Counting polyiamonds

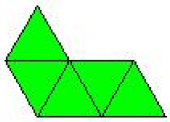
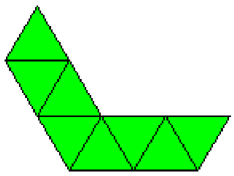
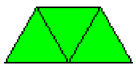
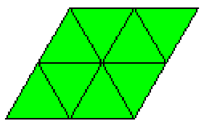
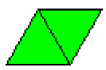
Contents

John Mason, October 2023

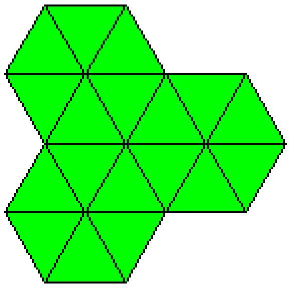
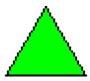
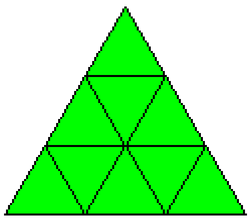
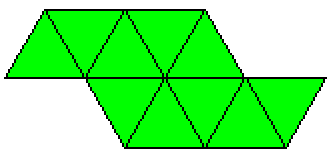
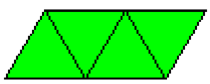
1 INTRODUCTION

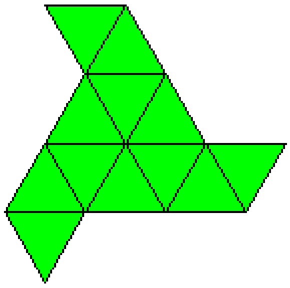
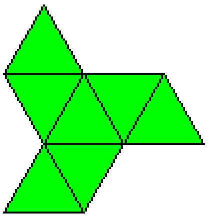
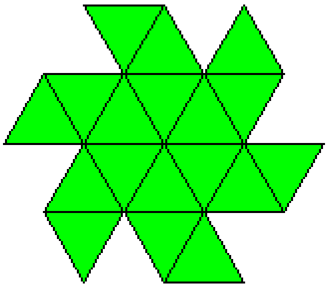
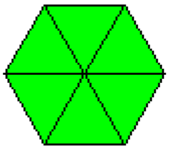
Free polyiamonds[1] are enumerated by OEIS sequence A000577. In addition, A001420 enumerates the fixed polyiamonds, and A006534 the one-sided.

The following table lists the possible symmetries. In each case, the symmetry X is defined to include those polyiamonds satisfying said symmetry, but not those that satisfy some greater symmetry that includes X.

Acronym	**Symmetry**	**Fixed multiplier**	**One-sided multiplier**
ASYM	No symmetry 	12	2
MA	A single reflective symmetry, on an axis aligned to an edge of a triangle in the underlying triangular lattice 	6	1
MU	A single reflective symmetry, on an axis not aligned to an edge of a triangle in the underlying triangular lattice 	6	1
M2	Two reflective symmetries (one aligned, one not aligned), in axes at 90° from each other, and 180° rotational symmetry. In particular, M2V: the axis of rotation coincides with a vertex of the triangular lattice:  or M2M: the axis of rotation coincides with the midpoint of the edge of a cell of the triangular lattice: 	3	1

[1] https://en.wikipedia.org/wiki/Polyiamond

M3A	Three reflective symmetries, on axes aligned to edges, at 60° from each other, and 120° rotational symmetry. The axis of rotation coincides with a vertex of a cell of the triangular lattice: 	2	1
M3U	Three reflective symmetries, on axes not aligned to edges, at 60° from each other, and 120° rotational symmetry. The axis of rotation coincides with the centre of a cell (M3UC):  or a vertex of the lattice (M3UV): 	2	1
R180	180° rotational symmetry. In particular, R180V: the axis of rotation coincides with a vertex of the triangular lattice:  or R180M: the axis of rotation coincides with the midpoint of the edge of a cell of the triangular lattice: 	6	2

R120	120° rotational symmetry. In particular, R120V: the axis of rotation coincides with a vertex of a cell of the triangular lattice:  or R120C: the axis of rotation coincides with the centre of a cell of the triangular lattice: 	4	2
R60	60° rotational symmetry 	2	2
ALL	With all symmetries (reflective symmetry in 6 axes, 60° rotational symmetry, consequent 120° and 180° rotational symmetries) 	1	1

Therefore, two simple formulae may be stated.

