# Efficient Distributed Scheduling in Cognitive Radio Networks in the Many-Channel Regime

Dongyue Xue and Eylem Ekici

Department of Electrical and Computer Engineering
Ohio State University, USA
Email: {xued, ekici}@ece.osu.edu

***Abstract*—The design of efficient and distributed scheduling algorithms is essential to garner the full potential of cognitive radio networks. In this paper, we propose a distributed OFDM-based scheduling algorithm, named *collision-queue-regulated algorithm*, which aims to limit the collision rate to a level imposed by primary users of a cognitive radio network. Via a novel equivalent-queue-system analysis, we prove that the proposed algorithm can achieve *at least* a constant fraction of the asymptotic capacity region in the many-channel regime. Our numerical studies indicate that the proposed distributed collision-queue-regulated algorithm achieves a throughput very close to that achievable by a centralized throughput-optimal back-pressure-based scheduling algorithm.**

## I. Introduction

Cognitive radio networks (CRNs) [1] allow unlicensed users, referred to as secondary users (SUs), to opportunistically exploit the unused spectrum allocated to licensed users, referred to as primary users (PUs). Different from traditional wireless networks, a typical design requirement for CRNs is that the collisions/interference between PUs and SUs transmission should be avoided or kept under a certain acceptable threshold.

Developing efficient scheduling algorithms for CRNs is essential to garner the full potential of cognitive radio networks. In recent years, centralized opportunistic scheduling algorithms have been developed for cognitive radio networks. Throughput-optimal cooperative scheduling has been studied in [2], [3], [4], where PUs are aware of SU activities and SUs cooperatively relay PU data. Non-cooperative scheduling has been studied in [5], [6], [7] to achieve optimal SU throughput/utility. However, the above algorithms, though throughput-optimal, are centralized with high time complexity and hence not suitable for practical implementations. In addition to computational complexity, these algorithms do not work when a global centralized component is not available (e.g., a scenario where SUs transmit peer-to-peer without centralized control and only local information is available). Therefore, low-complexity and distributed algorithms are needed to deploy efficient and high performance CRNs. While heuristic distributed solutions have been proposed in the literature (e.g., [8]), the design of distributed algorithms for CRNs with provable properties (i.e., provable throughput/utility performance) remains an open research problem.

Distributed scheduling algorithms for traditional wireless networks have been proposed in the literature over the last decade. Earlier examples [13], [14] achieve at least certain fractions of the optimal throughput in single-channel wireless networks. More recently, throughput optimality has been achieved with distributed queue-length-based scheduling algorithms [9]-[12] for the same setting. Among the few attempts to design distributed scheduling algorithms for multi-channel networks, [15] guarantees at least a certain fraction (dependent on the network interference model) of the optimal throughput. However, these algorithms have been designed for wireless networks *with a non-fading channel capacity*, and thus are not suitable for CRNs where channel states are modulated by PU activities.

In this paper, we propose a *distributed* scheduling algorithm, called *collision-queue-regulated algorithm*, for a CRN in the many-channel regime with a degree-$d$ interference graph model. We consider an OFDM setting (which is the basis for IEEE 802.22 standard for cognitive radio networks), where spectrum is partitioned into tens or hundreds of orthogonal sub-channels. With collision rate constraints imposed by PUs, the algorithm achieves *at least* $\frac{d^d L}{(1+d)^{1+d}\Gamma}$ fraction of the capacity region as the number of channels grows, where $L$ is the number of communication links in the network and $\Gamma$ is the size of the maximum independent set of the network's underlying interference graph. We also show via simulation results that the throughput performance is actually close to the optimal.

Salient contributions of our work are summarized in the following:
1) Under the collision-queue-regulated algorithm, the collision rates observed by the PUs are upper bounded by an arbitrary threshold;
2) We design a novel equivalent queue system such that the queues in the original CRN converge asymptotically (with respect to the number of channels) to this system;
3) The collision-queue-regulated algorithm achieves *at least* $\frac{d^d L}{(1+d)^{1+d}\Gamma}$ fraction of the asymptotic (with respect to the number of channels) capacity region.

The rest of the paper is organized as follows: The network model and the distributed collision-queue-regulated algorithm are described in Section II. This is followed by an asymptotic analysis on the queuing behavior and the throughput performance in Section III. Numerical results are provided in Section IV, and the paper is concluded in Section V.

## II. Network Model and Algorithm

### A. Network Elements

Consider a time-slotted cognitive radio network (CRN) composed of a PU system and an SU system. The SU system consists of a set $\mathcal{L}$ of single-hop directional SU communication links, with $|\mathcal{L}| = L$. We consider a degree-$d$ interference graph model for the SU system, where we define an interference set $\mathcal{N}_i \subset \mathcal{L}$ for each SU link $i \in \mathcal{L}$, such that (i) $|\mathcal{N}_i| = d$, and (ii) when the PU system is idle, the transmission of SU link $i$ over a channel fails if and only if there is a simultaneous transmission of some link $l \in \mathcal{N}_i$ over the same channel. An *independent set* is a set of SU links in $\mathcal{L}$, no two of which interfere with each other when transmitting over a same channel. The *maximum independent set* is the largest independent set with its size denoted by $\Gamma$.[1] Under a node-exclusive setting, typical network topologies of the degree-$d$ interference graph model include complete graph, cycle (or infinite tandem), torus (or infinite grid), etc. Some sample network topologies have been illustrated in Figure 1 under a node-exclusive setting, where an SU $i \in \mathcal{L}$ is represented by a link (a node pair) in the graph. Note that in a node-exclusive setting, adjacent links (links sharing a common node) cannot transmit simultaneously. Let $A_i(t)$ be the amount of data (in unit of bits) arriving at SU link $i$ at the beginning of time slot $t$, which is assumed to be i.i.d. over time with average $\bar{A}_i$, $\forall i \in \mathcal{L}$.

OFDM has proven to be one of the prime candidates for CRNs (e.g., IEEE 802.22 standard). The reason is two-fold: (i) We have irregular openings in the PU spectrum, and OFDM helps with collectively utilizing non-contiguous PU channels in one SU transmission; (ii) Inter-symbol interference (ISI) can be significantly reduced in an OFDM system by transmitting data in parallel over *a large number of low-rate subchannels* [16]. Thus, we consider an OFDM mechanism for SUs' channel access: an SU link can transmit its data opportunistically over multiple PU channels in a time slot.

