# JupyterLab_MacOS_arm64

September 6, 2023

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy
from scipy.linalg import expm

import control as ct
import control.optimal as obc

import time
```

[ ]:
```
import platform
print('Platform Info: ' + str(platform.system_alias(platform.system(),platform.
 ↪release(),platform.version())))
print('Platform Info: ' + platform.platform(aliased=0, terse=0))
print('Python Version: ' + platform.python_version())
print('Numpy Version: ' + np.__version__)
print('Scipy Version: ' + scipy.__version__)
print('Control Version: ' + ct.__version__)
```

```
Platform Info: ('Darwin', '22.6.0', 'Darwin Kernel Version 22.6.0: Wed Jul  5
22:22:52 PDT 2023; root:xnu-8796.141.3~6/RELEASE_ARM64_T8103')
Platform Info: macOS-13.5.1-arm64-arm-64bit
Python Version: 3.11.4
Numpy Version: 1.25.0
Scipy Version: 1.11.1
Control Version: 0.9.4
```

[ ]:
```
M = 0.5
m = 0.2
l = 0.3
I = 0.006
b = 0.2
g = 9.81


Ac = np.array([[0, 1, 0, 0],
    [0, b*(m*l*l - I)/(I*(M+m)-M*m*l*l), -m*m*l*l*g/(I*(M+m)-M*m*l*l), 0],
    [0, 0, 0, 1],
```

```python
    [0, -b*m*l/(I*(M+m)-M*m*l*l), m*g*l*(M+m)/(I*(M+m)-M*m*l*l), 0]])

Bc = np.array([[0],
    [(I-m*l*l)/(I*(M+m)-M*m*l*l)],
    [0],
    [m*l/(I*(M+m)-M*m*l*l)]])
```

[ ]:
```python
def convertToDiscreteTime(Ac,Bc,Ts):
    AB = expm(np.concatenate((np.concatenate((Ac, Bc),axis=1),np.
 ↪zeros((1,5))),axis=0) * Ts)
    A = np.array(AB[0:4,0:4])
    B = np.transpose(np.array([AB[0:4,-1]]))
    return A, B
```

[ ]:
```python
# sampling rate
Ts = 0.1

# convert to discrete time
A, B = convertToDiscreteTime(Ac,Bc,Ts)

# grab number of states
numState = len(A)
```

[ ]:
```python
# initial conditions
x_0 = np.array([[2],[0],[0],[0]])

# target state reference
xstar = np.array([[10],[0],[0],[0]])
```

[ ]:
```python
# include state constraints, set min max velocity and acceleration, assume
maxCranePosition = 10 # m
minCranePosition = -5 # m
maxVelocity = 3 # m/s
minVelocity = -maxVelocity # m/s
maxCraneAngle = 0.1 # rad
minCraneAngle = -maxCraneAngle # rad
maxCraneAngledot = 10
minCraneAngledot = -maxCraneAngledot

maxForce = 100 # N
minForce = -maxForce # N
```

[ ]:
```python
# Choose the finite horizon
finiteHorizon = 1 # seconds
# convert to discrete-time steps
finiteHorizon = finiteHorizon/Ts
```

[ ]:
```python
# Specify Q and R matrices
Q = np.eye(numState)
R = np.eye(1)*1
```

[ ]:
```python
# create system model (used to mimic the system)
system = ct.ss2io(ct.ss(A, B, np.eye(numState), 0, Ts))

# Assume full state knowledge
C = np.eye(numState)
# create mathematical model that will be the basis of the controller design
model = ct.ss2io(ct.ss(A, B, C, 0, Ts))

# create constraint vector
constraints = [obc.state_range_constraint(system, [minCranePosition,␣
 ↪minVelocity, minCraneAngle, minCraneAngledot], [maxCranePosition,␣
 ↪maxVelocity, maxCraneAngle, maxCraneAngledot]), obc.
 ↪input_range_constraint(system, [minForce], [maxForce])]