Formula 1:
Free(n) = ASYM(n)+MA(n)+MU(n)+M2V(n)+M2M(n)+M3A(n)+M3UC(n)+
M3UV(n)+R120C(n)+R120V(n)+R180M(n)+R180V(n)+R60(n)+ALL(n)

Formula 2:
Fixed(n) = 12*ASYM(n)+6*MA(n)+6*MU(n)+3*M2V(n)+3*M2M(n)+2*M3A(n)+2*M3UC(n)
+2*M3UV(n)+4*R120C(n)+4*R120V(n)+6*R180M(n)+6*R180V(n)+2*R60(n)+ALL(n)

As a consequence, eliminating ASYM(n), we have
Formula 3:
Free(n) = (6*MA(n)+6*MU(n)+9*M2V(n)+9*M2M(n)+10*M3A(n)+10*M3UC(n)
+10*M3UV(n)+8*R120C(n)+8*R120V(n)+6*R180M(n)+6*R180V(n)+10*R60(n)
+11*ALL(n)+Fixed(n))/12

We also have Formula 4:
Achiral(n) = MA(n)+MU(n)+M2V(n)+M2M(n)+M3A(n)+M3UC(n)+M3UV(n)+ALL(n)

Hence Formula 4a:
MA(n)+MU(n) = Achiral(n) - M2V(n)-M2M(n)-M3A(n)-M3UC(n)-M3UV(n)-ALL(n)

We can therefore deduce Formula 5:
Free(n) = (6*Achiral(n)+3*M2V(n)+3*M2M(n)+4*M3A(n)+4*M3UC(n)
+4*M3UV(n)+8*R120C(n)+8*R120V(n)+6*R180M(n)+6*R180V(n)+10*R60(n)
+5*ALL(n)+Fixed(n))/12

Also, by definition:
Chiral(n) = Free(n) – Achiral(n)
Asym(n) = Free(n) – Achiral(n) - R120C(n) - R120V(n) - R180M(n) - R180V(n) - R60(n)

Chessboard (A359689) polyiamonds may be calculated by:

CB(n) = 2*ASYM(n) + 2*MU(n) + MA(n) + M2M(n) + M2V(n) + M3A(n) + 2*M3UV(n) +
2*M3UC(n) + R180M(n) + 2*R120V(n) + R180V(n) + 2*R120C(n) + R60(n) + ALL(n)

As of July 2023, the principal sequences had been published through to the following maximum indexes:

Sequence	**Content**	**n**
A000577	Free	30
A001420	Fixed	75
A006534	One-sided	28
A030223	Achiral	60
A030224	Chiral	28

In order to calculate new values of Free(n) using Formula 5, it is necessary to enumerate each “at least rotational” symmetry through to size n.

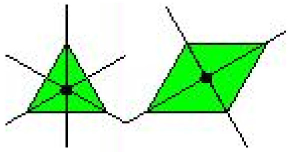
The enumeration was programmed according to the inner ring method outlined in Redelmeier’s paper[2] to which reference should be made for a more complete explanation.

[2] Counting Polyominoes: yet another attack. D. Hugh Redelmeier 1980

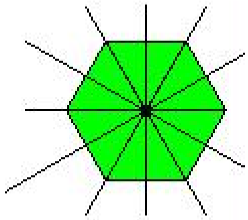
Non-trivial inner rings with rotational symmetry were identified as per the following table:

Size/Sym	ALL	R60	R180V	R180M	R120V	R120C	M3UV	M3UC	M3A	M2M	M2V
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	1	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0
15	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0
18	1	0	0	1	0	0	0	1	0	0	1
19	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	2	0	0	0	0	0	2	2
23	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	1	0	1	1	0	0
25	0	0	0	0	0	0	0	0	0	0	0
26	0	0	2	7	0	0	0	0	0	3	2
27	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0
30	2	0	7	22	1	2	1	3	0	5	4
31	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0
34	0	0	21	68	0	0	0	0	0	6	10
35	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	3	8	0	5	2	0	0
37	0	0	0	0	0	0	0	0	0	0	0
38	0	0	68	202	0	0	0	0	0	17	14
39	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0
42	2	1	207	608	9	23	5	7	3	22	24
43	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0
46	0	0	620	1811	0	0	0	0	0	45	46
47	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	32	72	0	12	7	0	0
49	0	0	0	0	0	0	0	0	0	0	0
50	0	0	1860	5402	0	0	0	0	0	69	78
51	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0

