X

JIMS 15(1923)

2411
217
1296
1297

The Operator $(xD)^n$

The object of this note is to prove that

$$(xD)^n = x^n D^n + \Sigma(n-1)\, x^{n-1} D^{n-1} + \Sigma(n-2)\,\Sigma(n-2)\, x^{n-2} D^{n-2} + \ldots$$
$$+ \Sigma(n-r)\,\Sigma(n-r) \ldots\ldots \Sigma(n-r).\; x^{n-r} D^{n-r} + \ldots + xD \quad (1)$$

the number of Σ's in the co-efficient of $x^{n-r} D^{n-r}$ being r, and these indicate multiple summation. (See page 55, Vol. XIV. *J. I. M. S.*).

The co-efficients are easily calculated by the help of the following table, which is self explanatory —

	217	2411	1296		1297		
n	Σn	$n\Sigma n$	$\Sigma n\Sigma n$	$n\Sigma n\Sigma n$	$\Sigma\Sigma n\Sigma n$	$n\Sigma n\Sigma n\Sigma n$	$\Sigma n\Sigma n\Sigma n\Sigma n$
1	1	1	1	1	1	1	1
2	3	6	7	14	15	30	31
3	6	18	25	75	90	270	301
4	10	40	65	260	350	1400	1701
5	15	75	140	700	1050	5250	
6	21	126	266	1596	2646		
7	28	196	462	3234	5880		
8	36	288	750	6000	11880.		

e. g. $\quad (xD)^4 = x^4 D^4 + 6x^3 D^3 + 7x^2 D^2 + xD.$

$\quad (xD)^5 = x^5 D^5 + 10x^4 D^4 + 25x^3 D^3 + 15x^2 D^2 + xD.$

Now obviously from the way in which the summation takes place, as illustrated in the table, we have the general formula of reduction

$$\Sigma(n-r)\,\Sigma(n-r)\ldots\Sigma(n-r) = (n-r)\,\Sigma(n-r)\,\Sigma(n-r)\ldots\Sigma(n-r)$$
$$+ (n-r-1)\,\Sigma(n-r-1)\,\Sigma(n-r-1)\ldots\ldots\Sigma(n-r-1)$$
$$+ \ldots\ldots + 2\,\Sigma 2\,\Sigma 2 \ldots\ldots \Sigma 2 + 1\,\Sigma 1\,\Sigma 1\,\Sigma 1 \ldots\ldots \Sigma 1, \quad (2)$$

the number of Σ's on the left is r, and in each of the terms on the right is $r-1$. This may also be written,

$$\Sigma(n-r)\,\Sigma(n-r)\ldots\ldots\Sigma(n-r) = (n-r)\,\Sigma(n-r)\,\Sigma(n-r)\ldots\ldots$$
$$+ \Sigma(n-r-1)\,\Sigma(n-r-1)\ldots\ldots\Sigma(n-r-1) \quad (3)$$

the number of Σ's on the left and in the second term on the right is r, and in the first term, it is $r-1$. Remembering (3), it is very easy to prove (1) by induction. For, from (1),

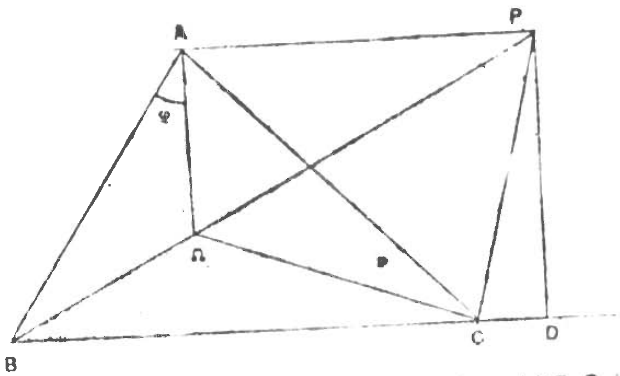
$$(xD)^{n+1} = x^{n+1} D^{n+1} + \{ n + \Sigma(n-1) \}\, x^n D^n + \ldots\ldots\ldots\ldots$$
$$+ x^{n-r+1} D^{n-r+1} \,\{ \Sigma(n-r)\,\Sigma(n-r)\ldots\ldots\Sigma(n-r)$$
$$+ (n-r+1)\,\Sigma(n-r+1)\,\Sigma(n-r+1)\ldots\Sigma(n-r+1) \} + \ldots + xD$$

there being r Σ's, in the first and $(r - 1)$ in the second term in the co-efficient of $x^{n-r+1}\, p^{n-r+1}$, whence (1) follows by induction.

C. KRISHNAMACHARI

A Geometrical Proof of the Property $\cot \omega = \cot A + \cot B + \cot C$.

Let $B\Omega$ produced cut the parallel through A to BC at P and let PD be perpendicular to BC produced.



Then, $\angle$ PCD $= \angle$ A (from construction since APC Ω is cyclic).

$$\therefore \quad \cot A = \cot PCD = CD/PD$$

$$\text{But} \quad \cot B + \cot C = BC/PD.$$

$$\therefore \quad \cot A + \cot B + \cot C = BD/PD$$

$$= \cot \omega.$$

V. V. SATHYANARAYANA MURTHY.
