Voter Control in k-Approval and k-Veto under Partial Information*

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Abstract

In constructive control, an external agent seeks to change the outcome of an election by adding, deleting or partitioning voters or candidates. Traditionally, the manipulative agent has full information, i.e. he knows the candidate set, each voter's votes and the voting rule. In this paper, we focus on constructive control by adding or deleting voters and examine the complexity of control under partial information for the k-Approval and k-Veto rules.

1 Introduction

For a long time, voting has been an important way to make collective decisions or aggregate individual preferences. Its application range varies from politics to computer science, for example design of recommender systems (Ghosh et al. 1999), ranking algorithms (Dwork et al. 2001) or machine learning (Xia 2013), to name a few. While for political elections, the number of candidates or voters is often manageable, we are dealing with huge data volumes in computer science applications. Therefore it seems natural to study the computational complexity of problems related to voting. One important decision problem in voting theory is *voter control* where an external agent – the chair – adds or deletes some voters in order to make a candidate c win the election. Other decision problems are *manipulation* and *bribery*. In manipulation, a group of agents cast their votes strategically (and possibly dishonestly) in order to make c win whereas in bribery, an external agent – the briber – changes some voters' votes. The seminal papers of Bartholdi et al. (Bartholdi, Tovey, and Trick 1989a; 1989b; 1992) suggest that computational hardness of such decision problems is a (worst case) barrier against manipulative attacks. Unfortunately, various decision problems in voting theory are easy for generic and frequently used voting rules such as scoring rules (each voter assigns points to some candidates according to a fixed scheme) (Lin 2011; Bartholdi, Tovey, and Trick 1989a; Faliszewski, Hemaspaandra, and Hemaspaandra 2009). In most settings, the manipulative agent has full information, i.e. he knows the preference orders of each voter about all candidates. However, in many real-world settings, this assumption is not realistic. For example, each voter ranks only a small subset of candidates. Recently, various papers dealt with problems under a certain kind of uncertainty.

In this paper, we will analyse the complexity for constructive control by adding (CCAV) and deleting voters (CCDV) in k-Approval and k-Veto under partial information, as these closed families of voting rules belong to the voting rules most popular and frequently used and one can very well observe the transition from P to NP-hardness. [1] In CCAV, the chair tries to add a certain number of new voters to an election such that a given candidate wins. In CCDV, the chair deletes some voters from the election. Examples are political elections with the voting age being lowered or voter suppression. For CCAV, three natural generalizations of full information arise. We will investigate if it makes a difference if the previous (*registered*) voters or the new (*unregistered*) voters are partial (or both groups). On the one hand, it seems reasonable that the chair has (due to surveys or former elections) full (or at least enough) knowledge about the registered voters, but he has only partial knowledge about the unregistered voters. On the other hand, it may occur that the chair personally knows the unregistered voters (at least some of them) or has some belief about them, but has only fragmentary knowledge about the registered voters. We will not study the complexity of control by adding/deleting *candidates*, as these problems are already hard for full information (Lin 2011).

Related Work First of all, for full information, constructive control by adding/deleting voters or candidates was introduced by Bartholdi et al. (Bartholdi, Tovey, and Trick 1992). Hemaspaandra et al. (Hemaspaandra, Hemaspaandra, and Rothe 2007) extended this line of research for destructive control problems. In (Erdélyi et al. 2015) and (Menton 2013), it has been shown that the voting rules *Fallback Voting* and *Normalized Range Voting* are the best voting rules in a sense that they offer the highest number of resistances or immunities against 22 control variants in total, i.e. these two voting rules provide the best (worst case) protection against

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[1] There is a minimal $\hat{k}$ such that control for k-Approval (-Veto) is hard for each $k \geq \hat{k}$ and easy for each $k < \hat{k}$.

control attacks up to now. [2] Control results especially for k-Approval and k-Veto were published in (Lin 2011). In contrast, we are investigating CCAV and CCDV in k-Approval and k-Veto in settings with partial votes.

Our paper fits in the line of research on the complexity analysis of manipulation, bribery and winner determination under some kind of uncertainty. First of all, the possible/necessary winner problems were introduced by Konczak and Lang (Konczak and Lang 2005). They generalized the canonical setting with votes as linear orders and allowed votes to be partial orders. In their paper, they investigated under which conditions a candidate is a winner in *at least one* (*possible winner*) or in *all* (*necessary winner*) complete extensions of the partial votes. Our problem is related to the possible/necessary winner problems in a sense that we consider nine different ways to display partial information and partial orders is only one of them. Particularly, our problem is related to the necessary winner problem in a way that we ask if the chair can make his favorite candidate a winner under *all* possible completions.

The possible winner problem and some similar problems were further studied by Xia and Conitzer (Xia and Conitzer 2011), Betzler and Dorn (Betzler and Dorn 2010), Baumeister and Rothe (Baumeister and Rothe 2012), and Chevaleyre et al. (Chevaleyre et al. 2010).

Finally the work of Conitzer et al. (Conitzer, Walsh, and Xia 2011) especially encouraged us to consider different notions and hierarchies of partial information, as in their model, a manipulator has partial information about the votes and they ask (with a slight abuse of notation) if the manipulator can cast a vote to improve the outcome of the election. In particular, they do not define a special partial information model, but an information set containing all possible profiles that can be achieved by completing the partial profile. Basically, their model is a generalization of all the models considered in this paper. We recently studied the complexity of bribery under nine different models of partial information (Briskorn, Erdélyi, and Reger 2015).