In this section, we consider a single-PU scenario. Note that the following analysis can be easily extended to the model of multiple PUs at the expense of notational complexity, and we provide a brief discussion on the multi-PU scenario in Section III-C. The CRN is synchronized with a time-slotted PU system comprised of $N$ orthogonal PU subchannels, which we refer to as *channels* for short in the following. Each channel has a capacity (i.e., maximum data rate in bit per time slot) equal to $\frac{K}{N}$, where we can consider $K$ (bit per time slot) as the total capacity of the considered PU system. Note that the growing number of channels leads to diminishing bandwidth per channel where the sum of all bands is constant $K$, which conforms to the setting of OFDM systems with a large number of low-rate channels [16]. We assume that the channels are occupied by a single PU. Specifically, we assume that the PU system evolves according to an ON-OFF Markovian process $C(t)$: At time slot $t$, we let $C(t) = 1$ if the PU system is busy (PU system is in ON state and occupies the entire set of channels) and $C(t) = 0$ if the PU system is idle (PU system is in OFF state and the entire set of channels are available to SUs). For analytical simplicity, we assume the process $C(t)$ starts with a steady state distribution at $t = 0$. We denote by $\mathcal{H}(t) = C(t-1)$ the channel availability information of SUs at time slot $t$. Note that the exact knowledge of $C(t)$ may not be available to SUs due to time-varying PU activities or sensing overheads. Thus, $S(t) \triangleq \mathbb{E}\{1 - C(t)|\mathcal{H}(t)\}$ defines the probability that the PU system is OFF given $\mathcal{H}(t)$, which is known to the SUs at time slot $t$. We note that $S(t)$ is simply the transition probability of $C(t)$ and can be obtained by SUs via the observation of PU data traffic statistics.

Let $\mu_{ij}(t) \in \{0, 1\}$ denote the schedule of SU link $i \in \mathcal{L}$ over channel $j$ at time slot $t$, with $j = 1, ..., N$. Specifically, $\mu_{ij}(t) = 1$ if SU link $i$ is scheduled over channel $j$; $\mu_{ij}(t) = 0$, otherwise. For analytical simplicity, we let $\mu_{ij}(0) = 0$, $\forall i, j$. Note that when $\mu_{ij}(t) = 1$, SU link $i$ is scheduled to transmit up to $\frac{K}{N}$ bits over channel $j$ in one time slot. We say a collision with the PU system occurs if $\mu_{ij}(t)C(t) = 1$, i.e., there is a scheduled SU data transmission when the PU system is busy. Thus, for each SU data queue $q_i(t)$, $i \in \mathcal{L}$, we have the following queue dynamics:

$$\begin{aligned} q_i(t) = {} & [q_i(t-1) \\ & - \frac{K}{N}\sum_{j=1}^{N}\mu_{ij}(t-1)(1 - C(t-1)) + A_i(t-1)]^+, \end{aligned} \tag{1}$$

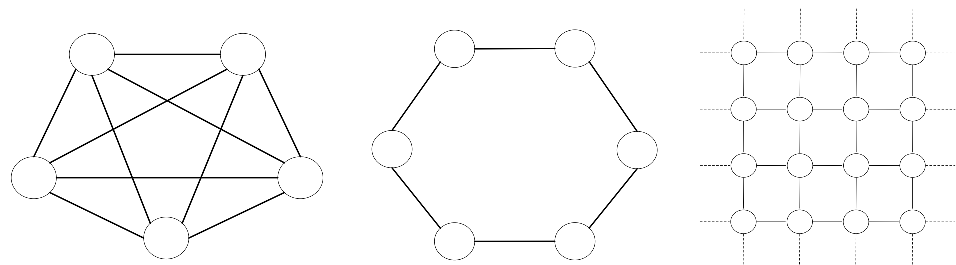
with $q_i(0) = 0$, $\forall i \in \mathcal{L}$. To constrain the potential interference caused by the SUs to the PU system, we require that the collision rate (caused by any SU link $i$) observed by the PU system be upper-bounded by a maximum collision rate $\rho$ (normalized by the number of channels):

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t)C(t) \le \rho, \forall i \in \mathcal{L}. \tag{2}$$

Note that under the degree-$d$ interference model, the accumulated collision rate in the neighborhood of any SU link is upper-bounded by $(d+1)\rho$.

As suggested in [17], for the considered OFDM-based CRN, we assume the existence of an out-of-band common control channel (CCC) which is not interrupted by PU activities. In the collision-queue-regulated algorithm proposed in Section II-B, the exchange of local control information is performed over the CCC at the beginning of each time slot. Since CCC is dedicated only to the transmission and reception of control messages, CCC can utilize the small portions of the guard bands between the licensed channels [17].

[1] In graph theory [20], the underlying degree-$d$ interference graph of the considered interference model is a *d-regular graph*, and the size of its maximum independent set $\Gamma$ is referred to as *the independence number*.



(a) Complete graph with $d = 6$ (b) Cycle with $d = 2$ (c) (Infinite) grid topology with $d = 6$

Fig. 1. Sample network topologies of degree-$d$ interference graph model with a node-exclusive setting

### *B. Collision-Queue-Regulated Algorithm*

In this section, we propose a distributed collision-queue-regulated algorithm. We will show in Section III that the proposed algorithm can achieve at least $\frac{d^d L}{(d+1)^{d+1}\Gamma}$ fraction of the capacity region asymptotically with respect to $N$ under the degree-$d$ interference graph model.

We maintain a virtual collision queue $X_i(t)$ at each SU link $i \in \mathcal{L}$, to assist the development of the proposed algorithm. Specifically, the queue dynamics of $X_i(t)$ is defined as, $\forall i \in \mathcal{L}$,

$$X_i(t) = \left[X_i(t-1) - \rho + \frac{1}{N}\sum_{j=1}^{N}\mu_{ij}(t-1)C(t-1)\right]^+, \quad (3)$$

with $X_i(0) = 0$. We note that the collision rate constraint (2) is satisfied if collision queues $X_i(t)$ are stable.