The following trivial rings (labelled T1, T2) are also needed:



T3 (see below) satisfies the criteria of being a ring (each cell touches precisely 2 other cells) even though it contains no empty space.



This table shows which rings are needed for each rotational symmetry:

Symmetry	**Required rings**				
ALL[3]	ALL				
R60	ALL	R60			
R180V	ALL	R60	M2V	R180V	
R180M	T2	M2M	R180M		
R120V	ALL	R60	M3A	R120V	M3UV
R120C	T1	M3UC	R120C		

This table shows the outputs of each program (which correspond to the non-trivial input rings):

Program	**Outputs**					
R180V	ALL	R60	M2V	R180V		
R180M	M2M	R180M				
R120V	ALL	R60	M3A	R120V	M3UV	
R120C	M3UC	R120C				
MA[4]	ALL	M3A	M2M	M2V	MA	
MU	ALL	M2M	M2V	M3UC	M3UV	MU

[3] Both R60 and ALL symmetries are enumerated by the R180V and R120V programs and so were not specifically implemented.

[4] The programs for MA and MU, implemented according to Redelmeier's paper, were not strictly necessary for Formula 5 but were used to perform consistency checks with the other programs. They were executed through to a smaller size than the other programs.

As discussed by Redelmeier, the algorithm for enumerating polyforms with rotational symmetry consists of identifying the inner rings, and then performing internal and external growth. The condition of performing internal growth is that any polyform generated from one specific inner ring must not have a more "internal" inner ring. For polyiamonds, this was implemented as follows:

a. Identify the distinct islands that the inner growth has produced.
b. No island may touch more than 2 triangles of the inner ring. If an island touches 2 triangles, they must be adjacent over an edge.

2 INTERMEDIATE RESULTS

The following tables show the results of each program.

2.1 R180V

Size	ALL	R60	R180V	M2V
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	1	0	0	0
7	0	0	0	0
8	0	0	0	1
9	0	0	0	0
10	0	0	1	1
11	0	0	0	0
12	1	0	4	1
13	0	0	0	0
14	0	0	12	2
15	0	0	0	0
16	0	0	34	5
17	0	0	0	0
18	1	1	100	6
19	0	0	0	0
20	0	0	290	11
21	0	0	0	0
22	0	0	845	20
23	0	0	0	0
24	1	2	2476	29
25	0	0	0	0
26	0	0	7256	55
27	0	0	0	0
28	0	0	21309	82
29	0	0	0	0
30	3	5	62621	150
31	0	0	0	0
32	0	0	184326	225
33	0	0	0	0
34	0	0	543317	425
35	0	0	0	0
36	4	15	1604026	617
37	0	0	0	0
38	0	0	4741921	1182
39	0	0	0	0
40	0	0	14036974	1728
41	0	0	0	0
42	7	41	41599779	3325
43	0	0	0	0
44	0	0	123417271	4859
45	0	0	0	0
46	0	0	366506222	9458
47	0	0	0	0
48	9	113	1089374334	13745
49	0	0	0	0
50	0	0	3240641098	26953
51	0	0	0	0
52	0	0	9647559400	39106

2.2 R180M

Size	R180M	M2M
1	0	0
2	0	1
3	0	0
4	1	0
5	0	0
6	2	1
7	0	0
8	7	0
9	0	0
10	18	2
11	0	0
12	52	1
13	0	0
14	145	5
15	0	0
16	416	3
17	0	0
18	1189	11
19	0	0
20	3434	7
21	0	0
22	9927	29
23	0	0
24	28848	23
25	0	0
26	84024	72
27	0	0
28	245538	63
29	0	0
30	719151	190
31	0	0
32	2111267	176
33	0	0
34	6209931	521
35	0	0
36	18298049	509
37	0	0
38	53999703	1427
39	0	0
40	159585806	1453
41	0	0
42	472223955	3948
43	0	0
44	1398965707	4153
45	0	0
46	4148828980	11016
47	0	0
48	12315907480	11922
49	0	0
50	36592769121	30927
51	0	0
52	108813465779	34262

