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R.G. Wilson, ✓
letter (long)

add to many
list not to A7138,
that is just a placeholder

Mira Asked for [illegible] (A7138)

A5596

A7348 – 7355

25 August 1993

Neil James Alexander Sloane
% Room 2C-376, Mathematics Research Center
AT&T Bell Telephone Laboratories Inc.
Murry Hill, New Jersey 07974
201+582-3000, ext. 2005

Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequences for inclusion in your forthcoming second edition of the above.

The first is the sequence of those primes for which the reciprocal ~~of the primes which~~ has ~~have~~ for the first time a recurring period of length **n**. This series is also the first factor of R_n (Repunits), for **n**>3, not previously used as a factor in R_j, j<n & j|n. Furthermore, this sequence is the first occurrence of order **n**. "[T]he order of such an X modulo d, that is the minimal value of n for which $X^n \equiv 1$ (mod d)." [Bruce] In this case we are using d=10 (X?) for the decimal system. The sequence is infinite, with no gaps, begins: 3, ~~R2 #~~ 11, 37, 101, 41, 7, 239, 73, 333667, 9091, 21649, 9901, 53, 909091, 31, 17, 2071723, 19, ~~R19 #~~ 1111111111111111111, 3541, 43, 23, ~~R23 #~~ 11111111111111111111111, 99990001, 21401, 859, 757, 29, 3191, 211, 2791, 353, 67, 103, 71, 999999000001, 2028119, 909090909090909091, 900900900900990990990991, 1676321, 83, 127, 173, 89, 238681, 47, 35121409, 9999999900000001, 505885997, 251, 613, 521, 107, 70541929, 1321, 7841, 21319, 59, 2559647034361, 61, 733, 9090909090909090909090909091, 10837, 19841, 162503518711, 599144041, 493121, 28559389, 277, 4147571, 241573142393627673576957439049, 3169, 12171337159, 7253, 151, 7228170363222379041, 5237, 157, 317, 5070721, 163, 2670502781396266997, 3367147378267, 226549, 262533041, 57009401, 4003, 617, 497867, 29611, 547, 1289, 900900900900900900900900900990990990990990990990990991,

A7138

%N Smallest ~~Largest~~ new factor of $10 sup n -1$.

%R CUNN.

%A njas, mb, rgw

%I 1,1

6299, 191, 97, 12004721, 197, 199, 60101, 45315301818166132345551908 41,

52986961, 1031, 158081, 30703738801, 909090909090909090909090909090-

9090909090909090909091, 643, 109, 1192679, 331, 3055705151864730 7, 113,

227, 21319, 31511, 349, 240396841140769, 1889, 923441, 100009999998 99-

9899990000001 0001, 15973, 81131, 1811791, 2049349, 751, 5274739, 18797,

• • •.

This second sequence is the above numbers less one divided by n and it is as follows: 2, 5, 12, 25, 8, 1, 34, 9, 37074, 909, 1968, 825, 4, 64935, 2, 1, 121866, 1, 58479532163742690, 177, 2, 1, 48309178743961352-6570, 4166250, 856, 33, 28, 1, 110, 7, 90, 11, 2, 3, 2, 27777750000, 54814, 23923444976076555, 23100023100025410025410, 41983, 2, 3, 4, 2, 5304, 1, 747264, 208333331250000, 10324204, 5, 12, 10, 2, 1306332, 24, 140, 374, 1, 4338384804, 1, 12, 14662756598240469208211143695, 172, 310, 2500054134, 9077940, 7360, 419991, 4, 59251, 34024386252623615996754568-88, 44, 166730646, 5711487482898, 2, 9510750477926040, 68, 2, 4, 63384, 2, 32567107090198378, 40568040702, 2697, 3088624, 662900, 46, 7, 5594, 329, 6, 14, 968710646130000968710646130006558171074300106558171074 30, 67, 2, 1, 123760, 2, 2, 601, 448666354635308241045068 40, 519480, 10, 1520, 292416560, 85763293310463121783876500857632933104631217838765, 6, 1, 10942, 3, 275288752420246, 1, 2, 187, 274, 3, 2054673855904, 16, 7760, 833416666658332499916666667 50, 132, 665, 14730, 16527, 6, 41863, 148, • • •.