Organization This paper is organized as follows. In Section 2, we provide some preliminaries and briefly introduce nine models of partial information before we define our problems studied in this paper in Section 3. Finally, in Section 4, we give our complexity results for voter control under partial information. Section 5 gives a short conclusion.

2 Preliminaries

Formally, an *election* is a pair $E = (C,V)$ where C is a finite *candidate set* and V is a finite multiset of *voters*. Usually, each voter v_i is given as a strict linear order $\succ_{v_i}$ over C which represents his preference order (i.e. it is total, transitive and asymmetric). In other words, there is full information for such voters. In this paper, however, the votes are often given partially in terms of a certain model which will be specified later. An *n-voter profile* $P := (v_1,\dots,v_n)$ on C consists of n voters $v_1,\dots,v_n$ given as strict linear orders or (for a *partial profile*) partial in terms of the same model. A *completion* or *extension* of a partial profile P is a complete profile P' not contradicting P. [3] We also use the game-theoretic notion *information set*, i.e. $P' \in I(P)$ where $I(P)$ is the information set of P containing all complete profiles not contradicting P.

Sometimes, we will use the notion $d \succ_v \overrightarrow{B}$ for some $d \in C \setminus B, B \subset C$ which is equivalent to $d \succ_v b \; \forall b \in B$ and the candidates in B are ordered among themselves in an arbitrary, but fixed way.

A *voting rule* $\mathscr{E}$ maps an election $E = (C,V)$ to some $W \subseteq C$ which is called the *winner set*. Actually, we are dealing with *voting correspondences*, but we use the expression *voting rule* throughout this paper with a slight abuse of notation. Note that we use the *non-unique winner model*, as we admit a voting rule to have exactly one, more than one or no winner at all. There is a variety of voting rules, but in this paper, we restrict ourselves to the following two classes of voting rules:

- In k-Approval, each voter assigns one point to his k most favorite candidates and zero points to the remaining candidates. $score(c_i)$ counts the number of points which candidate $c_i \in C$ receives in total. The candidates with the highest approval score are the winners.
- In k-Veto, each voters assigns zero points to his k least favorite candidates and one point to the remaining ones. $vscore(c_i)$ denotes the number vetoes (i.e. the number of voters not assigning a point to c_i) that candidate c_i gets. The candidates with the lowest number of vetoes win.

1-Approval is also known as Plurality. Moreover, we write Veto instead of 1-Veto. Note that for a fixed number m of candidates, k-Approval equals $(m-k)$-Veto. In our analysis, however, only k is fixed, but m is variable.

In our proofs, $pscore(d)$ sums up all potential (i.e. definite and possible, but unsure) approvals. Similarly, we define $pvscore(d)$. Sometimes, we also use $score_{(C,V)}(d)$ to gather all approvals of candidate d in the (sub)election (C,V). [4]

In the remainder of this section, we give a short survey about all nine models of partial information for which we will study the complexity of voter control and which have been already regarded in (Briskorn, Erdélyi, and Reger 2015) and the references therein. Besides, we briefly point out their interrelations. For each model of partial information mentioned in the following, we specify the structure of data given. Note that this is exactly the information given. The "real" vote, i.e. the actual complete ranking of the regarded voter, is among all potential completions of the given partial vote. Moreover, one can easily verify that full information can be displayed by each model. We let $m = |C|$.

▸ *Gaps* (GAPS)/*One Gap* (1GAP)

Our first model GAPS handles the case where there are *holes* in a vote, i.e. each vote has some fully ranked blocks and

[2] A control problem is said to be *resistant* if the decision problem is NP-hard and *immune* if control is not possible at all.

[3] In other words, each vote v_i is completed in a way that the partial structure in v_i is conserved.

[4] In a similar manner, we can define $pscore_{(C,V)}(d)$, $pvscore_{(C,V)}(d)$ and so on.

some blocks in between where it is only known which candidates belong to these blocks, but not how they are ordered among themselves. Examples could be nearly single-peaked elections (Erdélyi, Lackner, and Pfandler 2013), where for every candidate at least an approximate position is known, or cases where the voter is simply indifferent between alternatives.

Formally, for each vote v, we have a partition $C_1^v, \ldots, C_{2m+1}^v$ of the set of candidates and a total order for each C_k^v with k even. Note that possibly $C_k^v = \emptyset$ for some k.

If $C_k^v = C_{k+1}^v = \emptyset$, we can drop both partite sets without changing the information set. Therefore, we can restrict ourselves to at most $2m+1$ partite sets.

A special case is 1GAP, where in each vote some candidates are ranked at the top and at the bottom of the votes, and there is at most one hole in between. Formally, 1GAP refers to the special case of GAPS with $C_k^v = \emptyset$, for each $k \in \{1, 5, 6, \ldots, 2m+1\}$ and each voter v.

▸ *Top-/Bottom-Truncated Orders* (TTO/BTO)

TTO equals 1GAP with $C_1^v = C_4^v = \ldots = C_{2m+1}^v = \emptyset$ for each voter v. A natural application is an election where each voter assigns some points to his favorite alternatives and the remaining alternatives are known to be less preferred.