2.3 R120V

Size	ALL	R60	R120V	M3UV	M3A
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	1	0	0	0	0
7	0	0	0	0	0
8	0	0	0	0	0
9	0	0	0	1	0
10	0	0	0	0	0
11	0	0	0	0	0
12	1	0	1	0	0
13	0	0	0	0	0
14	0	0	0	0	0
15	0	0	3	1	0
16	0	0	0	0	0
17	0	0	0	0	0
18	1	1	7	1	1
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	22	3	0
22	0	0	0	0	0
23	0	0	0	0	0
24	1	2	61	4	2
25	0	0	0	0	0
26	0	0	0	0	0
27	0	0	177	10	0
28	0	0	0	0	0
29	0	0	0	0	0
30	3	5	499	13	6
31	0	0	0	0	0
32	0	0	0	0	0
33	0	0	1438	26	0
34	0	0	0	0	0
35	0	0	0	0	0
36	4	15	4093	37	16
37	0	0	0	0	0
38	0	0	0	0	0
39	0	0	11779	75	0
40	0	0	0	0	0
41	0	0	0	0	0
42	7	41	33870	108	44
43	0	0	0	0	0
44	0	0	0	0	0
45	0	0	97880	196	0
46	0	0	0	0	0
47	0	0	0	0	0
48	9	113	283270	301	124
49	0	0	0	0	0
50	0	0	0	0	0
51	0	0	822585	535	0
52	0	0	0	0	0

2.4 R120C

Size	R120C	M3UC
1	0	1
2	0	0
3	0	0
4	0	1
5	0	0
6	0	0
7	1	0
8	0	0
9	0	0
10	2	1
11	0	0
12	0	1
13	5	1
14	0	0
15	1	1
16	13	2
17	0	0
18	4	2
19	36	2
20	0	0
21	12	5
22	97	5
23	0	0
24	38	6
25	273	4
26	0	0
27	111	12
28	758	10
29	0	0
30	327	19
31	2121	10
32	0	0
33	960	32
34	5963	22
35	0	0
36	2814	57
37	16868	25
38	0	0
39	8245	98
40	47940	53
41	0	0
42	24162	169
43	136972	64
44	0	0
45	70876	288
46	392849	129
47	0	0
48	208077	494
49	1130730	162
50	0	0
51	611540	838
52	3264498	323

2.5 MA[5]

Size	ALL	M3A	M2M	M2V	MA
1	0	0	0	0	0
2	0	0	1	0	0
3	0	0	0	0	0
4	0	0	0	0	1
5	0	0	0	0	0
6	1	0	1	0	1
7	0	0	0	0	0
8	0	0	0	1	4
9	0	0	0	0	0
10	0	0	2	1	10
11	0	0	0	0	0
12	1	0	1	1	28
13	0	0	0	0	0
14	0	0	5	2	74
15	0	0	0	0	0
16	0	0	3	5	205
17	0	0	0	0	0
18	1	1	11	6	558
19	0	0	0	0	0
20	0	0	7	11	1552
21	0	0	0	0	0
22	0	0	29	20	4307
23	0	0	0	0	0
24	1	2	23	29	12072
25	0	0	0	0	0
26	0	0	72	55	33966
27	0	0	0	0	0
28	0	0	63	82	96163
29	0	0	0	0	0
30	3	6	190	150	273272
31	0	0	0	0	0
32	0	0	176	225	780055
33	0	0	0	0	0
34	0	0	521	425	2234027
35	0	0	0	0	0
36	4	16	509	617	6419240
37	0	0	0	0	0
38	0	0	1427	1182	18496510
39	0	0	0	0	0
40	0	0	1453	1728	53436175
41	0	0	0	0	0
42	7	44	3948	3325	154735315
43	0	0	0	0	0
44	0	0	4153	4859	449033383
45	0	0	0	0	0
46	0	0	11016	9458	1305609517
47	0	0	0	0	0
48	9	124	11922	13745	3803019212
49	0	0	0	0	0
50	0	0	30927	26953	11095818518

[5] MA and MU were calculated only through to size 50. Their sum, for sizes 51 and 52, can be deduced from Formula 4a (and, as MA(51) is certainly 0, MU(51) can be deduced precisely.)