# create stage cost function on the model, with given desired error signal and␣
 ↪zero control input
cost = obc.quadratic_cost(model, Q, R, x0=xstar.flatten(), u0=0)
# note: this uses the 'model' (which could be different to the system)

# online MPC controller object is constructed with finite horizon, cost and␣
 ↪constraints
ctrl = obc.create_mpc_iosystem(model, np.arange(0, finiteHorizon) * Ts, cost,␣
 ↪constraints)

# create closed-loop feedback 'loop' object using the feedback function to link␣
 ↪the controller to the system
loop = ct.feedback(system, ctrl, 1)
```

[ ]:
```python
res = obc.solve_ocp(model, np.arange(0, finiteHorizon) * Ts, x_0.flatten(),␣
 ↪cost, constraints, squeeze=True)

print(res)
```

```
Summary statistics:
* Cost function calls: 105
* Constraint calls: 126
* System simulations: 215
* Final cost: 568.2949584276911
 message: Optimization terminated successfully
 success: True
  status: 0
     fun: 568.2949584276911
       x: [ 8.085e-01  1.464e-02  4.912e-01  7.621e-01  5.672e-01
```

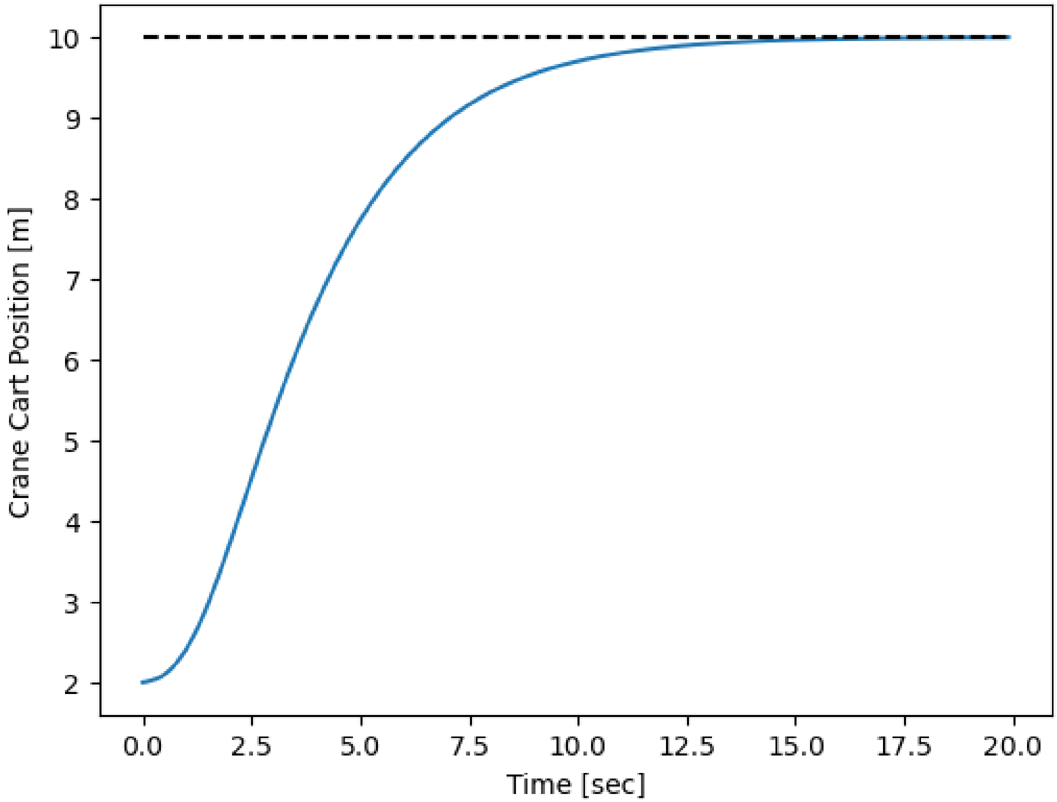
```
            5.080e-01  4.151e-01  2.398e-01  0.000e+00  0.000e+00]
     nit: 9
     jac: [-3.049e+00 -4.285e+00 -1.991e+00 -2.022e-03 -9.003e-04
            1.045e-03 -8.469e-04  1.541e-03  0.000e+00  0.000e+00]
    nfev: 105
    njev: 9
 problem: <control.optimal.OptimalControlProblem object at 0x11f5cba90>
    cost: 568.2949584276911
    time: [ 0.000e+00  1.000e-01  2.000e-01  3.000e-01  4.000e-01
            5.000e-01  6.000e-01  7.000e-01  8.000e-01  9.000e-01]
  inputs: [ 8.085e-01  1.464e-02  4.912e-01  7.621e-01  5.672e-01
            5.080e-01  4.151e-01  2.398e-01  0.000e+00  0.000e+00]
  states: [[ 2.000e+00  2.010e+00 …  2.216e+00  2.261e+00]
           [ 0.000e+00  1.856e-01 …  4.615e-01  4.397e-01]
           [ 0.000e+00 -4.623e-02 … -1.408e-02  1.761e-02]
           [ 0.000e+00 -8.497e-01 …  2.412e-01  3.458e-01]]
```

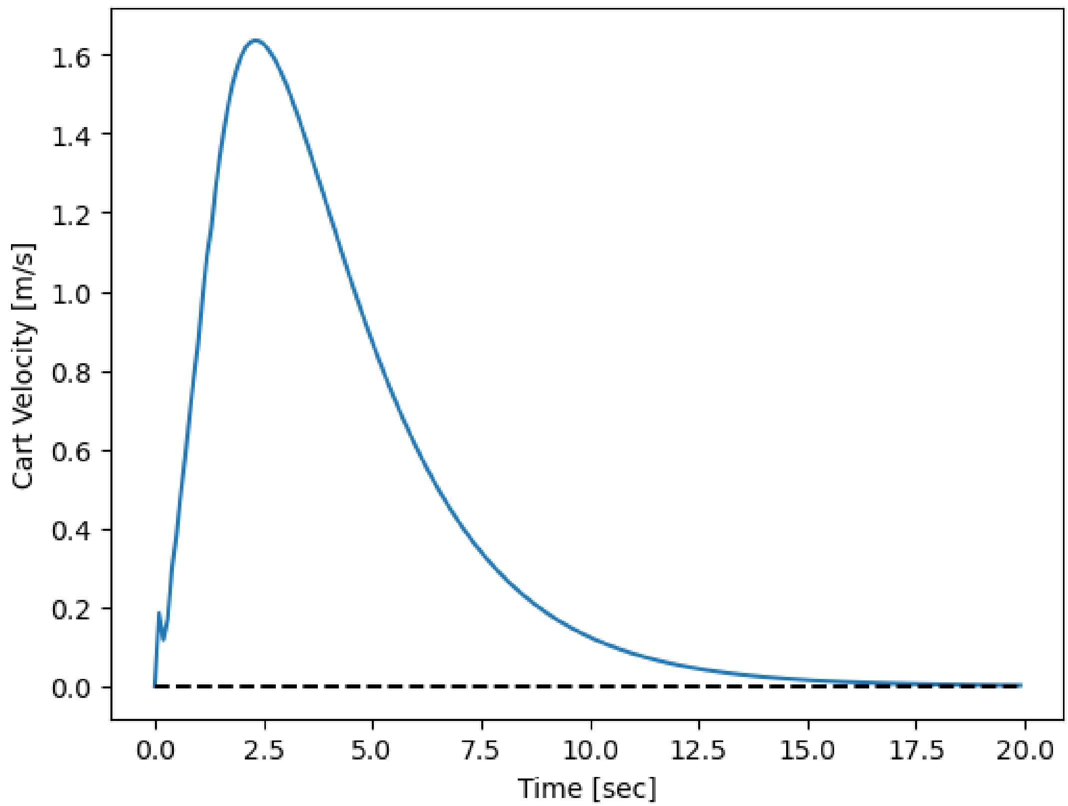
[ ]:
```
# set length of simulation
Nsim = 20
# convert to number of discrete-time steps
Nsim = Nsim/Ts