BTO refers to the special case of 1GAP where $C_3^v = \ldots = C_{2m+1}^v = \emptyset$ for each voter v. BTO could be used to represent negative properties or objections (and their extent) of different alternatives in multiagent settings.

▸ *Complete or empty votes* (CEV)

CEV handles the case where each vote is either complete or empty. Formally, CEV is a special case of TTO with either $C_2^v = \emptyset$ or $C_3^v = \emptyset$ for each voter v. This model represents the case where new voters join the election about whom there is no information at all.

▸ *Fixed Positions* (FP)

For each vote v we have a subset of candidates C^v with distinct positions in range between 1 and m assigned. An example for this model is the case, where there are three candidates c_1, c_2, and c_3. Candidates c_1 and c_3 have clearly opposing properties such that each voter prefers either favors c_1 most and c_3 least or the other way round. Candidate c_2 is thus fixed to position 2.

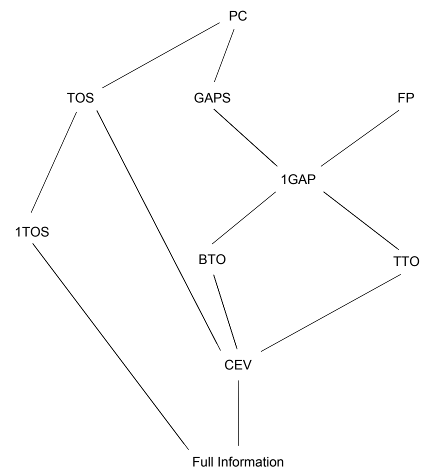
▸ *Pairwise Comparisons* (PC)

PC (aka *partial orders*) is probably the most natural way to display partial preferences. Formally, for each vote v we have a subset Π^v of $C \times C$ which we restrict to be asymmetric and transitive for matters of convenience.

▸ *(Unique) Totally Ordered Subset of Candidates* ((1)TOS)

In TOS, for each vote v, we formally have a subset $C^v \subseteq C$ and a complete ranking about C^v. As an example, one could imagine a film database where each user ranks a (tiny) subset of (very many) films. An important special case, 1TOS, requires that $C^v = C' \ \forall v \in V$. A natural example would be the addition of candidates to an election. For C', there is full information, but nothing is known about the new candidates.

The models of partial information defined above and their interrelations can be visualized in the following Hasse diagram:



3 Problem Settings

In the following, we let

$$\text{PIM} := \{\text{PC}, \text{GAPS}, \text{1GAP}, \text{FP}, \text{TOS}, \text{BTO}, \text{1TOS}, \text{CEV}, \text{TTO}\}$$

be the set of all nine partial information models defined above. Besides, we let $\overline{\text{PIM}} := \text{PIM} \cup \{\text{FI}\}$ where FI is the standard model of full information. The first problem defined in the following asks if the chair can add a certain number of new voters to the election such that c is a winner in the resulting election no matter how the partial votes are completed.

$\mathscr{E}$-(X,Y)-Constructive Control by Adding Voters	
Given:	An election $(C, V \cup W)$ with registered voters V according to model $X \in \overline{\text{PIM}}$, unregistered voters W according to model $Y \in \overline{\text{PIM}}$, a designated candidate $c \in C$, a non-negative integer $\ell \leq \lvert W \rvert$, and a partial profile P according to (X,Y)
Question:	Is it possible to choose a subset $W' \subseteq W$, $\lvert W' \rvert \leq \ell$ such that c wins the election $(C, V \cup W')$ under $\mathscr{E}$ for each complete profile $P' \in I(P)$?

This way, one could combine all models of full/partial information. Our complexity analysis however includes only the three problems (FI,X), (X,FI) and (X,X) for a given model $X \in$ PIM. (FI,FI) refers to both V and W representing full information which has been widely studied up to now and which is a special case of the other three problems. E.g. Plurality-(X,FI)-CCAV is the constructive control problem by adding voters where all votes in W are linear orders and all votes in V are partial according to model X. The other problem studied in this paper asks if the chair can delete some voters such that c is a winner of the resulting election for each completion of the (remaining) partial profile.

$\mathscr{E}$-X-CONSTRUCTIVE CONTROL BY DELETING VOTERS	
Given:	An election (C,V), a designated candidate $c \in C$, a non-negative integer $\ell \leq \|V\|$, and a partial profile P according to X.
Question:	Is it possible to choose a $V' \subseteq V$, $\|V \setminus V'\| \leq \ell$ such that c is a winner of the election (C,V') under $\mathscr{E}$ for each complete profile $P' \in I(P)$?

4 Complexity Results

Tables 1, 2 and 3 present the complexity results for the control problems studied in this paper. Column FI contains all results for full information given in (Lin 2011). Results in italic are hardness results that follow from hardness results for full information. Finally, results in boldface are new.

In hardness proofs, we use a reduction from X3C known to be NP-complete (Garey and Johnson 1979). [5]

EXACT COVER BY 3-SETS	
Given:	A set $B = \{b_1,\ldots,b_{3m}\}$ and a collection $\mathscr{S} = \{S_1,\ldots,S_n\}$ of 3-element subsets of B.
Question:	Does $\mathscr{S}$ contain an exact cover for B (i.e., a subcollection $\mathscr{S}' \subseteq \mathscr{S}$ such that every element of B occurs in exactly one member of $\mathscr{S}'$)?

Note that we use the indices m and n in the style of candidates and voters, as the elements in B represent candidates and the subsets S_i refer to voters.