2.6 MU

Size	ALL	M3UC	M3UV	M2M	M2V	MU
1	0	1	0	0	0	0
2	0	0	0	1	0	0
3	0	0	0	0	0	1
4	0	1	0	0	0	0
5	0	0	0	0	0	2
6	1	0	0	1	0	2
7	0	0	0	0	0	5
8	0	0	0	0	1	7
9	0	0	1	0	0	12
10	0	1	0	2	1	16
11	0	0	0	0	0	36
12	1	1	0	1	1	48
13	0	1	0	0	0	96
14	0	0	0	5	2	132
15	0	1	1	0	0	264
16	0	2	0	3	5	363
17	0	0	0	0	0	737
18	1	2	1	11	6	1009
19	0	2	0	0	0	2049
20	0	0	0	7	11	2838
21	0	5	3	0	0	5739
22	0	5	0	29	20	7972
23	0	0	0	0	0	16213
24	1	6	4	23	29	22600
25	0	4	0	0	0	45975
26	0	0	0	72	55	64274
27	0	12	10	0	0	130985
28	0	10	0	63	82	183584
29	0	0	0	0	0	374781
30	3	19	13	190	150	526079
31	0	10	0	0	0	1075783
32	0	0	0	176	225	1512737
33	0	32	26	0	0	3097357
34	0	22	0	521	425	4361792
35	0	0	0	0	0	8942350
36	4	57	37	509	617	12610608
37	0	25	0	0	0	25880342
38	0	0	0	1427	1182	36544442
39	0	98	75	0	0	75068772
40	0	53	0	1453	1728	106131215
41	0	0	0	0	0	218189681
42	7	169	108	3948	3325	308819974
43	0	64	0	0	0	635353500
44	0	0	0	4153	4859	900210950
45	0	288	196	0	0	1853264711
46	0	129	0	11016	9458	2628400342
47	0	0	0	0	0	5414289076
48	9	494	301	11922	13745	7685889357
49	0	162	0	0	0	15840716774
50	0	0	0	30927	26953	22506150406