At the beginning of each time slot $t$, the collision-queue-regulated algorithm consists of two phases: *Exchange Phase* and *Scheduling Phase*, the duration of which we assume is negligible compared to that of a unit time slot. The exchange phase is detailed as follows:

**Exchange Phase:**
The exchange phase takes place over the CCC. Specifically, the transmitter of each SU link $i \in \mathcal{L}$ broadcasts the following three binary vectors to all its neighbors (its intended receiver and all nodes in $\mathcal{N}_i$) over the CCC: its schedules at the previous time slot $(\mu_{ij}(t-1))_{j=1}^N$, a vector of contention variables $(a_{ij}(t))_{j=1}^N$, and a vector of transmission variables $(p_{ij}(t))_{j=1}^N$.

The *contention variables* $(a_{ij}(t))$ are i.i.d. over SUs $i$ and channels $j$ with

$$a_{ij}(t) = \begin{cases} 1, \text{ w.p. } \frac{1}{d+1}, \\ 0, \text{ w.p. } \frac{d}{d+1}. \end{cases}$$

The *transmission variables* $(p_{ij}(t))$ are i.i.d. over channels $j$ and independent over SUs $i$ with

$$p_{ij}(t) = \begin{cases} 1, \text{ w.p. } \frac{e^{y_i(t)} - 1}{e^{y_i(t)}}, \\ 0, \text{ w.p. } \frac{1}{e^{y_i(t)}}, \end{cases}$$

where *the collision-queue-regulated weight* $y_i(t)$ is defined as

$$y_i(t) \triangleq \left[q_i(t-1)S(t) - \gamma X_i(t-1)(1 - S(t))\right]^+, \quad (4)$$

and $\gamma > 0$ is a constant parameter that serves as a weight to the collision queue $X_i(t-1)$.

After the exchange phase, the transmitter and receiver of each SU link $i$ have the following information: $\left((\mu_{lj}(t-1))_{j=1,\dots,N}^{l\in\mathcal{N}_i\cup\{i\}}, (a_{lj}(t))_{j=1,\dots,N}^{l\in\mathcal{N}_i\cup\{i\}}, (p_{ij}(t))_{j=1}^N\right)$, which will be used to determine the transmission schedules for SU link $i$.

To assist the development of scheduling phase, we define the following three conditions, for any given SU link $i$ and channel $j$.

*Condition (i):* The "contention" of SU link $i$ for channel $j$ is successful, i.e., $a_{ij}(t)\Pi_{l\in\mathcal{N}_i}(1 - a_{lj}(t)) = 1$.

*Condition (ii):* $\sum_{l\in\mathcal{N}_i}\mu_{lj}(t-1) = 0$, i.e., none of the neighbors were scheduled at the previous time slot.

*Condition (iii):* The transmission variable $p_{ij}(t) = 1$.

The scheduling phase is introduced as follows:

**Scheduling Phase:**
The transmitter and the receiver of each SU link $i$ determine the schedules $\mu_{ij}(t)$, $j = 1, ..., N$, according to the following:
*Case 1:* $\mu_{ij}(t) = 1$ if Conditions (i)(ii)(iii) hold.
*Case 2:* If Condition (i) does not hold and Condition (iii) holds, then $\mu_{ij}(t) = \mu_{ij}(t-1)$.
*Case 3:* Otherwise, $\mu_{ij}(t) = 0$.

According to the scheduling phase, we conclude that, $\forall i \in \mathcal{L}$, $\forall j \in \{1, 2, ..., N\}$,

$$\begin{aligned}\mu_{ij}(t) &= p_{ij}(t) \\ &\times \{a_{ij}(t)\Pi_{l\in\mathcal{N}_i}(1 - a_{lj}(t))(1 - \sum_{l\in\mathcal{N}_i}\mu_{lj}(t-1)) \\ &+ [1 - a_{ij}(t)\Pi_{l\in\mathcal{N}_i}(1 - a_{lj}(t))]\mu_{ij}(t-1)\},\end{aligned} \quad (5)$$

where the first and second terms in the $\{\cdot\}$ in (5) correspond to Case 1 and Case 2 in the scheduling phase, respectively.

Since both the transmitter and the receiver of SU link $i \in \mathcal{L}$ have a copy of the schedule vector $(\mu_{ij}(t))_{j=1}^N$ when the scheduling phase ends, they will tune to the set of channels $\{j : \mu_{ij}(t) = 1\}$ for SU data transmission in the remaining time slot $t$.

We show in Proposition 1 that the collision-queue-regulated algorithm is feasible in that interfering links are never scheduled over a same channel in any time slot.

*Proposition 1:* The collision-queue-regulated algorithm provides a feasible schedule for each time slot $t$, i.e., $\forall i, j, t$: $\sum_{l\in\mathcal{N}_i} \mu_{lj}(t) = 0$, if $\mu_{ij}(t) = 1$.

Proposition 1 can be proved easily by mathematical induction over time slot $t$ and we omit the proof for brevity.

## III. Performance Analysis in the Degree-$d$ Interference Graph Model in a Many-Channel Regime

In Section III-A, we show that under the collision-queue-regulated algorithm, the original system of the queue lengths $q_i(t)$ and the collision queues $X_i(t)$ converge to an equivalent queue system as the number of channels $N$ grows. Based on the analysis of the equivalent queue system, we show in Section III-B that the algorithm achieves at least $\frac{d^d L}{(d+1)^{d+1}\Gamma}$ fraction of the capacity region asymptotically with respect to $N$. We provide a brief discussion on a multiple PU extension in Section III-C.

### A. Asymptotic Queuing Behavior of the Collision-Queue-Regulated Algorithm

In the following analysis, we assume the arrival processes follow:

$$A_i(t) \xrightarrow{P}_N \lambda,\ \forall i \in \mathcal{L}, \tag{6}$$

where $\xrightarrow{P}_N$ denotes the convergence in probability [18] as $N \to \infty$ and $\lambda$ can be considered as the arrival rate normalized with respect to the number of channels. We present the asymptotic queuing behavior of the collision-queue-regulated algorithm in Theorem 1.