~~This sequence taken from~~ The above has a one at **n** for **n** equaling: 6, 16, 18, 22, 28, 46, 58, 60, 96, • • •, and ~~this sequence has~~ a two for **n** equaling: 1, 15, 21, 33, 35, 41, 44, 53, 75, 78, 81, 95, 98, 99, • • •. These will show up later (as the above cited numbers times **n** plus

1) in the series ten sequence entitled the Primes of **n** Cyclic.

And this sequence, numbered three, is the first occurrence from the second sequence and it is a bit harder to generate and to explain from the other ones just stated. For the most part, the number presented at the $n^{\underline{th}}$ position will be the first number in sequence ten for the $n^{\underline{th}}$ Cyclic. In any case, you will find it in the $n^{\underline{th}}$ Cyclic series. Often it is the second number which qualifies but this is not always the case, as with **n**=121 were the third entry is necessary to meet the criteria. Recall that an **n** Cyclic is defined as **n=(p-1)/Order(p)**, **n** will always be a whole number of any prime **p** and will also be a divisor (not necessarily a prime or a proper factor) of **p-1**. Therefore **n*Order(p)+1=p**. However, if there exists a **k<n** for which the order of **k*Order(p)+1=p**, then **p** cannot be the first occurrence at **n**. It is as follows: 6, 1, 34, 13, 2, 91, 30, 5, 8, 52, 32, 3, 184, 92, 766, 118, 554, 137, 852, 156, 512, 226, 482, 55, 4, 275, 2128, 27, 1332, 377, 3666, 449, 26, 7, 788, 267, 6154, 362, 870, 229, 2900, 161, 424, 72, 1972, 87, 1818, 406, 1758, 803, 908, 2374, 2552, 405, 1774, 3083, 1368, 375, 1034, 514, 456, 144, 3472, 1995, 818, 1648, 94, 77, 3118, 169, 10346, 655, 1956, 1229, 3892, 1780, 2496, 347, 340, 906, 3096, 859, 5082, 149, 3052, 453, 964, 1561, 612, 31, 11032, 1631, 10306, 3498, 6134, 1275, 6438, 74, 8572, 2266, 6312, 351, 31596, 732, 3092, 1528, 6564, 356, 28008, 29, 4546, 535, 752, 407, 5332, 1137, 484, 4381, 25464, 813, 24730, 635, •••. If you compare this sequence modified by the above stated formula, you will arrive at the fifth sequence except as noted. This situation occurs for the following **n**s: 6, 10, 14, 15, 17, 20, 21, 27, 30, 31, 32, 37, 44, 47, 50, 52, 54, 55, 56, 60, 63, 64, 69,

71, 72, 75, 76, 78, 80, 81, 82, 83, 88, 91, 92, 93, 94, 96, 97, 99, 100, 101, 103, 104, 107, 109, 111, 115, 116, 118, 121, 124, 125, 130, 133, 135, 138, 143, 147, 149, 150, 151, 152, 154, 155, 156, 157, 159, 161, 162, 163, 166, 167, 168, 170, 171, 178, 184, 186, 190, 191, 193, 194, 195, 196, 197, 198, 200, 201, 204, 205, 206, 207, 208, 211, 214, 216, 218, 221, 230, 231, 232, 236, 239, 240, 242, 243, 244, 247, 249, •••. And for this series, the **k**s: 4, 1, 11, 2, 9, 11, 1, 2, 2, 18, 9, 1, 11, 3, 2, 21, 8, 1, 2, 1, 3, 3, 15, 15, 7, 21, 12, 4, 30, 3, 2, 3, 72, 3, 2, 2, 2, 2, 10, 1, 1, 5, 10, 18, 5, 9, 27, 49, 2, 3, 1, 2, 5, 4, 7, 6, 8, 1, 145, 80, 30, 2, 54, 4, 2, 6, 3, 16, 2, 35, 18, 12, 11, 78, 2, 15, 24, 6, 1, 4, 38, 1, 2, 9, 6, 2, 2, 30, 9, 4, 1, 6, 1, 6, 187, 4, 16, 26, 1, 6, 30, 34, 40, 61, 2, 2, 3, 1, 1, 1, •••.

