Historical Changes of Modes and their Substructure Modeled as Pitch Distributions in Plainchant from the 1100s to the 1500s

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Abstract. Large-scale quantitative investigations into the cultural evolution of music have mostly focused on only a limited range of time periods and genres. Here, we analyze more than 40 000 pieces of plainchant to better understand the evolution of modes and pitch distributions in a period of five centuries that saw the development of the Western modal practice. Specifically, we employ a hierarchical Markov mixture model to analyze the eight medieval modes and their substructure represented as pitch distributions and observe their historical changes. We found that the individual modes exhibit internal clusters, that the relative frequencies of the eight modes remained remarkably stable over time, and that there were comparatively large changes in the pitch distributions of individual modes. We discuss our results on the background of musicological insights and point to the need for further interdisciplinary work.

Keywords: computational musicology; cultural evolution; plainchant; statistical modeling; mode classification.

1 Introduction

Quantitative analysis of music evolution has been gaining increasing attention in recent years. Previous studies have observed trends and regularities in musical styles in several cultural domains such as Western classical music [1–6] and popular music [7–10]. There are also several studies on evolution of folk and world music [11,12]. Such studies inherently rely on the availability of large-scale music data that also include information about the time of composition to be used for evolutionary analysis. Most studies using quantitative methods in the Western classical context have focused on music from

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Number	Name	Type	Final	Reciting tone	Range
1	Dorian	authentic	D	A4	D4–D5
2	Hypodorian	plagal	D	F4	A3–A4
3	Phrygian	authentic	E	C5	E4–E5
4	Hypophrygian	plagal	E	A4	B3–B4
5	Lydian	authentic	F	C5	F4–F5
6	Hypolydian	plagal	F	A4	C4–C5
7	Mixolydian	authentic	G	D5	G4–G5
8	Hypomixolydian	plagal	G	C5	D4–D5

Table 1: The eight medieval modes. The reciting tone and range are represented in the standard pitch notation for clarity, but the pitches have only relative meanings here.

the Renaissance, Baroque, Classical, or Romantic periods, and thus covered both modal and tonal practices [13]. However, this concentration on the period from approximately the 16th to the 19th centuries ignores several preceding centuries in which the Western modal practice developed. Here, we draw our attention to medieval monophony and its manifestation in chants [14], in order to shed light on our understanding of the development of pitch organization in Western music from its earliest beginnings.

Arguably, the most fundamental concept of medieval music theory for the categorization of chants is that of a *mode*. In the liturgical practice since the eighth century, we can find so-called *tonaries*: books that categorize chants into eight modes [15]. These are then identified in later manuscripts with one of the *finals* D, E, F, and G, each coming in two variants: authentic (the final is usually the lowest note) and plagal (the final is usually the central note in terms of pitch height). The eight medieval modes are conventionally labeled with a number or a Greek name (Table 1). It is, however, not entirely clear whether the concept of mode was merely used as a classification system to organize existing musical material into distinct categories based on some set of shared features (e.g. pitch-related or other), or whether mode was, in contrast, a concept existing prior to composition, that allowed music to be sung "in a certain mode." It seems likely that these two conceptualizations were never strictly separated but that they are rather intricately entangled. Both the categorization and composition aspects probably played a role to varying degrees, as mode is a complex concept influenced by ancient music theory as well as medieval practice [16].

Commonly, the mode of a chant is determined on the basis of pitch-related features, e.g. which pitches are used in which frequency, which pitches are initial or final to a chant, etc. Consequently, modal characteristics should be reflected in pitch-distribution statistics, although it is not clear whether a mode can be modeled as a simple pitch distribution or a distribution with substructure reflecting other features such as the function of chant in liturgical use. These arguments indicate the importance of analyzing the evolution of modes and related pitch distributions and the need for addressing two major research questions: (i) How did the relative frequencies of modes change over time? (ii) How did the pitch distributions of individual modes change over time?

Our contribution shares a research interest with two prior studies on medieval mode. In their pioneering study on the pitch-class distributions of the eight modes, Huron and

Veltman [17] found a 'supra-modal group' consisting of modes 3, 5, and 8 (sharing reciting tone c), and another group consisting of modes 1, 4, and 6 (sharing reciting tone A), and suggested that this polarization facilitated the major-minor bifurcation in the 17th century. There has also been some musicological criticism on this work. Specifically, Wiering [18] noted that the abstraction of a pitch class ignores the different functions of octave-equivalent tones in medieval music. Moreover, the assumption of chromatic transpositions contradicts the medieval practice (and virtually all musical practice prior to the Romantic era). It was also noted that the melodic aspects were also ignored in the pitch-class profile approach, which does not account for pitch transitions and cannot represent subtle distinctions of modes based on melodic motions.

