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The New South Wales Institute of Technology
DEPARTMENT OF APPLIED MATHEMATICS

6 DEC '74

Dr N.J.A. Sloane,
Bell Laboratories,
600 Mountain Avenue,
Murray Hill, New Jersey, 07974,
UNITED STATES OF AMERICA.

Dear Dr Sloane,

Thank you for your copy of Supplement I to your Handbook of Integer Sequences.

Three other integer sequences are enclosed for your consideration, though they might not interest you or they might not fit into your scheme since they are more useful as two and three dimensional arrays.

I refer to the

(i) Fermatian sequences, $\{\underline{q}_n\}$, defined by

(1) $\underline{q}_n = 1 + q + q^2 + \cdots + q^{n-1}$, $n \geq 1$, $q \in Z_+$.

just the integer sum

These are useful as examples of divisibility sequences as in Ward [3];

(ii) Pellian sequences, $\{W^{(r)}_{s,n}\}$, defined by

(2)
$$\begin{cases} W^{(r)}_{s,n} = \sum_{j=1}^{r} \binom{r}{j} D^{r-j} W^{(r)}_{s,n-j}, & n > r, \\ W^{(r)}_{s,n} = \delta_{s,n+1} & \begin{cases} s \leq n+1 \\ 1 \leq n < r \end{cases} \\ W^{(r)}_{s,r} = D^{r-1} & \\ W^{(r)}_{s,n} = D\, W^{(r)}_{s-1,n} + W^{(r)}_{s-1,n-1} & \\ W^{(r)}_{r+s,n} = w^r\, W^{(r)}_{s,n} & \end{cases}$$

in which $D = [w]$, $w^r = m = D^r + d$, $m, d, D \in Z_+$.

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These provide solutions

$$(3)\qquad x_{ik} = \sum_{j=0}^{r-k} \binom{r-k}{j} D^j W^{(r)}_{i,n+j+k}$$

$$(i, k = 1, 2, \cdots, r)$$

for the generalized Pellian Diophantine equations

$$(4)\qquad \begin{vmatrix} x_{rr} & mx_{1r} & mx_{2r} & \cdots & mx_{r-1,r} \\ x_{r-1,r} & x_{rr} & mx_{1r} & \cdots & mx_{r-2,r} \\ & & \cdots & & \\ x_{1r} & x_{2r} & x_{3r} & \cdots & x_{rr} \end{vmatrix} = \pm 1$$

as in Bernstein [3];

(iii) __?__ sequences, $\{V^{(r)}_{n,m}\}$, defined by

$$(5)\qquad \begin{cases} V^{(r)}_{n,m} = V^{(r)}_{n-1,m-1} + P_{r,r-m} V^{(r)}_{n-1,r-1} & \begin{cases} 0 < m < r \\ n > 0 \end{cases} \\ V^{(r)}_{n,m} = 2 V^{(r)}_{n,0} & \begin{cases} m \geq r \\ n > 0 \end{cases} \\ V^{(r)}_{n,m} = 0 & \begin{cases} m < 0 \\ n < 0 \end{cases} \end{cases}$$

$$V^{(r)}_{0,m} = P_{r,r-m}$$

$$V^{(r)}_{n,0} = P_{rr} U^{(r)}_n$$

where $U^{(r)}_n = \sum_{j=1}^{r} P_{rj} U^{(r)}_{n-j}$, $\begin{cases} U^{(r)}_n = 0 \text{ for } 1 \leq n < r \\ U^{(r)}_r = 1 \end{cases}$

and the P_{rj} are arbitrary integers.

These provide contractions of Bernoulli's iteration for the solution of polynomial equations as in Shannon [2] ; that is, for the shift operator E,

$$E^r = \sum_{m=0}^{r-1} P_{r,r-m} E^m \quad \text{implies that} \quad E^{r+n} = \sum_{m=0}^{r-1} V_{n,m}^{(r)} E^m.$$

Examples of these from programs run on an HP9830 A are enclosed.

References :

1. Leon Bernstein, "The Jacobi-Perron Algorithm : Its Theory and Application", Lecture Notes in Mathematics, 207, Springer-Verlag, Berlin - Heidelberg-New York, 1971.

2. A.G. Shannon, "The Jacobi-Perron Algorithm and Bernoulli's Iteration", The Mathematics Student, in press.

3. Morgan Ward, "Divisibility Sequences", Bulletin of the American Mathematical Society, Vol. 42 (1936), 834-845.

With best wishes,

Yours sincerely,

Tony Shannon

A.G. SHANNON.