\mathcal{Z}. \end{aligned}$$

5: To solve problem (P3), we use polyblock outer approximation algorithm, i.e., Algorithm 1, that is used to solve a monotonic optimization problem.

6: By using Algorithm 1, we achieve the optimal vertex $\boldsymbol{z}^*$ of problem (P3).

7: From $\boldsymbol{z}^*$, the optimal subchannel allocation of problem (P2) is achieved as

$$a^{l*}_{(k,m)}=\begin{cases} 1, & z^{l*}_{(k,m)}>1, \\ 0, & z^{l*}_{(k,m)}=1 \end{cases}, \qquad \forall k\in\mathcal{K},\ \forall m\in\mathcal{M}(k),\ \forall l\in\mathcal{L}.$$

8: Furthermore, we achieve the optimal transmission power $\boldsymbol{p}^*$ of problem (P2) from results of Algorithm 1 when we achieve the optimal vertex $\boldsymbol{z}^*$.

9: From the solution of problem (P2), $(\boldsymbol{a}^*,\boldsymbol{p}^*)$, we can achieve the feasible solution of problem (P1), $(\tilde{\boldsymbol{a}}^*,\tilde{\boldsymbol{p}}^*)$, as

$$\tilde{\boldsymbol{a}}^*=\boldsymbol{a}^* \text{ and } \tilde{\boldsymbol{p}}^*=\left(\frac{p_{\max}}{\rho(\boldsymbol{p}^*)}\right)\boldsymbol{p}^*,$$

where $\rho(\boldsymbol{p}^*)=\max\left\{p^*_k \,\middle|\, p^*_k=\sum_{l\in\mathcal{L}}p^{l*}_k,\quad \forall k\in\mathcal{K}\right\}$.

1.5km, total bandwidth to be 10MHz, and the bandwidth of each subchannel to be 200kHz. Furthermore, we consider the pathloss model as $128+37.6\log(d)$ where d is the distance in kilometers, the fading model as Rayleigh fading, the noise power spectral density as -174dBm/Hz, and the noise figure as 10dB. Parameters and their values are summarized in Table I [28].

To evaluate the performance of our algorithm, we define the normalized difference, $\xi(\boldsymbol{a}^*,\boldsymbol{p}^*)$, between the upper and lower bounds of the maximum sum capacity as the ratio of the difference between upper and lower bounds to the lower bound as follows:

$$\xi(\boldsymbol{a}^*,\boldsymbol{p}^*)=\frac{\psi(\boldsymbol{a}^*,\boldsymbol{p}^*)-\psi(\tilde{\boldsymbol{a}}^*,\tilde{\boldsymbol{p}}^*)}{\psi(\tilde{\boldsymbol{a}}^*,\tilde{\boldsymbol{p}}^*)}.$$

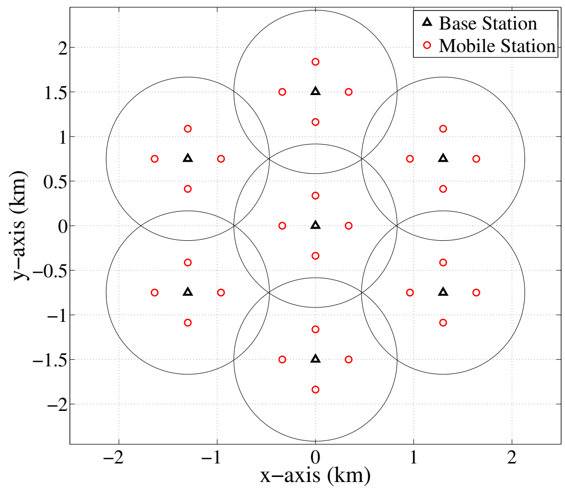

Fig. 1. Position of BSs and MSs in a multi-cell OFDMA system.

TABLE I
SYSTEM PARAMETERS

Parameter	Value
ISD	1.5 km
Total Bandwidth	10 MHz
Bandwidth of subchannel	200 kHz
Pathloss model	128+37.6log(d)
Fading	Rayleigh
Noise Power Spectral Density	-174 dBm/Hz
Noise Figure	10 dB

In Fig. 2, we provide the normalized difference varying the maximum total transmission power $p_{\max}$. The figure shows that as the maximum total transmission power increases, the normalized difference gets smaller. In addition, we can see that normalized differences are small in all cases, which implies that our algorithm provides good approximations to the optimal solution that results in tight upper and lower bounds on the maximum sum capacity of the multi-cell OFDMA system.

In Fig. 3, we provide the normalized difference varying the distance between MSs and the according BSs. The figure shows that as the distance between MSs and the according BSs gets closer, the normalized difference gets smaller. In addition, as in the previous case, the normalized differences are small in all cases, which also supports that our algorithm provides good approximations to the optimal solution that result in tight upper and lower bounds on the maximum sum capacity of the multi-cell OFDMA system.

V. CONCLUSION

In this paper, we studied the joint subchannel allocation and transmission power control problem for the downlink in the multi-cell OFDMA system. Through a monotonic optimization approach, we developed an algorithm that provides both

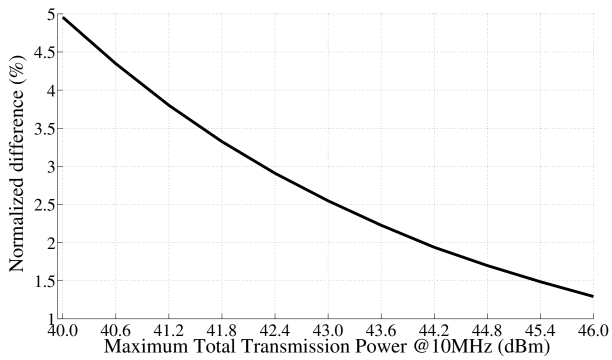

Fig. 2. Normalized difference according to the maximum total transmission power.

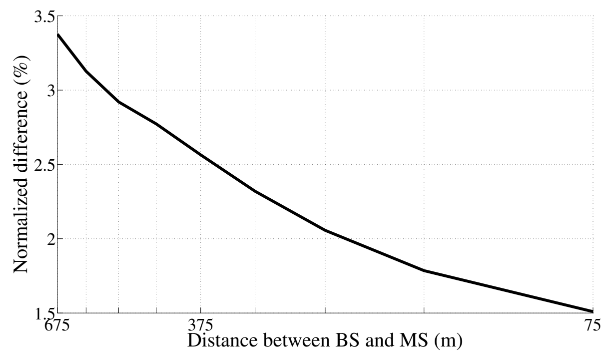

Fig. 3. Normalized difference according to the distance between BS and MS.

upper and lower bounds on the maximum sum capacity. We showed that those two bounds are reasonably close to each other. This indicates that they can be good approximations to the maximum sum capacity. Hence, with our algorithm, we can evaluate the maximum sum capacity of the multi-cell OFDMA system when transmission power and subchannels are optimally allocated. In addition, our algorithm can be used as a tool for a benchmark to evaluate the efficiency of other heuristic algorithms relative to the maximum sum capacity of the multi-cell OFDMA system.

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