In a more recent study, Cornelissen et al. [19] examined mode classification in medieval plainchant melodies using a distributional model that improved some of the shortcomings of the earlier study by including both pitch information (as opposed to pitch classes) and n-gram models. Using the tf-idf vectors of chants, their model achieved a classification F-score of 93–95% and maintained F-scores of 81–83% even without absolute pitch information. The result suggested that plainchant contains 'natural units' that lie somewhere between the levels of individual notes and complete phrases.

To address our two research questions about mode in medieval plainchant, we analyze a large corpus of monophonic melodies that were almost exclusively written for liturgical use (see Sec. 2.1). We analyze the historical changes in pitch distributions in the chants whose source manuscripts date back to the range between the 12th and the 16th centuries. To examine the substructure of modes, we go beyond the approaches in previous research and apply an elaborated technique of machine learning to infer internal clusters of pitch distribution from data. We formulate a hierarchical Markov mixture model for this purpose and study the inferred parameters in terms of mode classification ability and the relationship with chant genres. We then analyze the historical changes in the relative frequencies of these clusters to draw conclusions for our research questions.

2 Method

2.1 Data

Our data source is *Cantus* [20], a database for Latin ecclesiastical chant that was created with the goal of digitizing and distributing indices of medieval chant manuscripts and early printed books [21] (see Fig. 1a for an example). Developed by Steiner in the 1980s, the Cantus database continues to provide an essential resource for scholars and researchers studying the history and evolution of Latin ecclesiastical chant. The central focus of the Cantus database is the so-called *liturgical office*, which is, besides the eucharist (mass), an essential element of the liturgy in almost all Christian denominations. It is a shared act of prayer, typically sung, that involves reciting the psalms and other supplementary texts throughout the various times of day and days of the year (referred to as the canonical hours) [22].

For the present study, we draw on a publicly available data resource that contains a total of 63 628 chants from the Cantus database, including a rich set of metadata [19]. Three types of information are used in particular: melody, mode, and source date. We

(a)

(b)

(c) `D4 F4 G4 G4 G4 A4 G4 F4 A4 A4 C5 C5 G4 G4 F4 G4`

Fig. 1: Three different representations of a responsory chant incipit in mode 8 from the Cantus database (`https://cantus.uwaterloo.ca/chant/284929`). (a) Detail of the manuscript scan with only one staff line indicating the position of pitch F. (b) Volpiano encoding of the responsory assigning pitches to letters `a` to `p`, and modern staff notation. (c) Pitches in standard notation (note name + octave).

thus excluded in the following analyses chants with less than ten notes, without an annotated mode, or without source date information. Furthermore, we use the genre metadata with labels such as antiphon, responsory, and responsory verse[1]. Melodies are represented by a string in *Volpiano* encoding [23] (Fig. 1b). The alphabets represent pitches in the ascending order, and dashes indicate the hierarchical segmentation into words, syllables, and neumes. Conversion from this format to the standard pitch notation is straightforward (Fig. 1c).

A chant's metadata often contains a mode attribute extracted from the containing manuscript or assigned by experts. A majority of annotated modes are a single number from 1 to 8 corresponding to the eight modes explained in Sec. 1; only chants classified in these modes are used for analysis. Some of the other chants are transposed chants, indicated with a T, or verses that are sung with a special melody, indicated with an S. There are also chants whose mode is unknown or uncertain, indicated with a question mark. The date of a source manuscript, if it is given, is represented as a year range (e.g. 1201–1300). We use the middle values of these ranges as the time stamps of contained chants. After these data selection steps, we were left with 41 158 chants in total used for the following analysis.

[1] An antiphon is a short, mostly syllabic refrain that was commonly sung before and after a psalm in the liturgical chant. A responsory typically follows a scripture reading and comprises a verse sung by a soloist or small group, succeeded by a response from the choir or congregation. Its melodic structure is often more intricate and melismatic than an antiphon, and its content closely aligns with the theme of the reading it accompanies.

2.2 Markov mixture model

We use a Markov model to parameterize the pitch distribution of a certain set of chants, e.g. chants in one of the eight modes. To analyze the substructure of modes, we formulate a Markov mixture model to find internal clusters of chants according to their pitch distributions. This model can also be used to represent the different pitch distributions of the eight modes and to automatically estimate the mode of an unseen piece.