This fourth sequence in fact is taken from the sequence representing the number of primitive co-factors of R_n. And except for **n** =3, it is also the number of primes having a Period length of **n**, and it is as follows: 1, 2, 1, 1, 2, 2, 2, 2, 1, 1, 2, 1, 3, 1, 2, 2, 2, 2, 1, 2, 3, 3, 1, 1, 3, 2, 2, 3, 5, 3, 3, 5, 2, 3, 3, 1, 3, 1, 1, 2, 4, 3, 4, 3, 2, 4, 2, 1, 2, 3, 4, 2, 4, 2, 3, 2, 3, 2, 2, 3, 7, 1, 5, 4, 2, 2, 3, 3, 3, 2, 2, 2, 3, 3, 3, 2, 4, 4, 6, 2, 5, 2, 3, 2, 3, 3, 3, 2, 5, 3, 6, 3, 1, 3, 5, 4, ? it is between 3 and 8 for **n** =97 ?, 2, 4, 4, •••. And this sequence is where the above sequence above is equal to one and therefore; the index of the Only Primes with Period length of **n**, it will include the Prime Repunits, and it is as follows: 1, 2, 3, 4, 9, 10, 12, 14, 19, 23, 24, 36, 38, 39, 48, 62, 93, 106, 120, 134, 150, 196, 294, 317, 586, 597, •••?, 1031, •••. Once the question marks occur in the preceding lists, you are not guaranteed that there are not any intervening entries after that point.

The fifth sequence are those primes which have for the first time a period of length **n** =(**p**-1)/order(**p**) and thus of **n** cyclic. The sequence is infinite, with no gaps, and it begins: 7, 3, 103, 53, 11, 79, 211, 41, 73, 281, 353, 37, 2393, 449, 3061, 1889, 137, 2467, 16189, 641, 3109, 4973, 11087, 1321, 101, 7151, 7669, 757, 38629, 1231, 49663, 12289, 859, 239, 27581, 9613, 18131, 13757, 33931, 9161, 118901, 6763, 18233, 1409, 88741, 4003, 5171, 19489, 86143, 23201, 46309, 98801, 135257, 271, 2531, 4201, 77977, 21751, 61007, 15361, 27817, 8929, 168211, 2689, 53171, 108769, 6299, 5237, 401029, 11831, 115589, 2161,

2161, 142789, 90947, 142501, 4637, 192193, 6163, 26861, 30161, 100927, 6397, 176459, 12517, 259421, 38959, 83869, 57641, 54469, 2791, 228593, 1933, 68821, 65707, 582731, 101281, 90017, 7253, 251263, 129001, 5051, 35803, 2368589, 25169, 324661, 161969, 400823, 38449, 131891, 3191, 429127, 59921, 84977, 46399, 356501, 9397, 56629, 4013, 3030219, 97561, 1065527, 77471, 475273, 18973, 819251, 177787, 408433, 160001, 321469, 10271, 480509, 15973, 28463, 99563, 111781, 544001, 1672771, 51199, 4079651, 7841, 615607, 110477, 69499, 492769, 321611, 618311, 732943, 18797, 312007, 119551, 969421, 61561, 205633, 139987, 139501, 14197, 24179, 172853, 3270313, 134401, 498779, 162649, 261127, 294053, 107251, 71879, 2009011, 70729, 2586377, 418031, 280099, 10837, 1106509, 143551, 2403451, 473089, 3541, 957107, 1613149, 121321, 51767, 591319, 345139, 785129, 126541, 4093, 70313, 117877, 704593, 77711, 1065017, 773569, 905171, 196523, 522211, 17837, 74861, 82963, 1038383, 75401, 639181, 351077, 3793259, 39373, 85691, 224129, 1427887, 56369, 548417, 148471, 79337, 1176601, 1089709, 206083, 845381, 74521, 1919149, 59951, 947833, 133321, 4444753, 265957, 158777, 641761, 31051, 258319, 4741577, 1232797, 4545193, 329591, 725341, 1348849, 8097217, 478999, 1903501, 273997, 49297, 77351, 233743, 44641, 3931229, 972599, 1098847, 236681, 784981, 248707, 459421, 629921, 861541, 676751, ...