*Theorem 1:* Given $\mathcal{H}'(t) \triangleq (\mathcal{H}(t), \mathcal{H}(t-1), ..., \mathcal{H}(1))$, there exists an equivalent queuing system $(q(t), x(t))$ with an equivalent schedule variable $u(t)$, such that the following four arguments (I($t$), II($t$), III($t$), and IV($t$)) hold under the collision-queue-regulated algorithm for each time slot $t$:

I($t$): The queue lengths $q_i(t)$ and the collision queue lengths $X_i(t)$ converge to $q(t)$ and $x(t)$, respectively:

$$q_i(t) \xrightarrow{P}_N q(t),\ \text{and}\ X_i(t) \xrightarrow{P}_N x(t),\ \forall i \in \mathcal{L}. \tag{7}$$

II($t$): The schedules $\mu_{ij}(t)$ converge to the equivalent schedule variable $u(t)$:

$$\mu_{ij}(t) \xrightarrow{L}_N u(t),\ \forall i, j, \tag{8}$$

where $\xrightarrow{L}_N$ denotes the convergence in distribution [18] as $N \to \infty$.

III($t$): The schedules $\mu_{ij}(t)$ follow a Law of Large Numbers (LLN):

$$\frac{1}{N}\sum_{j=1}^{N} \mu_{ij}(t) \xrightarrow{P}_N \mathbb{E}\{u(t)|\mathcal{H}'(t)\},\ \forall i \in \mathcal{L}. \tag{9}$$

IV($t$): The schedules $\mu_{ij}(t)$ are asymptotically mutually independent. Specifically, for any given SU links $i_1, i_2 \in \mathcal{L}$, and any two distinct channels $j_1 \neq j_2 \in \{1, 2, ..., N\}$, the scheduling decisions are independent, i.e., $\forall k_1, k_2 \in \{0, 1\}$,

$$\begin{aligned}&\lim_{N\to\infty} \Pr\{\mu_{i_1 j_1}(t) = k_1, \mu_{i_2 j_2}(t) = k_2|\mathcal{H}'(t)\}\\ &= \Pr\{u(t) = k_1|\mathcal{H}'(t)\} \Pr\{u(t) = k_2|\mathcal{H}'(t)\}.\end{aligned} \tag{10}$$

The equivalent queuing system ($q(t)$, $x(t)$) and the equivalent schedule variable $u(t)$ evolve as follows,

$$\begin{aligned}q(t) = [&q(t-1)\\ &- K(1 - C(t-1))\mathbb{E}\{u(t-1)|\mathcal{H}'(t-1)\} + \lambda]^+,\end{aligned} \tag{11}$$

$$\begin{aligned}x(t) = [&x(t-1)\\ &- \rho + C(t-1)\mathbb{E}\{u(t-1)|\mathcal{H}'(t-1)\}]^+\end{aligned} \tag{12}$$

$$u(t) = U_1(t)u(t-1) + U_2(t)(1 - u(t-1)), \tag{13}$$

where $U_1(t)$ and $U_2(t)$ are independent over time and defined as follows:

$$U_1(t) = \begin{cases} 1, & \text{w.p. } (1 - d\beta)\dfrac{e^{y(t)} - 1}{e^{y(t)}}, \\ 0, & \text{otherwise}, \end{cases}$$

$$U_2(t) = \begin{cases} 1, & \text{w.p. } \beta\dfrac{e^{y(t)} - 1}{e^{y(t)}}, \\ 0, & \text{otherwise}, \end{cases}$$

with

$$\beta \triangleq \frac{d^d}{(d+1)^{d+1}},$$

$$y(t) \triangleq [q(t-1)S(t) - \gamma x(t-1)(1 - S(t))]^+.$$

The initial conditions of the equivalent queue system are set as:

$$q(0) = 0,\ x(0) = 0,\ \text{and}\ u(0) = 0. \tag{14}$$

*Proof:* The proof for Theorem 1 is provided in Appendix A. ■

*Remark 1:* According to (7) in Theorem 1, given $\mathcal{H}'(t)$, the data queues $q_i(t)$ and the collision queues $X_i(t)$ converge (in probability) asymptotically to two *deterministic* equivalent queues $q(t)$ and $x(t)$, respectively. By the dynamics of $u(t)$ in (13), we find the dynamics of $\mathbb{E}\{u(t)|\mathcal{H}'(t)\}$ as follows:

$$\begin{aligned}\mathbb{E}\{u(t)|\mathcal{H}'(t)\} = &\beta\frac{e^{y(t)} - 1}{e^{y(t)}}\\ &+ (1 - \beta - d\beta)\frac{e^{y(t)} - 1}{e^{y(t)}}\mathbb{E}\{u(t-1)|\mathcal{H}'(t-1)\},\end{aligned} \tag{15}$$

where we note that $u(t-1)$ is independent of $\mathcal{H}(t)$ given $\mathcal{H}'(t-1)$.

In Section III-B, we will study the stability of the equivalent queuing system $(q(t), x(t))$, which becomes the asymptotic network stability (i.e., the stability for the data queues $q_i(t)$ and the collision queues $X_i(t)$) under the collision-queue-regulated algorithm.

### B. Performance Analysis

We have shown through Theorem 1 that the equivalent system $(q(t), x(t))$ can represent the asymptotic queuing behavior of the data queues $q_i(t)$ and the collision queues $X_i(t)$. In this section, we will show that under the collision-queue-regulated algorithm, $(q(t), x(t))$ are stable for *at least* $\frac{d^d L}{(d+1)^{d+1}\Gamma}$ fraction of the asymptotic capacity region.

Specifically, we define the asymptotic capacity region $\Lambda$ (normalized with respect to the number of channels $N$) as

$$\Lambda = \{\lambda \geq 0 : \exists (\bar{A}_i)_{i\in\mathcal{L}} \text{ s.t. } \lim_{N\to\infty} \bar{A}_i = \lambda, \forall i \in \mathcal{L},$$
$$\text{and } (\bar{A}_i)_{i\in\mathcal{L}} \text{ is stabilizable by some scheduling algorithm}\},$$

where we recall $(\bar{A}_i)_{i\in\mathcal{L}}$ denotes the arrival rate vector. For any given $0 < \alpha < 1$, we let $\alpha\Lambda$ denote an $\alpha$ fraction of the capacity region such that

$$\alpha\Lambda \triangleq \{\lambda \geq 0 : \exists \lambda' \in \Lambda \text{ s.t. } \frac{\lambda}{\alpha} < \lambda'\}.$$

Before we present the asymptotic stability in Theorem 2, we introduce the following two lemmas to assist the proof of Theorem 2.

*Lemma 1:* For any given $0 < \delta < \frac{\beta L}{\Gamma}$, there exists $B_2(\delta) > 0$ such that for any time slot $t$, whenever $y(t) \geq B_2$, we have: $\Pr\{u(t) = 1\} \geq \beta - \frac{\Gamma\delta}{L}$.