We represent a piece as a sequence of pitches $\boldsymbol{x} = (x_\ell)_{\ell=1}^{L}$. A Markov model describes the generative probability of x_ℓ by the initial probability $\psi^{\rm ini}(q) = P(x_1 = q)$ and the transition probabilities $\psi(q', q) = P(x_\ell = q | x_{\ell-1} = q')$ as

$$P(\boldsymbol{x}|\boldsymbol{\psi}) = \psi^{\rm ini}(x_1) \prod_{\ell=2}^{L} \psi(x_{\ell-1}, x_\ell). \tag{1}$$

Given a set of pieces $(\boldsymbol{x}_n)_{n=1}^{N}$, the set of parameters $\boldsymbol{\psi} = \{\psi^{\rm ini}(q), \psi(q', q)\}$ can be optimized for maximizing the likelihood $\prod_{n=1}^{N} P(\boldsymbol{x}_n|\boldsymbol{\psi})$. The parameters learned in this way represent the pitch distribution in these pieces.

In a Markov mixture model, we consider a set of K Markov models parameterized by $\boldsymbol{\psi}_k$ ($k = 1, \dots, K$), each representing the pitch distribution of a class of data, and a mixture probability π_k representing the relative frequency of the k-th class. The probability of a pitch sequence is given as

$$P(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k P(\boldsymbol{x}|\boldsymbol{\psi}_k). \tag{2}$$

This model is not to be confused with a hidden Markov model, in which each latent variable is introduced for each pitch. In the Markov mixture model, the latent variable k is introduced for each sequence.

Supervised and unsupervised training methods can be derived for estimating the parameters π_k and $\boldsymbol{\psi}_k$ from data. In the supervised setup, we consider that we have data divided into K classes. We can then estimate the mixture probability π_k from the relative frequencies of the individual classes and parameters $\boldsymbol{\psi}_k$ from the subset of data in the k-th class. For example, using the mode label in the present data, we can train the Markov mixture model with eight classes corresponding to the eight modes. In the unsupervised setup, we can train a Markov mixture model from a dataset of pitch sequences without class labels. In this case, the number K of classes is an adjustable hyperparameter that defines a resolution of the analysis. The EM algorithm can be applied to estimate the parameters π_k and $\boldsymbol{\psi}_k$.

Given a Markov mixture model with trained parameters, we can estimate the posterior probability of the class of an unseen piece by the following equation:

$$P(k|\boldsymbol{x}) = \frac{P(\boldsymbol{x}, k)}{P(\boldsymbol{x})} \propto \pi_k P(\boldsymbol{x}|\boldsymbol{\psi}_k). \tag{3}$$

We can then take the class $\hat{k}$ that maximizes the posterior probability as the estimated class for the piece.

In our analysis, we use the Markov mixture model in a hierarchical manner. We train a Markov mixture model with K_m classes from a subset of data in mode m, thus obtaining parameters $\pi_k^{(m)}$ and $\boldsymbol{\psi}_k^{(m)}$. We combine the eight Markov mixture models to obtain a hierarchical Markov mixture model represented as

$$P(\boldsymbol{x}) = \sum_{m=1}^{8} \sum_{k=1}^{K_m} \pi_m \pi_k^{(m)} P(\boldsymbol{x}|\boldsymbol{\psi}_k^{(m)}). \tag{4}$$

As in Eq. (3), we can use this model to estimate the posterior mode probability of a piece $\boldsymbol{x}$ as

$$P(m|\boldsymbol{x}) \propto \sum_{k=1}^{K_m} \pi_m \pi_k^{(m)} P(\boldsymbol{x}|\boldsymbol{\psi}_k^{(m)}). \tag{5}$$

The Markov mixture model and its hierarchical version can be used for addressing our two research questions. First, since the mode-level mixture probabilities π_m represent the relative frequencies of modes, the first research question can be examined by analyzing the temporal changes in their values over time. Next, the component Markov models of the hierarchical Markov mixture model represent internal clusters within individual modes and can thus be used for analyzing the modes' substructure. Specifically, the second research question can be examined by analyzing the temporal changes in the relative frequencies of the internal clusters.

The set of pitches, or the state space of Markov models, was constructed from the Cantus database. There were 22 pitches ranging from the lowest pitch F3 to the highest pitch D6. To account for the specific statistical features of the last note of a piece, we also introduce an additional state 'end' in the Markov models so that the statistics of the last note is incorporated in the transition probabilities to the 'end' state. Therefore, the number of states of the Markov models was 23.

