

Chester SMA 18 (1952)

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limitations imposed above would require the formation of $m(3m + 2)$ two-factor products. For illustration let $2m = 78$ corresponding to a number of 702 or 780 digits. Hence $m = 39$, $m(m + 1) = 1560$, $m(3m + 2) = 4641$, and $4641/1560 = 2.975 = 2.975$, roughly 3 to 1.

The author hopes that it is not inappropriate to call attention to two of his earlier papers because they preceded and led up to the present contribution. The first one has the title "Many-Figure Approximations to $\sqrt{2}$, and Distribution of Digits in $\sqrt{2}$ and $1/\sqrt{2}$." The second paper bears the title "Approximations Exceeding 1300 Decimals for $\sqrt{3}$, $1/\sqrt{3}$, $\sin(\pi/3)$ and Distribution of Digits in Them." Both appeared in volume 37 of the *Proceedings of the National Academy of Sciences*, p. 63-67 for January and 443-447 for July 1951.

Please enter 2

TABLE I

$\sqrt[3]{2} =$

1, 23992	10498	94873	16476	72106	07278	22835	05702	51464	70150
78900	81975	11215	52996	76513	95918	37293	96562	43625	50941
54310	25603	56156	65259	39902	40406	13737	22845	91103	01269
35524	69606	42616	62500	09774	74526	56548	03068	67185	40551
86892	45872	51676	41993	73709	69509	83827	83161	39915	51293
13695	36618	39474	63448	57657	03031	19095	89598	47411	05981
16290	70535	90816	47801	14735	21325	48477	12978	80242	20858
20532	57972	52666	22026	69005	66560	81994	71562	81764	05060
66482	67735	72670	41948	62076	21442	96569	42050	79319	17244
14809	20448	23284	01274	70321	96428	20812	01905	71418	89964
59998	31750	38018	88689	59420	20559	22021	15472	99738	48802
60736	36974	17887	79215	79846	75099	53963	00782	60959	62420
34832	38660	13985	73634	33909	73712	65279	95991	96996	83779
13168	16815	44288	50279	65152	92781	07679	71400	20406	05674
80393	85610...								

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1, 2, 5, 9, 9, 2, 1, 6, 4, ... ✓

$\sqrt[3]{4} =$

1, 58740	10519	68199	47475	17056	39272	30826	03914	93327	89985
30098	08285	76182	52165	05624	21917	32735	41213	26222	09570
22934	76168	13220	17903	49765	98981	52752	27814	00110	44541
14661	93755	18278	56243	68732	90512	48507	22923	37449	77427
53646	07167	53710	66638	18475	71175	23515	75061	70456	02727
92426	50876	78157	14469	40695	09962	44845	05096	38778	70022
31729	23226	09424	96708	47966	04499	14155	08010	99401	07853
08121	03380	12450	30858	26825	36745	90167	17639	48817	72523
60208	91295	49173	22106	54798	94788	52586	81912	02688	44539
18502	08494	06558	44314	27295	56827	07329	92965	17272	25319
09901	90636	47232	32845	67870	18093	53877	31289	77245	94919
39332	05408	57255	22667	62209	81783	23741	93326	27117	91189
76413	98026	34411	98200	33655	19420	92292	45789	89600	30635
04131	23593	93069	58065	81392	11620	17755	50328	20345	26672
12584	8993...								

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$\sqrt[3]{3} =$

1, 41224	95703	07408	38232	16383	10780	10958	83918	69253	49935
05775	46416	19454	16875	96829	99733	98547	55479	70564	52566
86835	08085	44895	49966	42542	39461	10259	71486	89501	57185
23722	70903	32023	84759	84450	61085	54002	72600	88145	49887
27513	67355	35246	78660	74715	68843	92233	18918	20170	38998
23822	33212	96166	35508	52626	73491	33501	66545	48957	88175
85527	41755	93363	13187	41467	20060	46384	66647	56937	42641
97555	74942	49068	20810	91267	12359	06265	76368	96463	73616
17821	65584	25874	82385	65952	35871	90319	61040	71395	30602
81028	53508	44363	80351	94550	133(8)				

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TABLE 1 (Continued)

$\sqrt[3]{9} =$

2.08008	38230	51904	11453	00568	24357	88538	63378	05340	37326
21096	97591	08020	01063	11397	26877	36060	56636	79075	74807
28671	59208	65715	20538	90780	65514	32406	43515	59564	14938
59704	47434	29752	21629	41129	64022	76685	76697	47769	14303
36888	04813	53602	15995	45811	69669	65360	66894	99477	09982
60332	10379	04324	02956	47501	09882	07691	29996	31382	19077
69406	76055	30360	76927	98678	41312	93359	16330	86534	76940
86927	21478	72409	34971	04836	55139	43658	20070	26049	17311
41474	31700	98780	47273	12794	42970	02343	78721	53048	69234
55801	22409	06823	44009	22859	951(9)				

TABLE 2

n	$\sqrt{2}$		$\sqrt{4}$	
	x_n^2	P_n	x_n^2	P_n
50	4.4	0.8799	3.6	0.9337
100	5.4	0.7932	4.8	0.8452
150	7.46	0.5896	6.8	0.6573
200	12.7	0.1903	8.3	0.5044
250	10.	0.3630	9.28	0.4231
300	9.53	0.4023	8.6	0.4734
350	6.971*	0.6103	9.657*	0.3910
400	11.6	0.2404	8.3	0.5044
450	12.93	0.1797	7.95	0.5396
500	11.04	0.2786	10.88	0.2918
550	10.72	0.3043	10.290	0.3401
600	11.23	0.2628	11.46	0.2465
650	9.6	0.3968	10.584*	0.3160
700	10.914*	0.2889	10.171*	0.3499

TABLE 3

n	$\sqrt[3]{3}$		$\sqrt[3]{9}$	
	x_n^2	P_n	x_n^2	P_n
50	6.4	0.6987	17.2	0.0467
100	5.	0.8279	14.	0.1311
150	8.93	0.4515	10.6	0.3093
200	9.3	0.4214	4.4	0.8799
250	6.16	0.7233	6.08	0.7315
300	6.46	0.6919	6.2	0.7192
350	7.485*	0.5877	10.4	0.3312
400	7.35	0.6016	9.7	0.3886
450	8.17	0.5169	9.02	0.4442

4. A procedure for

The homogeneous eq

is satisfied if

$$y = p^2 - t$$

It remains only to so

This is a Pell equation if one initial solution is

Designating by p_1, q_1 number of solution by t

The solution of (1) is We shall give here two properties of squares ex Example 1. The equ

$$x = 2,$$

subject to the condition

Some of the solutions

The recurrence formul

For $p = 7, q = 2$, we Example 2. The equ