Or we may sort the above in numerical order to produce the following: 7, 11, 13, 37, 41, 53, 73, 79, 101, 103, 137, 211, 239, 271, 281, 353, 449, 641, 757, 859, 1231, 1321, 1409, 1889, 1933, 2161, 2393, 2467, 2531, 2689, 2791, 3061, 3109, 3191, 3541, 4003, 4013, 4093, 4201, 4637, 4649, 4973, 5051, 5171, 5237, 6163, 6299, 6397, 6763, 7151, 7253, 7669, 7841, 8779, 8929, 9091, 9161, 9397, 9613, 9901, 10271, 10837,

11087, 11831, 12289, 12517, 13757, 14197, 15361, 15973, 16189, 16763, 17837, 18131, 18233, 18797, 18973, 19489, 19841, 21319, 21401, 21649, 21751, 23201, 23311, 24179, 25169, 25601, 26861, 27581, 27817, 27961, 28463, 29611, 30161, 31051, 31511, 33931, 34849, 35803, 38237, 38449, 38629, 38861, 38959, 39373, 42043, 43037, 44641, 45613, 46309, 46399, 49297, 49663, 51199, 51767, 52009, 52579, 53171, 54469, 56369, 56629, 57641, 59281, 59921, 59951, 60101, 61007, 61561, 62003, 62921, 63799, 63841, 65707, 68389, 68821, 69499, 69857, 70313, 70729, 71879, 72559, 74521, 74687, 74861, 75401, 77351, 77471, 77711, 77977, 79337, 80173, 81131, 82963, 83869, 84977, 85691, 86143, 87211, 88741, 90017, 90679, 90947, 97561, 98641, 98801, 99563, 100927, ···.

The sixth sequence is the same as the unsorted third, but this time, we are not restricted to the primes (the composites are **highlighted** and <u>underlined</u>). It is as follows: 7, 13, 103, 53, 11, 79, 211, 41, 73, 281, 353, 37, 2393, 449, **<u>91</u>**, **<u>33</u>**, 137, 2467, **<u>7107</u>**, 641, 3109, 4973, 11087, 1321, 101, 7151, 7669, 757, 38629, 1231, 49663, 12289, 859, 239, **<u>561</u>**, 9613, 18131, 13757, **<u>703</u>**, 9161, 118901, 6763, **<u>259</u>**, 1409, **<u>451</u>**, 4003, 5171, 19489, **<u>99</u>**, 23201, 46309, 98801, 135257, 271,