*Proof:* Let $B_2 \triangleq \log(\frac{L\beta}{\Gamma\delta})$. By taking the expectation of both sides of (15) over $\mathcal{H}'(t)$ conditioned on $y(t) \geq B_2$, we have

$$\begin{aligned}
&\mathbb{E}\{u(t)|y(t) \geq B_2\}\\
=&\beta\mathbb{E}\{\frac{e^{y(t)}-1}{e^{y(t)}}|y(t) \geq B_2\}\\
&+ (1-\beta(d+1))\mathbb{E}\{\frac{e^{y(t)}-1}{e^{y(t)}}u(t-1)|y(t) \geq B_2\}\\
\geq&\beta\frac{e^{B_2}-1}{e^{B_2}} = \beta - \frac{\Gamma\delta}{L}.
\end{aligned}$$

■

We show that for any $\lambda' \in \Lambda$, there exists an (auxiliary) random variable $\mu^{STAT}(t)$ for each time slot $t$ satisfying the properties described in Lemma 2.

*Lemma 2:* For any $\lambda' \in \Lambda$, there exists a random variable $\mu^{STAT}(t) \in \{0, 1\}$ that is dependent only on $S(t)$ for each time slot $t$, such that the following holds:

$$K\mathbb{E}\{\mu^{STAT}(t)S(t)\} = \lambda', \tag{16}$$

$$\mathbb{E}\{\mu^{STAT}(t)(1 - S(t))\} \leq \rho, \tag{17}$$

$$\mathbb{E}\{\mu^{STAT}(t)|S(t)\} \leq \frac{\Gamma}{L}, \forall S(t). \tag{18}$$

*Proof:* Proof of Lemma 2 is provided in [21]. ■

Utilizing Lemma 1 and Lemma 2, we show in Theorem 2 that the equivalent system is stable for at least $\frac{d^d L}{(d+1)^{d+1}\Gamma}$ fraction of the asymptotic capacity region $\Lambda$ under the collision-queue-regulated algorithm.

*Theorem 2:* $\forall\lambda \in \alpha\Lambda$, with $\alpha = \frac{d^d L}{(d+1)^{d+1}\Gamma}$, $q(t)$ and $x(t)$ are stable under the collision-queue-regulated algorithm, i.e.,

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{q(t) + x(t)\} \leq \frac{B_3}{\epsilon_2}, \tag{19}$$

where positive constants $B_3$ and $\epsilon_2$ will be defined in the proof.

*Proof:* The proof of Theorem 2 is provided in Appendix B, where we have employed Lemma 1 and Lemma 2. ■

Since the queue lengths $q_i(t)$ and the collision queues $X_i(t)$ converge to $(q(t), x(t))$ asymptotically with respect to $N$, by Theorem 2, the collision-queue-regulated algorithm achieves at least $\alpha = \frac{d^d L}{(d+1)^{d+1}\Gamma}$ fraction of the asymptotic capacity region in the many-channel regime.[2]

This fraction $\alpha$, referred to as the *efficiency factor* of the asymptotic capacity region, is illustrated in Table I given some typical network topologies under a node-exclusive setting. For comparison, we also illustrate in Table I the efficiency factor of the distributed PLDS algorithm [15] proposed for a general multi-radio multi-nonfading-channel wireless network. Note that although both algorithms are proposed for a multi-channel scenario, the setting for the collision-queue-regulated algorithm is more stringent than that for PLDS, in that non-fading channels are assumed in [15] while we consider channels modulated by PU activities in this work. Yet, the provable efficiency factor of the collision-queue-regulated algorithm is larger than that of PLDS given the network topologies in Table I.

### C. Further Discussion in a multiple PU setting

In the CRN model introduced in Section II-A, we have assumed that there is only one PU, the spectrum of which is partitioned into $N$ subchannels and modulated by the activities of this PU. This model can be readily extended to the scenario where PU spectrum is modulated by a finite number of $M$ PUs. Specifically, when there are $M$ PUs in the PU system, the licensed spectrum of each PU $k$ is partitioned into a set $\mathcal{I}_k$ of subchannels, with $\mathcal{I}_k$ mutually disjoint and $\lim_{N\to\infty} \frac{|\mathcal{I}_k|}{N} = n_k, \forall k$, where constants $n_k$ satisfy $\sum_{k=1}^{M} n_k = 1$. We assume that a collision rate constraint $\rho_k$ is imposed by each PU $k = 1, ..., M$. Similar to the virtual queue analysis in Section II-B, we can construct $M$ virtual collision queues $(X_{ik}(t))_{k=1}^{M}$ for each SU link $i \in \mathcal{L}$ as follows:

$$\begin{aligned}
&X_{ik}(t)\\
&= \left[X_{ik}(t-1) - \rho_k + \frac{1}{|\mathcal{I}_k|}\sum_{j\in\mathcal{I}_k} \mu_{ij}(t-1)C_k(t-1)\right]^+,
\end{aligned}$$

[2]In graph theory [20], the ratio $\frac{\Gamma}{L}$ is referred to as *the independence ratio* of the underlying interference graph of the network.

TABLE I

EFFICIENCY FACTOR UNDER NODE-EXCLUSIVE SETTING, WHERE $e$ DENOTES EULER'S NUMBER

| Efficiency factor | collision-queue-regulated | PLDS |
|---|---|---|
| Fully-connected network (e.g., WLANs, cellular networks) | $\left(\frac{d}{d+1}\right)^d \geq \frac{1}{e}$ | $\frac{1}{3e}$ |
| Cycle (e.g., Figure 1(b)) | $\frac{4L}{27\lfloor \frac{L}{2} \rfloor} \geq \frac{8}{27}$ | $\frac{1}{4e}$ |
| (Infinite) grid topology (e.g., Figure 1(c)) | 0.2267 | $\frac{1}{4e}$ |

where $C_k(t) = 1$ if PU $k$ is busy at time slot $t$, and $C_k(t) = 0$ otherwise. Therefore, the stability of the collision queues $(X_{ik}(t))$ implies the collision rate constraints being satisfied. The collision-queue-regulated algorithm can be modified such that the weight $y_i(t)$ in (4) is replaced by $y_{ij}(t)$, for each channel $j \in \mathcal{I}_k$:

$$y_{ij}(t) \triangleq [q_i(t-1)S_k(t) - \gamma X_{ik}(t-1)(1 - S_k(t))]^+ ,$$

where $S_k(t) \triangleq \mathbb{E}\{1 - C_k(t)|C_k(t-1)\}$.