3 Result

3.1 Internal clusters of modes

We trained the hierarchical Markov mixture model with $K_m = 3$ for all modes m (Fig. 2). The obtained internal clusters of a mode are ordered in the average time of appearance weighted by the relative frequencies, from the earliest to the latest (see Sec. 3.3). We can observe that the three internal clusters exhibit notable differences in pitch-class transition probabilities in mode 1, 3, and 5, whereas the substructures are less visible in the other modes.

The result of hierarchical clustering of mode-level pitch distributions is also shown in Fig. 2. The same tree structure was obtained when the symmetric Kullback–Leibler divergence and the squared distance were used as the distance measure. This result is similar to the result of [17], which used a single source and a subset of the data we used. Therefore, the structure with two supra-modal groups, one with mode 1, 4, and 6 and the other one with mode 3, 5, and 8 as core members, is shown to be a general characteristic over the time period from the 12th century to the 16th century.

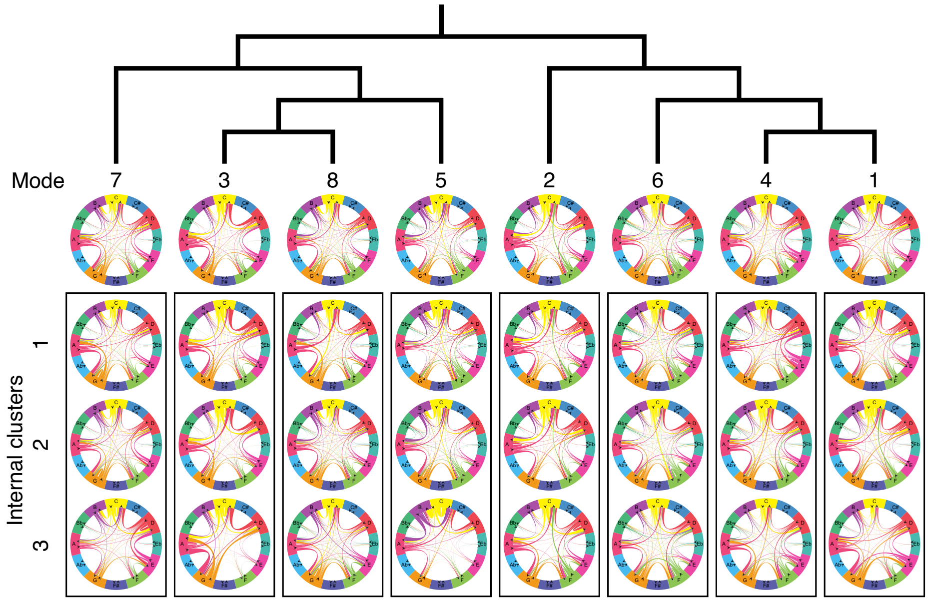

Fig. 2: Visualization of pitch distributions of the eight modes and their internal clusters. Here, the pitch bigram probabilities are reduced to pitch class bigram probabilities visualized as the band widths. The pitch classes are represented by different and arbitrary colors and the colors of the bands indicate the pitch class from which the corresponding pitch transitions occur. The dendrogram was obtained by complete-linkage hierarchical clustering.

To quantitatively measure the effect of the internal clusters for mode classification, we evaluated the accuracy of mode classification by the Markov mixture models with and without internal clusters. In this analysis, we randomly split the data into training (70%) and test (30%) data and used the former data for unsupervised training of the model and the latter data for evaluation. The accuracy was 84.0% without internal clusters and 85.3% with internal clusters, showing the positive effect of more precisely representing the distribution of pitch distributions using internal clusters. It was also confirmed that the accuracy further increased with $K_m = 4$ and 5, indicating the existence of finer-grained internal clusters. As a reference, a previous study [19] reported an F-score of 88–90% by a classification method using the pitch profile (unigram distribution) and 91–92% using pitch bigram features, evaluated on a smaller subset of data with a larger average piece length. We expect that the present Markov model without internal clusters has an equivalent classification ability with these methods when compared in the same setup since the pitch transition probabilities are generally more informative than the pitch profile and essentially equivalent to the bigram probabilities.