2531, 4201, 77977, 21751, 61007, 15361, 27817, 8929, 168211, 2689, 53171, 108769, 6299, 5237, 215143, 11831, 115589, 2161, 142789, 90947, 142501, 4637, 192193, 6163, 26861, **<u>481</u>**, 110927, **<u>657</u>**, 176459, 12517, 259421, 38959, 83869, 57641, 54469, 2791, 228593, 1933, 68821, **<u>2821</u>**, 582731, **<u>1729</u>**, 90017, 7253, 251263, 129001, 5051, 35803, 2368589, 25169, 324661, 161969, 400823, 38449, 131891, 3191, 429127, 59921, 84977, 46399, 356501, 9397, 56629, 4013, 3030217, 97561, 1065527, 77471, 475273, 18973, 819251, 177787, 408433, 160001, **<u>52633</u>**, **<u>5461</u>**, 480509, **<u>15841</u>**, 28463, 99563, 111781, 544001, 1672771, **<u>24013</u>**, 4079651, 7841, 615607, **<u>6533</u>**, 69499, 492769, 321611, **<u>321201</u>**, 732943, 18797, 312007, 119551, 969421, 61561, 205633, **<u>1233</u>**, 139501, 14197, 24179, 172853, **<u>12403</u>**, 134401, 498779, 162649, 261127, 294053, 107251, 71879, 2009011, 70729, 2586377, 418031, 280099, 10837, 1106509, 143551, 2403451, 473089, 3541, 957107, 1613149, 121321, 51767, 591319, 345139, 785129, 126541, 4093, 70313, 117877, 704593, 77711, 1065017, 773569, 905171, 196523, 522211, 17837, 74861, 82963, **<u>19503</u>**, 75401, 639181, 351077, 3793259, 39373, 85691, 224129, **<u>4141</u>**, 56369, 548417, 148471, 79337, 1176601, 1089709, **<u>99297</u>**, 845381, 74521, 1919149, **<u>6541</u>**, 947833, 133321, **<u>2165801</u>**, 265957, **<u>51291</u>**, 641761, 31051, 258319, **<u>909</u>**, 1232797, 4545193, 69921, 725341, 1348849, 8097217, 478999, 1903501, 273997, 49297, 77351, 233743, 44641, 3930229, 972599, 1098847, 236681, **<u>14701</u>**, 248707, 459421, 629921, **<u>7471</u>**, 676751, ···.

Or we may sort the above to produce the following: 7, 11, 13, **<u>33</u>**, 37, 41, 53, 73, 79, **<u>91</u>**, **<u>99</u>**, 101, 103, 137, 211, 239, **<u>259</u>**, 271, 281, 353, 449, **<u>451</u>**, **<u>481</u>**, **<u>561</u>**, 641, **<u>657</u>**, **<u>703</u>**, 757, 859, 1231, **<u>1233</u>**, 1321, 1409, **<u>1729</u>**, 1933, 2161, 2393, 2467, 2531, 2689, 2791, **<u>2821</u>**, 3109, 3191, 4003, 4013, 4201, 4637, 4973, 5051, 5171, 5237, **<u>5461</u>**, 6163, 6299, **<u>6533</u>**,

6763, **<u>7107</u>**, 7151, 7253, 7669, 7841, 8929, 9161, 9397, 9613, 11087, 11831, **<u>12403</u>**, 12517, 12289, 13757, 14197, 15361, **<u>15841</u>**, 18131, 18797, 18973, 19489, 21751, 23201, **<u>24013</u>**, 24179, 25169, 26861, 27817, 28463, 35803, 38449, 38629, 38959, 46309, 46399, 49663, **<u>52633</u>**, 53171, 54469, 56629, 57641, 59921, 61007, 61561, 68821, 69499, 77471, 77977, 83869, 84977, 90017, 90947, 97561, 98801, 99563, 108769, 110927, 111781, 115589, 118901, 119551, 129001, 131891, 134401, 135257, 139501, 142501, 142789, 160001, 161969, 168211, 172853, 176459, 177787, 192193, 205633, 215143, 228593, 251263, 259421, 312007, **<u>321201</u>**, 321611, 324661, 356501, 400823, 408433, 429127, 475273, 480509, 492769, 544001, 582731, 615607, 732943, 819251, 969421, 1065527, 1672771, ...

The seventh sequence is the index where sequence five and sequence six are different, ie, sequence five is represented by a composite number at the $n^{\underline{th}}$ position. It is as follows: 15, 16, 19, 35, 39, 43, 45, 49, 80, 82, 94, 96, 129, 130, 132, 138, 142, 146, 154, 159, 199, 207, 214, 218, 221, 223, 227, 230, 245, 249, ... These are the numbers: 91, 33, 7107, 561, 703, 259, 451, 99, 481, 657, 2821, 1729, 52633, 5461, 15841, 24013, 6533, 321201, 1233, 12403, 19503, 4141, 99297, 6541, 2165801, 51291, 909, 69921, 14701, 7471, ...