With the above modifications in model and algorithm, similar to the analysis of the equivalent queue system in Section III-A, we can construct an equivalent $(M+1)$-queue system $(q(t), (x_k(t))_{k=1}^M)$, such that $q_i(t)$ converges to $q(t)$ and $X_{ik}(t)$ converges to $x_k(t)$ in probability, $\forall i, k$. The throughput analysis follows that of Section III-B.

## IV. NUMERICAL RESULTS

In this section, via simulation, we compare the throughput performance of the proposed algorithm with a back-pressure-based *centralized throughput-optimal* algorithm, denoted as the BP algorithm. The BP algorithm is based on the throughput-optimal back-pressure algorithm [19] where we substitute the (generic) weight in [19] with the collision-queue-regulated weight $y_i(t)$ defined in (4) for each $i \in \mathcal{L}$. It can be shown, similar to the analysis [3][4] that the BP algorithm is optimal given the collision rate constraints. We consider the SU network topology of 10 SUs (represented by 10 SU links) in Figure 1(a) with a node-exclusive setting. Specifically, in Figure 1(a), each SU link interferes with its 6 adjacent links, i.e., $d = 6$. We set the parameter in (4) as $\gamma = 1$ and use $N = 50$. The channel state evolves according to the transition diagram in Figure 2, where the "busy" and the "idle" states are represented as $C(t) = 1$ and $C(t) = 0$, respectively. Note that in Figure 2, $p_{01}$ and $p_{10}$ represent the transition probability from the idle state to the busy state and that from the busy state to the idle state, respectively. In the numerical evaluation, we let $p_{01} = 0.3$, $p_{10} = 0.7$.

We illustrate the stability of queues through Figure 3 under the collision rate constraint $\rho = 0.05$. We let the arrival processes be: $A_i(t) = \lambda + \frac{0.2\lambda}{\sqrt{N}} rand(1)$, $\forall i \in \mathcal{L}$, $\forall t$, where $rand(1)$ outputs a random value uniformly distributed over the interval $(0, 1)$ independently across time slots and SUs. In our numerical studies, we have observed that both algorithms stabilize data and collisions queues for $\lambda = 0.110$. Again, both algorithms fail to stabilize the system for $\lambda = 0.118$, where the data and collision queues both keep growing, indicating both network instability and collision rate violation. Since

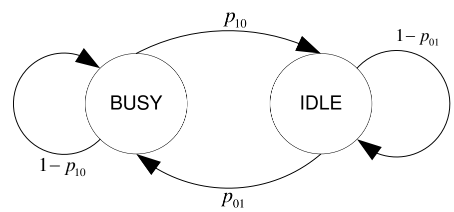



Fig. 2. Transition diagram of PU activity

the BP algorithm is throughput-optimal, we can expect that the maximum stabilizable $\lambda$ is in between $0.110$ and $0.118$. Hence, the collision-queue-regulated algorithm achieves at least $0.110/0.118 = 93\%$ of the throughput optimality under this simulation setting. Note that $0.93$ is significantly higher than the efficiency factor $\alpha = \frac{d^d L}{(d+1)^{d+1}\Gamma} = 0.2833$ in Theorem 2.

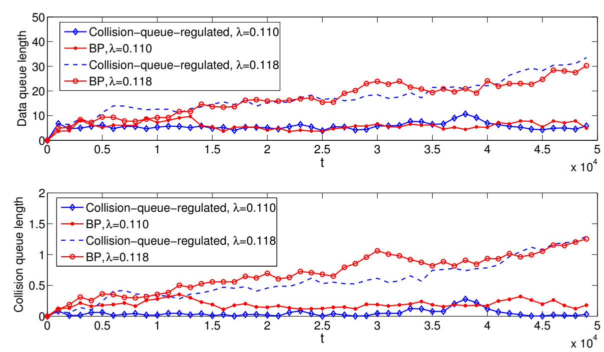



Fig. 3. Queue dynamics, $\rho = 0.05$.

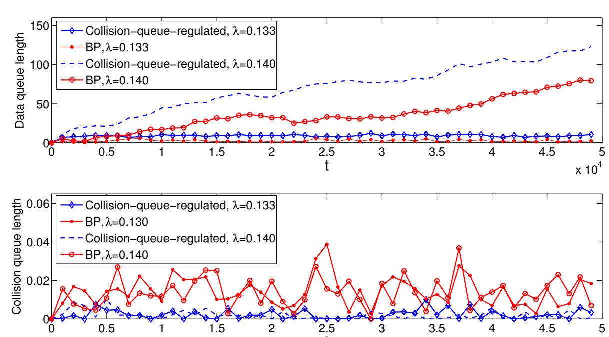



Fig. 4. Queue dynamics, $\rho = 0.1$.

We relax the collision rate constraint as $\rho = 0.1$ in Figure IV. Under both algorithms, at $\lambda = 0.133$, the data queues and collision queues are stable; at $\lambda = 0.140$, while the collision queues are stable, the data queue lengths are increasing over the time slots $t$, indicating network instability. That is, both algorithms can stabilize $\lambda = 0.133$ but cannot stabilize $\lambda = 0.140$. We can expect that the collision-queue-regulated algorithm achieves at least $0.133/0.140 = 95\%$ of the throughput optimality under this simulation setting.

## V. Conclusions and Future Works

In this paper, we proposed a distributed collision-queue-regulated scheduling algorithm for cognitive radio networks. We proved theoretically that the proposed algorithm can achieve at least $\frac{d^d L}{(d+1)^{d+1}\Gamma}$ of the capacity region asymptotically in the many-channel regime via a novel equivalent queue system analysis. We also illustrated through numerical evaluation that the throughput performance of the proposed algorithm is close to optimal.

We have assumed the degree-$d$ interference graph model in this work. The proposed algorithm can be readily extended to general interference-graph-based network topologies (e.g., see [9]-[12]). Thus, our future work involves performance analysis for a more general scenario: a general interference graph model with heterogenous arrival processes, where we can expect that the wireless system will converge (in the number of channels) to an equivalent queue system composed of $2L$ queues ($L$ equivalent queues for data queues and $L$ equivalent queues for collision queues).