In contrast, we derive some P results by solving an equivalent problem that can be decided in polynomial time. Such problems are *b-edge cover* and *b-edge matching*. (Ahn and Guha 2014; Gabow 1983):

b-EDGE-MATCHING	
Given:	An undirected multigraph $G = (V,E)$ with vertexes V and edges E, capacity constraints $b(v) \in \mathbb{N}_0$ $(v \in V)$, and $K \in \mathbb{N}_0$.
Question:	Does G have a maximal b-edge matching of size K, i.e. can we find a subset $E' \subseteq E$, $\|E'\| = K$ such that $v \in V$ is incident to at most $b(v)$ edges in E'?

We obtain the definition of b-edge cover by exchanging *maximal* by *minimal* and *at most* by *at least*.

4.1 Constructive Control by Adding Voters

In this section, we will investigate the complexity of CCAV for the whole classes of k-Approval and k-Veto. For CCAV, we will further distinguish if the registered or the unregistered voters (or both) are partial and observe that it makes a difference which group of voters is actually partial and which one not. If not mentioned other, throughout this section, we will have an input election $(C, V \cup W)$ with m candidates C, n registered voters V, unregistered voters W, a designated candidate c, and an $\ell \in \mathbb{N}_0$ as the maximal number of voters which may be added from W to the election (C,V). For CCDV, ℓ is the deletion limit and we do not require unregistered voters. Moreover we assume that $|C| > k$ when regarding k-Approval or k-Veto, as otherwise all candidates and especially c are winners for each completion even without any voters being added or deleted.

First of all, we will assume merely the registered voters to be partial. Our focus lies only on voting rules for which CCAV is in P under full information. The next result is however somewhat discouraging if one hopes for the complexity to increase under partial information.

Theorem 4.1. *$\mathscr{E} - (X,FI)$-CCAV is in P if and only if $\mathscr{E} - (FI,FI)$-CCAV is in P ($\mathscr{E} \in \{k$-Approval$, k$-Veto$\}$ $(k \in \mathbb{N})$).*

We omit the proof due to space restrictions. It suffices to regard PC and FP as the most general models. Clearly, we count all definite approvals (potential vetoes) for c and all possible approvals (definite vetoes) in (C,V) for each $d \neq c$. [6] Obviously it is in P for both models to determine the possible, definite and excluded approval (veto) candidates in each vote. As the votes in W are complete and the chair's decision depends only on the (partial) *scores* in (C,V), the arguments for full information can be applied.

Interestingly, by allowing the set of unregistered voters to be partial, many P results turn to hardness results, as we will see in the following. However, for Plurality, CCAV remains easy nevertheless.

Theorem 4.2. PLURALITY-(FI,X)-CCAV *and* PLURALITY-(X,X)-CCAV *are in* P *for every model $X \in PIM$.*

Proof. It suffices to show the result for registered and unregistered voters according to the same model $X \in \{\text{FP}, \text{PC}\}$, as these are the most general cases for which we like to prove our theorem, and all other problems thus inherit the polynomial time upper bound. Both for FP and PC, it is easy to determine for each vote which candidates score definitely, possibly but not definitely and not all. For c, we count only the definite points in V (and W), for each non-distinguished candidate d, we count all potential points (i.e. the maximal possible number of points that d can achieve in any extension). Obviously, the only reasonable strategy for the chair is to add voters from W who definitely rank c first and to add as many as possible of these voters, namely exactly $\min(\ell, score_{(C,W)}(c))$. It remains to verify after adding these voters if $score_{(C,V)}(c) + \min(\ell, score_{(C,W)}(c)) \geq pscore_{(C,V)}(d)\ \forall d \in C \setminus \{c\}$. ❑

In contrast, there are hardness results for 2-Approval.

Theorem 4.3. 2-Approval-(X,X)-CCAV *and* 2-Approval-(FI,X)-CCAV *are* NP*-complete for every model $X \in \{PC, GAPS, 1GAP, FP\}$.*

Proof. We prove our theorem only for (FI,1GAP) because this is the most special problem mentioned above.

[5] We assume the reader to be familiar with the notions P and NP-*completeness*.

[6] Note that for PC, this is barely correct: It may occur that some unsure approvals do not count for $d \neq c$: Suppose that c is possibly and not definitely approved by a voter v and we know that $c \succ_v d$ holds. Then, d is fixed behind c on a disapproval position although d may be possibly approved of, too. This holds as d cannot achieve a higher score than c for this vote in any extension.

	FI	GAPS	FP	TOS	PC	CEV	1TOS	TTO	BTO	1GAP
Plurality	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Approval	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
3-Approval	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
4-Approval	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
3-Veto	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*

Table 1: (X, FI)-CCAV

	FI	GAPS	FP	TOS	PC	CEV	1TOS	TTO	BTO	1GAP
Plurality	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Approval	P	**NPc**	**NPc**	**P**	**NPc**	**P**	**P**	**P**	**P**	**NPc**
3-Approval	P	**NPc**	**NPc**	**NPc**	**NPc**	**P**	**P**	**P**	**P**	**NPc**
4-Approval	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
3-Veto	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*