3 FINAL RESULTS

Summary of symmetry enumeration

Size	M2V	M2M	M3A	M3UV	M3UC	R120C	R120V	R180M	R180V	R60	ALL	MA	MU
1	0	0	0	0	1	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	0	0	1	0	0	1	0	0	0	1	0
5	0	0	0	0	0	0	0	0	0	0	0	0	2
6	0	1	0	0	0	0	0	2	0	0	1	1	2
7	0	0	0	0	0	1	0	0	0	0	0	0	5
8	1	0	0	0	0	0	0	7	0	0	0	4	7
9	0	0	0	1	0	0	0	0	0	0	0	0	12
10	1	2	0	0	1	2	0	18	1	0	0	10	16
11	0	0	0	0	0	0	0	0	0	0	0	0	36
12	1	1	0	0	1	0	1	52	4	0	1	28	48
13	0	0	0	0	1	5	0	0	0	0	0	0	96
14	2	5	0	0	0	0	0	145	12	0	0	74	132
15	0	0	0	1	1	1	3	0	0	0	0	0	264
16	5	3	0	0	2	13	0	416	34	0	0	205	363
17	0	0	0	0	0	0	0	0	0	0	0	0	737
18	6	11	1	1	2	4	7	1189	100	1	1	558	1009
19	0	0	0	0	2	36	0	0	0	0	0	0	2049
20	11	7	0	0	0	0	0	3434	290	0	0	1552	2838
21	0	0	0	3	5	12	22	0	0	0	0	0	5739
22	20	29	0	0	5	97	0	9927	845	0	0	4307	7972
23	0	0	0	0	0	0	0	0	0	0	0	0	16213
24	29	23	2	4	6	38	61	28848	2476	2	1	12072	22600
25	0	0	0	0	4	273	0	0	0	0	0	0	45975
26	55	72	0	0	0	0	0	84024	7256	0	0	33966	64274
27	0	0	0	10	12	111	177	0	0	0	0	0	130985
28	82	63	0	0	10	758	0	245538	21309	0	0	96163	183584
29	0	0	0	0	0	0	0	0	0	0	0	0	374781
30	150	190	6	13	19	327	499	719151	62621	5	3	273272	526079
31	0	0	0	0	10	2121	0	0	0	0	0	0	1075783
32	225	176	0	0	0	0	0	2111267	184326	0	0	780055	1512737
33	0	0	0	26	32	960	1438	0	0	0	0	0	3097357
34	425	521	0	0	22	5963	0	6209931	543317	0	0	2234027	4361792
35	0	0	0	0	0	0	0	0	0	0	0	0	8942350
36	617	509	16	37	57	2814	4093	18298049	1604026	15	4	6419240	12610608
37	0	0	0	0	25	16868	0	0	0	0	0	0	25880342
38	1182	1427	0	0	0	0	0	53999703	4741921	0	0	18496510	36544442
39	0	0	0	75	98	8245	11779	0	0	0	0	0	75068772
40	1728	1453	0	0	53	47940	0	159585806	14036974	0	0	53436175	106131215
41	0	0	0	0	0	0	0	0	0	0	0	0	218189681
42	3325	3948	44	108	169	24162	33870	472223955	41599779	41	7	154735315	308819974
43	0	0	0	0	64	136972	0	0	0	0	0	0	635353500
44	4859	4153	0	0	0	0	0	1398965707	123417271	0	0	449033383	900210950
45	0	0	0	196	288	70876	97880	0	0	0	0	0	1853264711
46	9458	11016	0	0	129	392849	0	4148828980	366506222	0	0	1305609517	2628400342
47	0	0	0	0	0	0	0	0	0	0	0	0	5414289076
48	13745	11922	124	301	494	208077	283270	12315907480	1089374334	113	9	3803019212	7685889357
49	0	0	0	0	162	1130730	0	0	0	0	0	0	15840716774
50	26953	30927	0	0	0	0	0	36592769121	3240641098	0	0	11095818518	22506150406
51	0	0	0	535	838	611540	822585	0	0	0	0	0	46407853035
52	39106	34262	0	0	323	3264498	0	108813465779	9647559400	0	0	-	-

Each of these columns is a potential OEIS sequence. If they are published, it must be decided whether or not to aggregate, for example, R180M and R180V. In the case of polyominoes, sequences exist for both the aggregated and non-aggregated symmetries.

OEIS sequences. New values are shown in bold.