## References


[1] I. F. Akyildiz, W. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey, Computer Networks", in *Computer Networks Journal (Elsevier)*, vol. 50, no. 13, pp. 2127-2159, Sep. 2006.

[2] K. Khalil, M. Karaca, O. Ercetin, and E. Ekici, "Optimal scheduling in cooperate-to-join cognitive radio networks", *in Proc. IEEE INFOCOM'11*, pp. 3002-3010, April 2011.

[3] R. Urgaonkar and M. Neely, "Opportunistic cooperation in cognitive femtocell networks", *to appear in IEEE Journal on Selected Areas in Communications (JSAC)*, 2012.

[4] D. Xue and E. Ekici, "cross-layer scheduling for cooperative multi-hop cognitive radio networks", *to appear in IEEE JSAC*, 2013.

[5] R. Urgaonkar and M. Neely, "Opporunistic scheduling with reliability guarantees in cognitive radio networks", in *IEEE Transactions on Mobile Computing*, vol. 8, no. 6, pp. 766-777, June 2009.

[6] M. Lotfinezhad, B. Liang and E. Sousa, "Optimal control of constrained cognitive radio networks with dynamic population size", in *Proc IEEE INFOCOM 2010*, pp. 1-9, March 2010.

[7] D. Xue and E. Ekici, "Guaranteed opportunistic scheduling in multi-hop cognitive radio networks", *in IEEE INFOCOM 2011*, pp. 2984-2992, April 2011.

[8] Y. Shi, Y. Hou, H. Zhou, and S. Midkiff, "Distributed cross-layer optimization for congitive radio networks", in *IEEE Transactions on Vehicular Technology*, vol. 59, no. 8, pp. 4058- 4069, Oct. 2010.

[9] L. Jiang and J. Walrand,"A distributed algorithm for throughput and utility maximization in wireless networks", in *IEEE/ACM Transactions on Networking*, vol. 18, no.3, pp. 960-972, June 2010.

[10] J. Ni, B. Tan, and R. Srikant, "Q-CSMA: queue-length based CSMA/CA algorithms for achieving maximum throughput and low delay in wireless networks", in *Proc. IEEE INFOCOM 2010 Mini-Conference*, pp. 1-5, April 2010.

[11] S. Rajagopalan, D. Shah, and J. Shin, "Network adiabatic theorem: an efficient randomized protocol for contention resolution", in *SIGMETRICS'09: Proceeding of the eleventh international joint conference on measurement and modeling of computer systems*, pp. 133-144, June 2009.

[12] J. Ghaderi and R. Srikant, "On the design of efficient CSMA algorithms for wireless networks", in *Proc. IEEE conference on Decision and Control (CDC 2010)*, pp. 954-959, December 2010.

[13] X. Wu, R. Srikant and J. Perkins, "Scheduling efficiency of distributed greedy scheduling algorithms in wireless networks", in *IEEE Trans. Mobile Comput.*, vol. 6, no. 6, pp. 595-605, June 2007.

[14] P. Chaporkar, K. Kar and S. Sarkar, "Throughput guarantees through maximal scheduling in wireless networks", in *Proc. 43rd Annual Allerton Conference on Communicastions, Control and Computing*, pp. 1-11, 2005.

[15] B. Han, V. Kumar, Madhav. Marathe, S. Parthasarathy, and A. Srinivasan, "Distributed strategies for channel allocation and scheduling in software-defined radio networks", *Proc. INFOCOM 2009*, pp. 1521-1529, April 2009.

[16] S. Ye, R. Blum, and L. Cimini, "Adaptive OFDM systems with imperfect channel state information", in *IEEE Transactions on Wireless Communications*, vol. 5, no. 11, pp. 3255-3265, Nov. 2006.

[17] K. R. Chowdhury and I. F. Akyildiz, "OFDM-based common control channel design for cognitive radio ad hoc networks", in *IEEE Transactions on Mobile Computing*, vol. 10, no. 2, pp. 228-238, Feb. 2011.

[18] A. Karr, *Probability*, Springer-Verlag, New York, 1993.

[19] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks", in *IEEE Trans. Autom. Control*, vol. 37, no. 12, pp. 1936-1948, Dec. 1992.

[20] B. Bollobs, "Extremal graph theory", in *London Math. Soc. Monographs*, no. 11, Academic Press, London and New York, 1978.

[21] D. Xue and E. Ekici, "Efficient distributed scheduling in cognitive radio networks in the many-channel regime", Technical Report, Ohio State University, available: http://www.ece.osu.edu/∼xued/multichannelCR.pdf


## Appendix A
## Proof of Theorem 1

We prove Theorem 1 by mathematical induction over time slot $t$. Given the initial conditions (14), the base case holds for time slot $t = 0$. Suppose the induction hypothesis (I$(t-1)$, II$(t-1)$, III$(t-1)$, and IV$(t-1)$) holds, we prove I$(t)$ holds in the following, and prove II$(t)$, III$(t)$, and IV$(t)$ hold in [21].

Given any SU link $i \in \mathcal{L}$, according to (6), I$(t-1)$ and III$(t-1)$, we have

$$q_i(t-1) - \frac{K}{N}(1 - C(t-1)) \sum_{j=1}^{N} \mu_{ij}(t-1) + A_i(t-1)$$
$$\xrightarrow{P}_N q(t-1) - K(1 - C(t-1))\mathbb{E}\{u(t-1)|\mathcal{H}'(t-1)\} + \lambda.$$

By queue dynamics (1)(11) and the continuity of $[\cdot]^+$, we conclude that $q_i(t) \xrightarrow{P}_N q(t)$, $\forall i \in \mathcal{L}$.

Similarly, we have

$$X_i(t-1) - \rho + \frac{1}{N}\sum_{j=1}^{N} \mu_{ij}(t-1)C(t-1)$$
$$\xrightarrow{P}_N x(t-1) - \rho + C(t-1)\mathbb{E}\{u(t-1)|\mathcal{H}'(t-1)\}.$$

By queue dynamics (3)(12) and the continuity of $[\cdot]^+$, we conclude that $X_i(t) \xrightarrow{P}_N x(t)$, which completes the proof of I$(t)$.

We have also provided proof for II$(t)$, III$(t)$, and IV$(t)$ in [21], i.e., the induction step holds, which completes the proof of Theorem 1.