Table 2: (FI, X)-CCAV/(X, X)-CCAV

	FI	GAPS	FP	TOS	PC	CEV	1TOS	TTO	BTO	1GAP
Plurality	P	**NPc**	**NPc**	**NPc**	**NPc**	**P**	**P**	**P**	**NPc**	**NPc**
2-Approval	P	**NPc**	**NPc**	**NPc**	**NPc**	**P**	**P**	**P**	**NPc**	**NPc**
3-Approval	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
3-Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
4-Veto	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*

Table 3: X-CCDV

For the remaining problems, the hardness lower bound immediately follows. Our proof uses a reduction from X3C. Given an X3C instance $(B, \mathscr{S})$ with $B = \{b_1, \ldots, b_{3m}\}$ and $\mathscr{S} = \{S_1, \ldots, S_n\}$, we construct our election $(C, V \cup W)$ as follows:

Our candidate set is $C = \{c\} \cup B \cup D$ where c is our designated candidate, B a set of candidates initially (possibly) winning the election, and $D = \{d_1, \ldots, d_{3mn+n-4m}\}$ a set of dummy candidates. Our set of registered voters V consists of complete votes which approve of a candidate from $B \cup \{c\}$ and a dummy candidate each. In total, there are $3mn - 4m + n$ voters such that $score_{(C,V)}(b) = n - 1$ ($b \in B$) and $score_{(C,V)}(c) = n - m$. For the dummy candidates, we have $score_{(C,V)}(d) = 1$ ($d \in D$). The unregistered voters W are defined as follows. There are n voters, and voter w_i definitely approves of c and possibly approves of the candidates in $S_i \subset B$ ($i = 1, \ldots, n$). As W is according to 1GAP, we have $C_2^{w_i} = \{c\}$, $C_3^{w_i} = S_i$ and $C_4^{w_i} = C \setminus (\{c\} \cup S_i)$ for $w_i \in W$. The chair may add m voters.

Now, it remains to argue that X3C has a solution iff the chair can make c a winner by adding at most m voters $W' \subseteq W$. Obviously an exact cover yields a successful control by adding exactly the w_i according to the cover. This leads to $score_{(C,V \cup W')}(c) = n = pscore_{(C,V \cup W')}(b) > 1 = pscore_{(C,V \cup W')}(d)$ for each $b \in B, d \in D$ and c is thus a winner. Conversely, suppose that c can be made a necessary winner and a cover does not exist. By adding m voters, c reaches n points, but there is some $\hat{b} \in B$ who is approved of twice or more in the votes added. Thus c is not a necessary winner which is a contradiction. ❑

In contrast, there are five models for which both remaining control problems are in P if we admit partial votes.

Theorem 4.4. 2-Approval-(FI,X)-CCAV *and* 2-Approval-(X,X)-CCAV *are in* P *for every model* $X \in \{TTO, BTO, TOS, CEV, 1TOS\}$.

Proof. It suffices to regard the models TTO, BTO and TOS for partial V and W. Clearly the scores are easy to compute. Obviously c definitely scores in each added vote and the chair adds as many voters as possible, namely precisely $\min(\ell, score_{(C,W)}(c))$ [7].

- For $X =$TOS, the chair adds only voters $w \in W$ with $C^w = C$ and c being among the two most preferred candidates or $|C^w| = |C| - 1$ and c being the top candidate in C^w. In the first type of votes, the other approval is fixed to another candidate d, whereas in the second kind of vote, either the second most preferred candidate b in C^w or the candidate in $C \setminus C^w$ is (possibly) approved. Now apparently c can be made a necessary winner after $\min(score_{(C,W)}(c), \ell)$

[7] As we will see later, this is barely correct for TTO, as there are voters assigning a (possible) point to each candidate including c and these votes can thus be ignored regarding the selection of the votes to be added.

voters being added iff the following b-edge matching has a solution of size $\min(score_{(C,W)}(c), \ell)$: We are given an undirected multigraph $G = ((C \setminus \{c\}) \cup \{*\}, E)$ with edges E defined as follows. For each voter $w \in W$ with $C^w = C$ and c and d being the first two candidates in C^w, there is an edge connecting $*$ and d, where $*$ is an artificial node. For every w with $|C^w| = |C| - 1$, there is an edge between a and d where $d \in C \setminus C^w$ and a is ranked second behind c in C^w. The upper capacities are $b(d) = score_{(C,V)}(c) + \min(score_{(C,W)}(c), \ell) - pscore_{(C,V)}(d)$ for $d \neq c$ and $b(*) = \infty$. (If some d cannot even be caught by adding $\min(score_{(C,W)}(c), \ell)$ voters, the capacity constraints are negative and the edge matching problem trivially has no solution.)

- For BTO and TTO, a simple greedy algorithm can be applied. For both structures, one can easily compute $score(c)$ and $pscore(d)$ for $d \neq c$ both in V and in W. For BTO, c can be definitely approved at all if $|C_1^v| \leq 2$. Otherwise there are at least two candidates preferred to c in the top set for some completion. For $|C_1^v| \leq 2$, we have full information in a sense that the score of each candidate can be uniquely determined in v. As there is exactly one $d \neq c$ approved of by each such voter in W, the chair greedily adds $\min(\ell, score_{(C,W)}(c))$ voters with the other approved candidate as bad as possible.

  For TTO, the chair greedily adds only votes with $|C_2^v| \geq 2$ and c is among the first two candidates in C_2^v. For $C_2^v \in \{\emptyset, \{d\}\}$ with $d \neq c$, each candidate except c is (possibly) scoring. For $C_2^v = \{c\}$, all candidates $d \neq c$ are (possibly) approved and c is definitely approved. Adding these voters does not make sense either.