This eighth sequence is the sorted first occurrence of **n** cyclics of just the composite numbers. It is as follows: 9, 33, 91, 99, 259, 451, 481, 561, 657, 703, 909, 1233, 1729, 2821, 4141, 5461, 6533, 6541, 6601, 7107, 7471, 12403, 12801, 13833, 14701, 15841, 19503, 24013, 34113, 34133, 51291, 52633, 69921, 97681, 99297, ...

This ninth sequence is the order of all odd **n**s $\neq$ 0 mod 5, ie **n** $\neq$

10. It is as follows: 2, 6, 2, 2, 6, 16, 18, 6, 22, 3, 28, 15, 2, 3, 6, 5, 21, 46, 42, 16, 13, 18, 58, 60, 6, 33, 22, 35, 8, 6, 13, 9, 41, 28, 44, 6, 15, 96, 2, 4, 34, 53, 108, 3, 112, 6, 48, 22, 5, 42, 21, 130, 18, 8, 46, 46, 6, 42, 148, 75, 16, 78, 13, 66, 81, 166, 78, 18, 43, 58, 178, 180, 60, 16, 6, 95, 192, 98, 99, 33, 84, 22, 18, 30, 35, ••• .

The tenth sequence is really several series:

Cyclic number of the Primes (not included are the Primes 2 & 5): 2, 1, 5, 2, 1, 1, 1, 1, 2, 12, 8, 2, 1, 4, 1, 1, 2, 2, 9, 6, 2, 2, 1, 25, 3, 2, 1, 1, 3, 1, 17, 3, 1, 2, 2, 2, 1, 4, 1, 1, 2, 1, 2, 2, 7, 1, 2, 1, 1, 34, 8, 5, 1, 1, 1, 54, 4, 10, 2, 2, 2, 2, 1, 4, 3, 1, 2, 3, 11, 2, 1, 2, 1, 1, 1, 4, 2, 2, 1, 3, 2, 1, 2, 2, 14, 3, 1, 3, 2, 2, 1, 1, 1, 1, 1, 10, 2, 1, 6, 2, 2, 2, 1, 1, 2, 1, 2, 2, 3, 12, 7, 1, 2, 20, 6, 1, 2, 1, 3, 3, 2, 2, 3, 1, 1, 2, 1, 12, 3, 1, 6, 28, 2, 4, 4, 2, 4, ••• . The first occurrence of a particular number in this series is cited in the fifth sequence.

Cycle One: 7, 17, 19, 23, 29, 47, 59, 61, 97, 109, 113, 131, 149, 167, 179, 181, 193, 223, 229, 233, 257, 263, 269, 313, 337, 367, 379, 383, 389, 419, 433, 461, 487, 491, 499, 503, 509, 541, 571, 577, 593, 619, 647, 659, 701, 709, 727, 743, 811, 821, 823, 857, 863, 887, 937, 941, 953, 971, 977, 983, 1019, 1021, 1033, 1051, 1063, 1069, ••• . This series is also represented by primes with +10 as a primitive root and is presented as SSN 1823.

A5596

E. Artin's const. A = 0.37395 58136 19202 28805 47280 54346 41641 51116 29249 ••• . See Mr. Yate's second reference for the applicable use of this constant to the above sequence. In a nutshell, it says that

Mira, 2 to enter

about three eighths of all primes have full length reciprocals; ie, they are "Cycle One" primes.

Please enter A7348

Primes with -10 as a primitive root: 3, 17, 29, 31, 43, 61, 67, 71, 83, 97, 107, 109, 113, 149, 151, 163, 181, 191, 193, 199, 227, 229, 233, 257, 269, 283, 307, 311, 313, 337, 347, 359, 389, 431, 433, 439, 443, 461, 467, 479, 509, 523, 541, 563, 577, 587, 593, 599, 631, 683, 701, 709, 719, 787, 821, 827, 839, 857, 883, 911, 919, 937, 941, 947, 953, 977, 991, 1021, ...