# Simulate the closed-loop MPC
start = time.time()
tout, xout = ct.input_output_response(loop, np.arange(0, Nsim) * Ts, 0, x_0.
 ↪flatten())
end = time.time()
print("Computation time = %g seconds" % (end-start))
```

```
/opt/homebrew/Caskroom/miniforge/base/envs/testControl/lib/python3.11/site-
packages/control/optimal.py:928: UserWarning: unable to solve optimal control
problem
scipy.optimize.minimize returned Positive directional derivative for linesearch
  warnings.warn(
/opt/homebrew/Caskroom/miniforge/base/envs/testControl/lib/python3.11/site-
packages/control/optimal.py:928: UserWarning: unable to solve optimal control
problem
scipy.optimize.minimize returned Iteration limit reached
  warnings.warn(

Computation time = 63.719 seconds
```

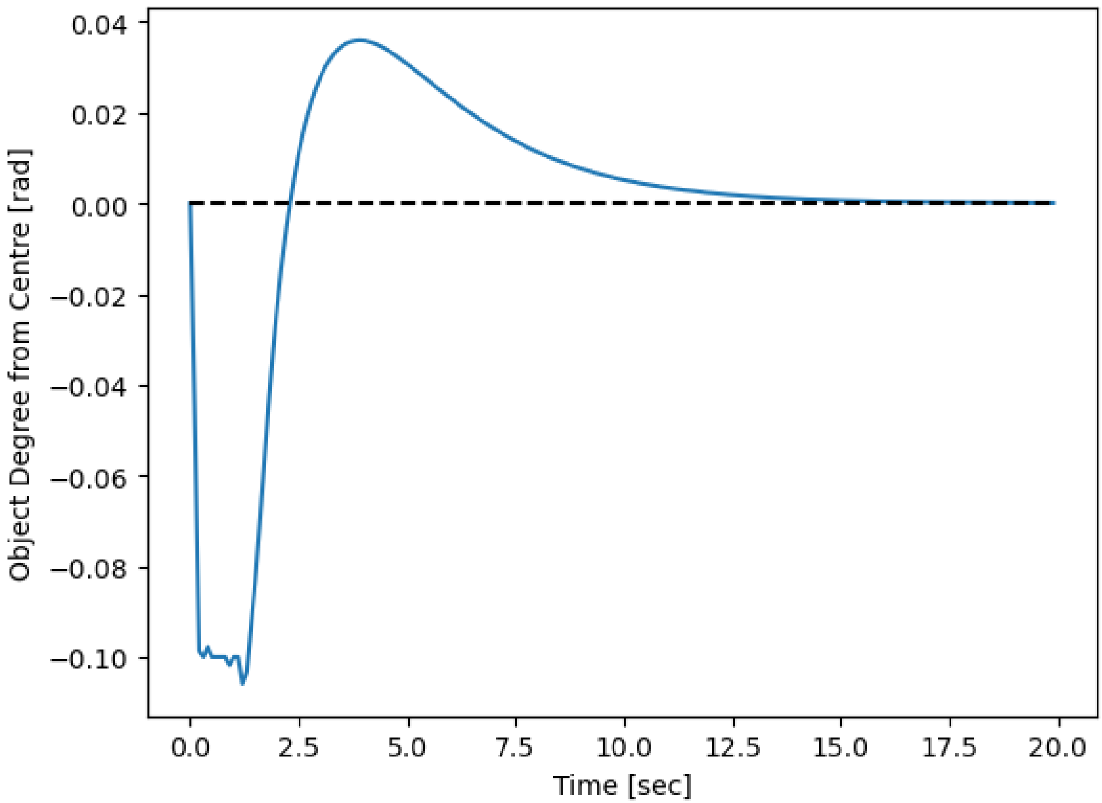
[ ]:
```
figlabels = ['Crane Cart Position [m]','Cart Velocity [m/s]','Object Degree␣
 ↪from Centre [rad]','Object Degree rate [rad/s]']
for i, y in enumerate(xout):
    plt.figure()
    plt.plot(tout, y)
    plt.plot(tout, xstar[i] * np.ones(tout.shape), 'k--')
```

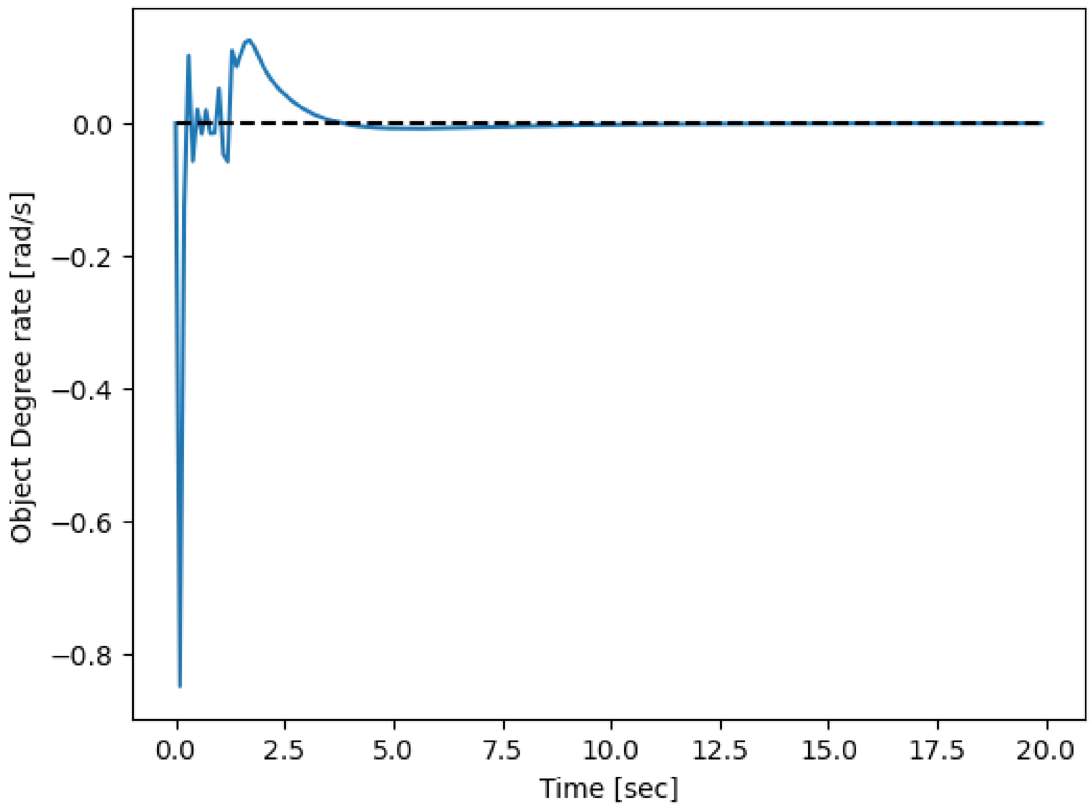
```
    plt.ylabel(figlabels[i])
    plt.xlabel('Time [sec]')
```





Cart Velocity [m/s]
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Time [sec]


0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
Object Degree from Centre [rad]
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Time [sec]


Object Degree rate [rad/s]
0.0
−0.2
−0.4
−0.6
−0.8
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Time [sec]