❑

It is noteworthy that TOS and BTO produced P results where by contrast, GAPS and FP led to hard decision problems. This is rather astounding, as for bribery in 2-Approval under TOS and BTO, these two models yield hardness results, too (Briskorn, Erdélyi, and Reger 2015). As for both models only votes are added for which c and at most two other candidates are approved of, the X3C argument fails in this context. Interestingly, as we will see next, CCAV with unregistered voters according to TOS becomes hard for 3-Approval whereas the P result yet remains for BTO. The reason is that for TOS, the third approval position enables that three candidates (instead of two) are potentially approved of aside from c by the same voter. With this property, the X3C proof for GAPS etc. can be extended to TOS.

Theorem 4.5. 3-Approval-(FI,X)-CCAV *and* 3-Approval-(X,X)-CCAV *are* NP*-complete for every model* $X \in \{PC, GAPS, 1GAP, FP, TOS\}$.

We omit the proof due to space constraints, as it works similarly to the proof for 2-Approval. The construction for 2-Approval can be easily adjusted by assigning the third approval to some dummy candidate in each vote in V such that each of these dummy candidates is approved exactly once. As we could observe, the complexity jumped from P to NP-completeness for TOS compared to 2-Approval. For the remaining models, the problem remains in P resp. NP-complete.

Theorem 4.6. 3-Approval-(FI,X)-CCAV *and* 3-Approval-(X,X)-CCAV *are in* P *for every model* $X \in \{BTO, TTO, CEV\}$.

Proof. It suffices to restrict ourselves to both registered and unregistered voters being partial according to the same model $X \in \{\mathrm{BTO}, \mathrm{TTO}\}$. Clearly c definitely scores in an added vote.

- For BTO, the chair adds only voters with at most three candidates in the top set. Otherwise c is not definitely approved by the voter even if c belongs to the top set, as there are at least three other candidates in the top set who are ranked in front of c for some extension. It can be easily verified that c definitely scores in a vote v iff $|C_1^v| = k$ and either $c \in C_1^v$ or c is among the $3-k$ most preferred candidates in the (totally ordered) block C_2^v ($k = 0, \ldots, 3$). Note that the three approved candidates are uniquely determined for each added vote. It can be easily shown that a successful control equals a solution of the following b-edge matching problem. We are given a multigraph $G = (C \setminus \{c\}, E)$ with vertices $C \setminus \{c\}$. Each voter in W who approves of c, d and e yields an edge (d, e) in E. As upper capacities, we are given $b(d) = score_{(C,V)}(c) + \min(score_{(C,W)}(c), \ell) - pscore_{(C,V)}(d)$ for $d \in C \setminus \{c\}$. Note that these numbers could be negative which means that d cannot be caught by adding up to ℓ voters from W approving of c. Then the matching problem trivially has no solution. Clearly a successful control equals the existence of a matching of (at least) $\min(score_{(C,W)}(c), \ell)$ edges as this means that the chair can add the maximal possible number of c voters from W without c being beaten by any d, i.e. each candidate $d \neq c$ receives at most the feasible number $b(d)$ of additional approvals from the voters added from W.
- For TTO, voters with at most two candidates in the top set increase the pscore value of each $d \neq c$ by one and should not be added [8]. Thus the chair only adds voters v with $|C_2^v| \geq 3$ and $c \in C_2^v$. Similarly to BTO, we reduce our problem to a (polynomial time solvable) matching problem.

❑

Another P result for 3-Approval is obtained for 1TOS.

Theorem 4.7. 3-Approval-(FI,1TOS)-CCAV *and* 3-Approval-(1TOS,1TOS)-CCAV *are in* P.

Proof. We only prove the more general case with partial V and W according to 1TOS. Our algorithm checks the following cases (C' denotes the totally ordered subset).

[8]There are votes where each candidate $d \neq c$ is (at least) possibly and c is definitely approved, e.g. if c is the only top set candidate. Although c is actually definitely approved, we may assume that the chair does not add such voters, as they do not change the score differences between any candidates. In other votes, all candidates except c are potentially approved (and c not), e.g. for $c \in C_3^v$ and $|C_2^v| = 2$.

- $c \in C \setminus C'$ or ($c \in C'$ and $|C \setminus C'| \geq 3$). In this case, we have $score_{(C,V \cup W)}(c) = 0$. Clearly, adding voters from W does not makes sense as c receives no definite approval at all. Hence c is a necessary winner (with not any voters being added) iff $V = \emptyset$.
- $C = C'$. This condition equals full information which is known to be in P. (Lin 2011).
- $c \in C', |C \setminus C'| = 2$. For c, only first places in (C', V) count. We call this number σ_1. As $p \notin C'$ receives full score for some completion and c can at most tie with p in V or W, c can only become a winner if c already receives full score in V, i.e. we have $\sigma_1 = |V|$. In this case, c already beats or ties with all candidates in C' and ties with all candidates in $C \setminus C'$.
- $c \in C'$ and $|C \setminus C'| = 1$. Again, c needs full score in V, but now first and second places in (C', V) contribute to the score of c. Thus c can become a winner (even without any voters being added) if and only if $\sigma_1 + \sigma_2 = |V|$, where σ_2 is the number of second positions in (C', V).