Fig. 3 shows the confusion matrix of mode classification by the hierarchical Markov mixture model ($K_m = 3$). The result shows that the classification errors generally occur within each of the two supra-modal groups, as expected. We can also find relatively high

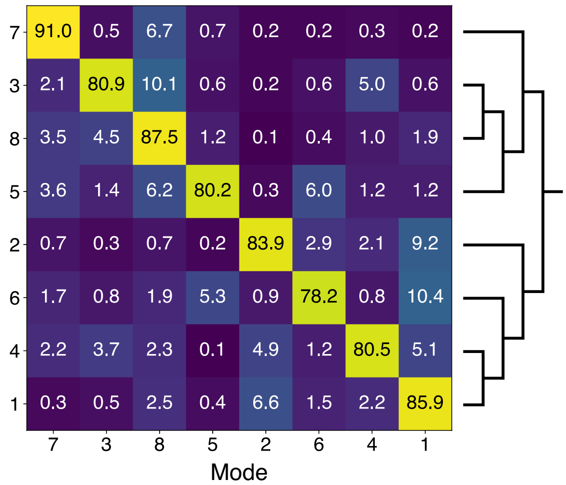

Fig. 3: Confusion matrix of mode classification rates (%) by the hierarchical Markov mixture model.

error rates across these groups between pairs of modes sharing the final, that is, between modes 3 and 4 (final E) and between modes 5 and 6 (final F).

3.2 Internal clusters and genres

To investigate the relationship between the internal clusters and the genres of chants, we analyzed their correlations. We focus on the three main genres, antiphon, responsory, and responsory verse, which cover 91% of the data, and analyzed the proportion of genres $P(g|k)$ in each internal cluster k. More specifically, we used the posterior probability $P(k|n)$ of internal clusters k for piece n and its annotated genre g_n to calculate the genre probability $P(g) \propto \sum_n \delta(g_n, g)$, the conditional internal cluster probability $P(k|g) \propto \sum_n \delta(g_n, g)P(k|n)$, and the proportion of genres $P(g|k) \propto P(k|g)P(g)$ in internal cluster k.

The result in Fig. 4 shows that, for all modes, most of the pieces in genre 'responsory verse' belong to the third internal cluster. Since the labels for the internal clusters only indicate the order in the average time of appearance weighted by the relative frequencies, such a relationship indicates the heterogeneous time distributions of the genres as well as their distinctive features. The distinctive features of the genre 'responsory verse' are not surprising because its musical structure is fundamentally different from that of antiphon or responsory. For example, a responsory verse follows a psalm formula [24] and does not necessarily close with the mode's final. Additionally, all the modes except mode 6 have an internal cluster dominated by the genre 'antiphon' and another one dominated the genre 'responsory'. These results indicate that the three genres tend to have pitch distributions with different characteristics in most modes, and at the same time, that the internal clusters obtained by unsupervised learning do not perfectly match the genres.

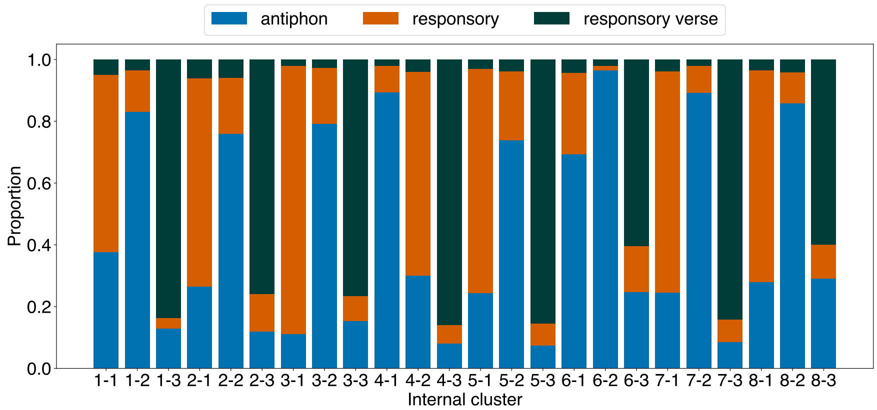

Fig. 4: Proportion of genres in each internal cluster. Internal cluster k of mode m is labeled as m-k.

3.3 Time evolution of mode frequency

The time evolution of the relative frequencies of modes and internal clusters is shown in Fig. 5. To obtain this result, we first calculated the posterior probability $P(k|n)$ of internal clusters for each piece n using its mode annotation as a constraint, and used this probability and the time stamp t_n to calculate the relative frequency of internal clusters in each century.