Please enter A7349

Primes with both ± 10 as a primitive root: 17, 29, 61, 97, 109, 113, 149, 181, 193, 229, 233, 257, 269, 313, 337, 389, 433, 461, 509, 541, 577, 593, 701, 709, 821, 857, 937, 941, 953, 977, 1021, 1033, 1069, 1097, 1109, 1153, 1181, 1193, 1217, 1229, 1297, 1301, 1381, 1429, 1433, 1549, 1553, 1621, 1697, 1709, 1741, 1777, 1789, 1861, 1873, 1913, 1949, 2017, 2029, ...

Cycle Two: 3, 13, 31, 43, 67, 71, 83, 89, 107, 151, 157, 163, 191, 197, 199, 227, 283, 293, 307, 311, 347, 359, 373, 401, 409, 431, 439, 443, 467, 479, 523, 557, 563, 569, 587, 599, 601, 631, 653, 677, 683, 719, 761, 787, 827, 839, 877, 881, 883, 911, 919, 929, 947, 991, 1039, 1049, 1117, 1123, 1129, 1151, 1163, 1187, 1277, 1279, 1283, 1307, 1319, 1361, 1373, 1399, ...

Cycle Three: 103, 127, 139, 331, 349, 421, 457, 463, 607, 661, 673, 691, 739, 829, 967, 1657, 1669, 1699, 1753, 1993, 2011, 2131, 2287, 2647, 2659, 2749, 2953, 3217, 3229, 3583, 3691, 3697, 3739, 3793, 3823, 3931, 4273, 4297, 4513, 4549, 4657, 4903, 4909, 4993, 5011, 5023, 5101, 5113, 5407, 5647, 5851, 5953, 6091, 6229, 6379, 6421, 6451, 6577, 6607,

. . . .

Cycle Four: 53, 173, 277, 317, 397, 769, 773, 797, 809, 853, 1009, 1013, 1093, 1493, 1613, 1637, 1693, 1721, 2129, 2213, 2333, 2477, 2521, 2557, 2729, 2797, 2837, 3329, 3373, 3517, 3637, 3733, 3797, 3853, 3877, 4133, 4241, 4253, 4373, 4493, 4729, 4733, 4877, 5081, 5333, 5437, 5477, 5569, 5693, 5717, 5801, 5849, 6133, 6277, 6361, 6449, 6569, 6689, ...

Cycle Five: 11, 251, 1061, 1451, 1901, 1931, 2381, 3181, 3491, 3851, 4621, 4861, 5261, 6101, 6491, 6581, 6781, 7331, 8101, 9941, 10331, 10771, 11251, 11261, 11411, 12301, 14051, 14221, 14411, 15091, 15131, 16061, 16141, 16301, 16651, 16811, 16901, 17021, 18371, 18541, 18701, 18731, 19211, 19301, 20341, 20731, 20771, 20981, 21061, 21221, 21341, 21491, 22091, 22621, ...

Cycle Six: 79, 547, 643, 751, 907, 997, 1201, 1213, 1237, 1249, 1483, 1489, 1627, 1723, 1747, 1831, 1879, 1987, 2053, 2551, 2683, 3049, 3253, 3319, 3613, 3919, 4159, 4507, 4519, 4801, 4813, 4831, 4969, 5119, 5443, 5557, 5791, 6079, 6151, 6271, 6373, 6427, 6529, 6547, 6907, 7027, 7351, 7603, 7723, 8089, 8191, 8599, 8803, 8923, 9133, 9151, 9283, 9403, ...

Cycle Seven: 211, 617, 1499, 2087, 2857, 6007, 6469, 7127, 7211, 7589, 9661, 10193, 13259, 13553, 14771, 18047, 18257, 19937, 20903, 21379, 23549, 26153, 27259, 27539, 32299, 33181, 33461, 34847, 35491, 35897, 41651, 42407, 42491, 43051, 43793, 44269, 44633, 45767, 46229, 47699, 47741, 49057, 49939, 50891, 51647, 55021, 55819, 55903, 56701, 59263, 59753, 60859, 63127, ...