## APPENDIX B
## PROOF OF THEOREM 2

For notational simplicity, we define $\Delta(t) \triangleq \mathbb{E}\left\{\frac{1}{2K}[q(t)^2 - q(t-1)^2] + \frac{\gamma}{2}[x(t)^2 - x(t-1)^2]\right\}$. By squaring both sides of the queue dynamics (11)(12), we have

$$\begin{aligned}
&\Delta(t) \\
\le& B_1 + \frac{\lambda}{K}\mathbb{E}\{q(t-1)\} - \gamma\rho\mathbb{E}\{x(t-1)\} \\
&- \mathbb{E}\{u(t-1)[(1-C(t-1))q(t-1) \\
&\quad - \gamma C(t-1)x(t-1)]\} \\
\le& B_1 + \max\{K,\gamma\} \\
&+ \frac{\lambda}{K}\mathbb{E}\{q(t-1)\} - \gamma\rho\mathbb{E}\{x(t-1)\} \\
&- \mathbb{E}\{u(t-1)[(1-C(t-1))q(t-2) \\
&\quad - \gamma C(t-1)x(t-2)]\} \\
\overset{(c)}{\le}& B_1 + \max\{K,\gamma\} + \frac{\lambda}{K}\mathbb{E}\{q(t-1)\} \\
&- \gamma\rho\mathbb{E}\{x(t-1)\} - \mathbb{E}\{u(t-1)y(t-1)\},
\end{aligned} \tag{20}$$

where $B_1 \triangleq \frac{1}{2K}\lambda^2 + \frac{\gamma\rho^2}{2} + \max\{\frac{K}{2}, \frac{1}{2}\gamma\}$. Note that $(c)$ follows from the following equality

$$\begin{aligned}
&\mathbb{E}\{u(t-1)[(1-C(t-1))q(t-2) \\
&\quad - \gamma C(t-1)x(t-2)]|\mathcal{H}'(t-1)\} \\
=& \mathbb{E}\{u(t-1)|\mathcal{H}'(t-1)\}y(t-1),
\end{aligned}$$

where we utilized the fact that $q(t-2)$, $x(t-2)$, and $u(t-1)$ are independent of $C(t-1)$ given $\mathcal{H}'(t-1)$ by their dynamics (11)(12)(13).

Since $\lambda \in \frac{d^d L}{(d+1)^{d+1}\Gamma}\Lambda$, there exists $\epsilon_1 > 0$ such that $\lambda' \triangleq \frac{\lambda\Gamma}{\beta L} + \epsilon_1 \in \Lambda$ by definition. We define $\delta$ in Lemma 1 as follows:

$$0 < \delta \triangleq \frac{\beta L \epsilon_1}{2(\epsilon_1 + \frac{\lambda\Gamma}{\beta L})\Gamma} < 1.$$

By Lemma 1, we have

$$\begin{aligned}
&\mathbb{E}\{u(t-1)y(t-1)|y(t-1) \ge B_2\} \\
\ge& (\beta - \frac{\delta\Gamma}{L})\mathbb{E}\{y(t-1)|y(t-1) \ge B_2\}.
\end{aligned}$$

Employing the above inequality to (20), we obtain

$$\begin{aligned}
&\Delta(t) \\
\le& B_1 + \max\{K,\gamma\} + \frac{\lambda}{K}\mathbb{E}\{q(t-1)\} - \gamma\rho\mathbb{E}\{x(t-1)\} \\
&- \Pr\{y(t-1) \ge B_2\} \\
&\quad \times \mathbb{E}\{u(t-1)y(t-1)|y(t-1) \ge B_2\} \\
&- \Pr\{y(t-1) < B_2\} \\
&\quad \times \mathbb{E}\{u(t-1)y(t-1)|y(t-1) < B_2\} \\
\le& B_1 + \max\{K,\gamma\} + \frac{\lambda}{K}\mathbb{E}\{q(t-1)\} - \gamma\rho\mathbb{E}\{x(t-1)\} \\
&+ B_2(\beta - \frac{\delta\Gamma}{L}) - (\beta - \frac{\delta\Gamma}{L})\mathbb{E}\{y(t-1)\}.
\end{aligned} \tag{21}$$

Since $\lambda' = \frac{\lambda\Gamma}{\beta L} + \epsilon_1 \in \Lambda$, by Lemma 2, there exists $\mu^{STAT}(t)$ such that (16)(17)(18) hold for each time slot $t$ for this $\lambda'$. According to (18), we have

$$-\frac{\Gamma}{L}\mathbb{E}\{y(t-1)\} \le -\mathbb{E}\{\mu^{STAT}(t-1)y(t-1)\}. \tag{22}$$

By applying (22) to (21) and employing (16)(17), we obtain

$$\begin{aligned}
&\Delta(t) \\
\le& B_3 + \frac{\lambda}{K}\mathbb{E}\{q(t-1)\} - \gamma\rho\mathbb{E}\{x(t-1)\} \\
&- (\frac{\beta L}{\Gamma} - \delta)\mathbb{E}\{\mu^{STAT}(t-1) \\
&\quad \times [S(t-1)q(t-1) - \gamma(1-S(t-1))x(t-1)]\} \\
=& B_3 - \mathbb{E}\{q(t-1) \\
&\quad \times \left[(\frac{\beta L}{\Gamma} - \delta)\mu^{STAT}(t-1)S(t-1) - \frac{\lambda}{K}\right]\} \\
-& \mathbb{E}\{x(t-1) \\
&\quad \times \gamma[\rho - (\frac{\beta L}{\Gamma} - \delta)(1-S(t-1))\mu^{STAT}(t-1)]\} \\
\le& B_3 - \epsilon_2\mathbb{E}\{q(t-1) + x(t-1)\},
\end{aligned} \tag{23}$$

where $B_3 \triangleq B_1 + B_2(\beta - \frac{\delta\Gamma}{L}) + \max\{K,\gamma\} + \max\{\lambda,\gamma\rho\}(\frac{\beta L}{\Gamma} - \delta)$, and

$$\epsilon_2 \triangleq \min\left\{\frac{\epsilon_1\beta L}{2K\Gamma}, \gamma\rho(1 - \frac{\beta L}{\Gamma} + \delta)\right\} > 0.$$

From (23), by taking the time-average over $t = 0, 1, ..., T-1$ and taking limsup of $T$, we can prove (19), completing the proof of Theorem 2.