Size	A000577 Free	A006534 One-sided	A030223 Achiral	A030224 Chiral
1	1	1	1	0
2	1	1	1	0
3	1	1	1	0
4	3	4	2	1
5	4	6	2	2
6	12	19	5	7
7	24	43	5	19
8	66	120	12	54
9	160	307	13	147
10	448	866	30	418
11	1186	2336	36	1150
12	3334	6588	80	3254
13	9235	18373	97	9138
14	26166	52119	213	25953
15	73983	147700	266	73717
16	211297	422016	578	210719
17	604107	1207477	737	603370
18	1736328	3471067	1589	1734739
19	5000593	9999135	2051	4998542
20	14448984	28893560	4408	14444576
21	41835738	83665729	5747	41829991
22	121419260	242826187	12333	121406927
23	353045291	706074369	16213	353029078
24	1028452717	2056870697	34737	1028417980
25	3000800627	6001555275	45979	3000754648
26	8769216722	17538335077	98367	8769118355
27	25661961898	51323792789	131007	25661830891
28	75195166667	150390053432	279902	75194886765
29	220605519559	**441210664337**	374781	**220605144778**
30	647943626796	**1295886453860**	799732	**647942827064**
31	**1905104762320**	**3810208448847**	1075793	**1905103686527**
32	**5607039506627**	**11214076720061**	2293193	**5607037213434**
33	**16517895669575**	**33035788241735**	3097415	**16517892572160**
34	**48703335271549**	**97406663946311**	6596787	**48703328674762**
35	**143722104802828**	**287444200663306**	8942350	**143722095860478**
36	**424452856377543**	**848905693723998**	19031088	**424452837346455**
37	**1254463258387951**	**2508926490895535**	25880367	**1254463232507584**
38	**3710155826313807**	**7420311597584053**	55043561	**3710155771270246**
39	**10980356266433181**	**21960712457797417**	75068945	**10980356191364236**
40	**32517440340934543**	**65034880522298462**	159570624	**32517440181363919**
41	**96355943401696748**	**192711886585203815**	218189681	**96355943183507067**
42	**285686988076989945**	**571373975690417000**	463562890	**285686987613427055**
43	**847501802385792459**	**1695003601103026354**	3668558564	**847501798717233895**
44	**2515464697928558319**	**5030929394507863293**	1349253345	**2515464696579304974**
45	**7469872299937684934**	**14939744598022104673**	1853265195	**7469872298084419739**
46	**22193002589716420893**	**44386005175498811324**	3934030462	**22193002585782390431**
47	**65965683484417184393**	**131931366963420079710**	5414289076	**65965683479002895317**
48	**196160209444205830212**	**392320418876922725260**	11488935164	**196160209432716895048**
49	**583561791526546290158**	**1167123583037251863380**	15840716936	**583561791510705573222**
50	**1736755023169729753352**	**3473510046305857479900**	33602026804	**1736755023136127726548**
51	**5170817249048194069283**	**10341634498049980284158**	46407854408	**5170817249001786214875**
52	**15400763370656316920275**	**30801526741214222140872**	98411699678	**15400763370557905220597**

Further:

Size	ASYM[6]	A359689 Chessboard
1	0	2
2	0	1
3	0	2
4	0	4
5	2	8
6	5	19
7	18	48
8	47	120
9	147	320
10	397	864
11	1150	2372
12	3197	6581
13	9133	18470
14	25796	52094
15	73713	147966
16	210256	421931
17	603370	1208214
18	1733438	3470789
19	4998506	10001186
20	14440852	28892674
21	41829957	83671476
22	121396058	242823392
23	353029078	706090582
24	1028386555	2056861981
25	3000754375	6001601254
26	8769027075	17538308071
27	25661830603	51323923796
28	75194619160	150389970179
29	220605144778	441211039118
30	647942044461	1295886198194
31	1905103684406	3810209524640
32	5607034917841	11214075937205
33	16517892569762	33035791339150
34	48703321915551	97406661554877
35	143722095860478	287444209605656
36	424452817437458	848905686432610
37	1254463232490716	2508926516775902
38	3710155712528622	7420311575386871
39	10980356191344212	21960712532866362
40	32517440007693199	65034880454806950
41	96355943183507067	192711886803393496
42	285686987099545248	571373975485413476
43	847501798717096923	1695003598705174918
44	2515464695056921996	5030929393885691265
45	7469872298084250983	14939744599875369868
46	22193002581266662380	44386005173611876593
47	65965683479002895317	131931366968834368786
48	196160209419311121774	392320418871203333485
49	583561791510704442492	1167123583053092580316
50	1736755023096294316329	3473510046288530220087
51	5170817249001784780750	10341634498096388138566
52	15400763370439440930920	-

[6] At the time of writing, there is no OEIS sequence for this column

4 CONCLUSIONS

The approach is heavily dependent on the calculation of Fixed(n) and Russell's enumeration of achiral polyiamonds.

The level of confidence in the results depends mainly on the following factors:

1. Consistency with existing enumerations of free and one-sided polyiamonds.
2. The built-in check of Formula 5. If the sum is not a multiple of 12, the calculation is wrong.
3. Consistency with results of MA and MU symmetry enumerations, not used in the calculation, but useful during the debugging phase.

The following are the most important runtimes:

1. R180V(52) – 9hr
2. R180M(52) – 5hr
3. MA(50) – 1hr25min
4. MU(50) – 5hr30min

Runtimes are more or less proportional to the number of polyiamonds enumerated. In most cases, the times are roughly triplicated for each increase in size of 2.

For any further information, or to point out errors, please contact me at this email address: TheIllustratedPolyomino (at) gmail (dot) com.