❑

As we could observe, many P results for k-Approval turned to hardness results by allowing some information to be missing. In contrast, our hope of increasing complexity will not be satisfied by Veto and 2-Veto.

Theorem 4.8. *k-Veto-(FI,X)-CCAV and k-Veto-(X,X)-CCAV are in P for $k \leq 2$ and each model $X \in PIM$.*

Proof. It suffices to prove our theorem for $X \in \{\text{FP,PC}\}$ and both registered and unregistered voters being partial according to the same model in X, as this is the most general case.

Clearly, we fix c on a veto position if c is potentially vetoed and count all $pvscore_{(C,V)}(c)$ (potential) vetoes of c. For all candidates $d \neq c$, only definite vetoes ($vscore_{(C,V)}(d)$) count for the rewritten election. The same procedure is applied to W.

For Veto and 2-Veto, the chair only adds voters definitely vetoing non-distinguished candidate(s) and he adds as many voters as possible of them.

- For Veto, c can be made a winner if and only if $\ell \geq \sum_{d \neq c} \max(0, pvscore_{(C,V)}(c) - vscore_{(C,V)}(d))$ and $vscore_{(C,W)}(d) \leq \max(0, pvscore_{(C,V)}(c) - vscore_{(C,V)}(d))$ $\forall d \in C \setminus \{c\}$, i.e. the total lead of initially better candidates can be equalized at all *and* there are enough vetoes for each $d \neq c$ in W.
- For 2-Veto, the chair adds either voters vetoing d, e ($d \neq c \neq e$) or vetoing $d \neq c$ and the other veto is in jeopardy (and particularly c is definitely not vetoed there). Moreover there may be voters for which both vetoed candidates are not definitely known (and c is not even potentially vetoed). These votes do not hurt c, but it does not make sense to add them either. Let's suppose that there are v_0 of such voters. For $\ell \geq |W| - pvscore_{(C,W)}(c) - v_0$, the chair simply adds all $|W| - pvscore_{(C,W)}(c) - v_0$ voters not vetoing c and at least one $d \neq c$ is definitely vetoed. It remains to argue for $\ell < |W| - pvscore_{(C,W)}(c) - v_0$. Now it can be easily shown that c can be made a necessary winner by adding (at most) ℓ voters if and only if the following b-edge cover problem has a solution of size (at most) ℓ:
  We are given an undirected multigraph $G = ((C \setminus \{c\}) \cup \{*\}, E)$ where the vertices are the non-distinguished candidates plus an artificial vertex $*$ representing all vetoes in jeopardy. Each voter definitely vetoing the candidates d and e yields an edge between d and e. If a voter definitely vetoes d and the other veto candidate is not definitely known, there is an edge connecting d and $*$. Besides, there are minimal capacities $b(d) := \max(0, pvscore_{(C,V)}(c) - vscore_{(C,V)}(d))$ and $b(*) = 0$. Obviously, the edge cover guarantees that each candidate receives the required number of additional vetoes from W, and the chair adds exactly the voters according to the edges of a minimal cover (in particular no more than ℓ voters).

❑

4.2 Constructive Control by Deleting Voters

First we show that some P results turn to hardness results for k-Approval ($k \leq 2$).

Theorem 4.9. *k-Approval-X-CCDV for $k \leq 2$ is NP-complete for every model $X \in \{GAPS, FP, PC, TOS, BTO, 1GAP\}$*

Proof. We will prove our theorem only for Plurality, as the proof can be easily adjusted for 2-Approval using dummy candidates. It suffices to regard $X \in \{TOS, BTO\}$. To show hardness, we reduce X3C to our problem. Given an X3C instance $(B, \mathscr{S})$, we construct the following election. Our candidate set is $C = \{c\} \cup B$ with c as the distinguished candidate. There are $3mn + 3m - n$ voters divided into three groups:

- For $1 \leq i \leq n$, there is a voter v_i who possibly votes for exactly the candidates in S_i. In terms of BTO, we have $C_1^{v_i} = S_i$ and $C_2^{v_i} = C \setminus S_i$, for TOS, we have $C^{v_i} = C \setminus \{b_{i2}, b_{i3}\}$ (with $S_i = \{b_{i1}, b_{i2}, b_{i3}\}$) as the totally ordered subset. Voter v_i votes $b_{i1} \succ \overrightarrow{C \setminus S_i}$. This makes precisely the candidates in S_i the potential scorers. [9]
- For each $j = 1, \dots, 3m$, there are exactly $n + 1 - l_j$ voters who definitely vote for b_j, where $l_j := |\{S_i :\ S_i \ni b_j\}|$.
- There are n voters who definitely vote for c. [10]

Note that the latter two groups of votes are actually complete votes and can be displayed by all nine structures.

Finally, the maximal number of voters which may be deleted is m. In our election, we have $score(c) = n$ and $pscore(b_j) = n + 1$ ($1 \leq j \leq 3m$).

It can be easily shown that the chair can make c a winner by deleting ℓ voters iff X3C has a solution. The reason is

[9] For 2-Approval, we have $C^{v_i} = C \setminus \{b_{i3}\}$ and $b_{i1} \succ b_{i2} \succ \overrightarrow{C \setminus S_i}$.