Some interesting observations can be made from the result. First, we find that the changes in the mode frequencies are remarkably small throughout the analyzed time period. The largest relative changes can be observed between the 12th and 13th centuries and between the 13th and 14th centuries, but the relative changes of the mode frequencies are less than 100%. The observed stability of relative frequencies of clusters of musical styles is in stark contrast with the transitions of musical styles in Western classical music since the 16th century [1, 2] and popular music [7, 9].

Second, compared to the overall mode frequencies, the internal cluster frequencies have larger changes over time. For example, the frequency of cluster 3-3 increased by more than 100% from the 12th century to the 15th century, whereas the frequency of cluster 1-1 decreased by more than 50% in this period. With the result in Sec. 3.1, this means that there is some amount of internal changes in the average pitch distribution within each mode. We can also find some systematic tendencies across modes: the frequencies of the first internal clusters tend to decrease and those of the third internal clusters tend to increase. With the result in Sec. 3.2, we can relate these tendencies to the overall decrease of the proportion of genre ‘responsory’ and the overall increase of the proportion of genre ‘responsory verse’.

Finally, all internal clusters had a non-negligible frequency in the 11th century and emergences of new clusters were not observed. This means that there is no significant innovation in the pitch distribution in this data.

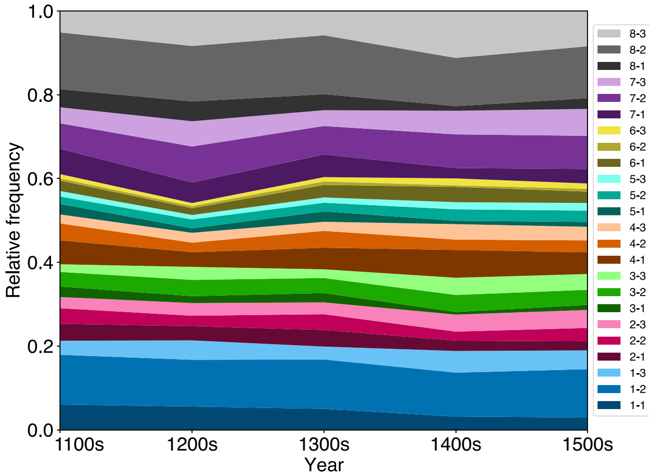

Fig. 5: Relative frequencies of modes and internal clusters over time.

4 Discussion and conclusions

We here summarize and discuss our results. First, we found that the eight medieval modes exhibit internal clusters of pitch distributions, which correlate with the three major chant genres (antiphon, responsory, and responsory verse). Although we focused on the case of three internal clusters per mode, the experimental result suggested the existence of finer-grained internal clusters, which are also expectable from musicological considerations. For example, there are different types of responsories, such as responsorium prolixum, responsorium breve, and responsorium graduale, that were sung on different occasions or in distinct liturgical contexts and have different melodic features [25]. Incorporating deeper musicological insights into specific genre forms and their implications on mode is likely to shed further light on the substructure of the medieval modes.

Second, we found that the mode frequencies remained remarkably stable over five centuries. This can be explained by the fact that the responsories and antiphons were seen as divine texts that should by all means be authentically preserved. Incidentally, this was also a main driving force behind the development of music notation in the West and the emergence of other genres such as sequences and tropes that granted greater creative flexibility. It would be interesting to further study the Cantus database to reveal how such stability was attained when chants were transmitted across different geographical locations in relation to notational practice. It is also important to examine possible reasons for the observed small variations in mode frequencies, such as changing preferences to write chants in certain modes, changes in categorization practices, and artifacts/biases of sampling manuscripts in different time periods. Such a study will be facilitated when more manuscripts will be digitized and made available for computational analyses.

Third, we found considerable changes in the frequencies of internal clusters, which are related to changes in the proportion of the chant genres in the data. Additionally, the analysis conducted at the resolution of three internal clusters per mode revealed a lack of substantial innovation in the pitch distribution. As a caveat, we note that the Cantus database has a limited number of manuscript sources, and further study should also inspect possible biases by employing strategies such as downsampling and generating synthetic pieces to ensure a more balanced analysis.

Moreover, this study demonstrates how researchers can employ large datasets and computational modeling for investigating music-theoretical concepts and their cultural evolution. Finally, our work points to the necessity of increasing collaboration and exchange between researchers from the humanities and computer science. This pertains not only to the interpretation of quantitative results *post factum*. Rather, it is important to engage in interdisciplinary dialog early on in the research process, in particular when constructing and evaluating computational models. We believe that the vibrant field of cultural evolution provides an ideal forum for such exchanges to take place.

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