[10] For 2-Approval, we can easily adapt the proof by giving the second approval to dummy candidates in the complete votes in the second and third group. Each dummy candidate is approved of only once.

that the chair must take away one point of each candidate in B and $3m$ points in total. Obviously this is merely possible by deleting voters from the first group and if the S_i form an exact cover. In this case, c and all other candidates end with exactly m (potential) points. ❑

In contrast, the complexity remains polynomial for the remaining models.

Theorem 4.10. *k-Approval-X-CCDV for $k \leq 2$ is in P for $X \in \{TTO, CEV\}$.*

Proof. We regard only TTO. For Plurality, there are either votes with non-empty top set which can be treated like complete votes (with unique top candidate) or votes with empty top set where each candidate except c possibly scores. Thus, these votes should be deleted with highest priority. We call this number of "empty" votes n_0. Apparently, the chair's best strategy is to delete $\min(n_0, \ell)$ voters with empty top set and $\ell - \min(n_0, \ell)$ voters with non-empty top set. These latter votes can be deleted greedily by deleting a voter voting for a candidate with the currently highest pscore value. For 2-Approval, votes with at least two candidates in the top set can be treated as complete votes. Besides, there are voters with empty top set or only $d \neq c$ in the top set. In these votes, c is the only candidate whose score does not increase by one. Hence, these votes must be deleted with highest priority. The remaining kind of votes with merely c in the top set give a (p)score of 1 to each candidate and can be thus ignored. The chair deletes as many votes as possibly with c being the only disapproved candidate. If there are votes left, it can be easily shown that c can be made a necessary winner iff a certain b-edge cover problem has a solution (each $d \neq c$ must lose a certain number of points). ❑

Another polynomial time results follows almost trivially for 1TOS.

Theorem 4.11. *k-Approval-1TOS-CCDV is in P for $k \leq 2$.*

Proof. For Plurality, we have $score(c) = 0$ unless $C' = C$ is the totally ordered subset of each voter. Hence, c can only be made a winner iff the chair can delete all voters. For 2-Approval, almost the same argument holds. The only exception arises for the case $c \in C' \ \wedge\ |C'| = |C| - 1$, where c receives exactly the points from voters ranking c first in the subelection (C', V). In this case, c can be made a winner iff the chair can delete all voters not ranking c first in this subelection, as there is a $p \in C \setminus C'$ who is (possibly) approved of by each voter. In other words, c can be made a necessary winner by deleting ℓ voters iff $\ell \geq n - score(c)$ holds. ❑

Last but not least, our last result discloses that k-Veto preserves P results under partial information.

Theorem 4.12. *k-Veto-X-CCDV for $k \leq 3$ is in P for every model $X \in PIM$.*

Proof. It suffices to regard FP and PC. If c is potentially, but not definitely vetoed, we fix c as bad as possible [11], as all potential vetoes count for c in some extension. For $d \neq c$, only definite vetoes count in the rewritten election. For each of the three voting rules mentioned above, the chair deletes only voters potentially vetoing c and he deletes as many as possible (namely $\min(pvscore(c), \ell)$) of them. However the selection of these votes is different for each of these voting rules.

- For Veto, the chair simply deletes $\min(pvscore(c), \ell)$ arbitrary voters who possibly or definitely give their veto to c. c can be thus made a winner iff $pvscore(c) - \min(pvscore(c), \ell) \leq vscore(d)\ \forall d \in C \setminus \{c\}$.
- For 2-Veto, the chair deletes as many voters as possible with c being (potentially) vetoed alone and the other veto is in jeopardy. Possibly there are some vetoes for c left and the chair may delete some further voters. Then, in each step, the chair greedily deletes a voter giving a veto to c and to some $d \neq c$ with the currently highest number of vetoes.
- For 3-Veto, there are three possible kinds of voters:
  1. c is vetoed, the other two vetoes are in jeopardy.
  2. c and $d \neq c$ are vetoed, the candidate who receives the third veto is not definitely known.
  3. c, d and e ($d \neq c \neq e$) are vetoed.

  Obviously the first group of vetoes should be deleted with highest priority, as only c loses a veto then and the relative gain of c against all other candidates is one by deleting such a voter. However, one cannot decide a priori if the chair should delete a voter from the second or third group. It can be shown that the other votes to delete can be found by finding an equivalent b-edge matching, as at most two candidates are vetoed together with c and the chair may catch each $d \neq c$ at most a certain number of times in the deleted votes.

❑

[11] Again, for PC and k-Veto ($k = 2, 3$), some candidate $d \neq c$ may in this way be fixed behind c on a veto position although the veto for d has been in jeopardy in the original election.

5 Conclusion

We have defined control by adding/deleting voters under partial information and studied the complexity for the two classes of voting rules, k-Approval and k-Veto for these problems (see the tables in Section 4). Interestingly – similar to bribery – voter control for k-Approval tends to be harder than for k-Veto under partial information. As we could observe in this paper, all P results for k-Veto under full information remain P results for each model of partial information. An interesting direction for future research could be the extension of our analysis to other voting rules, e.g. one could try to obtain dichotomy results for the whole class of scoring rules. Besides, as we could see above, it may make a difference for CCAV if the unregistered voters are linear orders or not. It would be interesting to see if a similar result can be achieved for the registered voters. Further research directions are destructive control, control by partitioning voters/candidates and control problems related to *Possible Winner* which ask if the chair can make a distinguished candidate a winner for *at least one* completion of a partial profile.

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