Cycle Eight: 41, 241, 1601, 1609, 2441, 2969, 3041, 3449, 3929, 4001, 4409, 5009, 6089, 6521, 6841, 8161, 8329, 8609, 9001, 9041, 9929, 13001, 13241, 14081, 14929, 16001, 16481, 17489, 17881, 18121, 19001, 20249, 20641, 20921, 21529, 22481, 23801, 24169, 24809, 24889, 26041, 26729, 26801, 26881, 26921, 27241, 27529, 28001, 30089, 30809, 32969, 33049, 33641, 34961, ...

Cycle Nine: 73, 1423, 1459, 2377, 2503, 3457, 7741, 9433, 10891, 10909, 16057, 17299, 17623, 20269, 21313, 22699, 24103, 26263, 28621, 28927, 29629, 30817, 32257, 34273, 34327, 35461, 35731, 36343, 36793, 37549, 37567, 37657, 38737, 39367, 39979, 40429, 43633, 48673, 49069, 49393, 50023, 50221, 51949, 58771, 59221, 59743, 62659, 63901, 64189, 64621, 64927, 65701, 67699, ...

Cycle Ten: 281, 521, 1031, 1951, 2281, 2311, 2591, 3671, 5471, 5711, 6791, 7481, 8111, 8681, 8761, 9281, 9551, 10601, 11321, 12401, 13151, 13591, 14831, 14951, 15671, 16111, 16361, 18671, 21191, 21521, 21881, 24281, 24551, 25391, 25801, 25841, 26161, 26431, 26591, 26711, 28031, 28151, 28591, 29231, 29881, 30881, 33071, 33151, 35201, 36761, 36871, 38231, 42391, 43391, ...

Cycle Eleven: 353, 3499, 10429, 13619, 15269, 20091, 25741, 30713, 35509, 38567, 45233, 49171, 57179, 57223, 60149, 63691, 63977, 67783, 77023, 85229, 88463, 90619, 91367, 93941, 96779, 108967, 109913, 110221, 112069, 115259, 117503, 120473, 120847, 121727, 126743, 132331, 135851, 137633, 138469, 143419, 144167, 150833, 151537, 152219, 159457, 159589, 163417, 167971, 175781, 176903, 185021, 190367, 193447, 199783, 200927, 202291, 203017, 204623, 208099, 209771, 218417, 220771, 224863, 226007, NO

231519, ...

Cycle Twelve: 37, 613, 733, 1597, 2677, 3037, 4957, 5197, 5641, 7129, 7333, 7573, 8521, 8521, 8677, 11317, 14281, 14293, 15289, 15373, 16249, 17053, 17293, 17317, 19441, 20161, 21397, 21613, 21997, 23053, 23197, 24133, 25357, 25717, 26053, 26293, 27277, 27397, 29437, 29569, 30649, 31081, 31237, 31477, 33721, 35437, 35533, 37561, 37813, 38557, 40609, 40933, 42013, ...

Primes with odd cycles: 7, 11, 17, 19, 23, 29, 47, 59, 61, 73, 97, 101, 103, 109, 113, 127, 131, 137, 139, 149, 167, 179, 181, 193, 211, 223, 229, 233, 251, 257, 263, 269, 313, 331, 337, 349, 353, 367, 379, 383, 389, 419, 421, 433, 457, 461, 463, 487, 491, 499, 503, 509, 541, 571, 577, 593, 607, 617, 619, 647, 659, 661, 673, 691, 701, 709, 727, 739, 743, 811, 821, ...

Primes with even cycles: 3, 13, 31, 37, 41, 43, 53, 67, 71, 79, 83, 89, 107, 151, 157, 163, 173, 191, 197, 199, 227, 239, 241, 271, 277, 281, 283, 293, 307, 311, 317, 347, 359, 373, 397, 401, 409, 431, 439, 443, 449, 467, 479, 521, 523, 547, 557, 563, 569, 587, 599, 601, 613, 631, 641, 643, 653, 677, 683, 719, 733, 751, 757, 761, 769, 773, 787, 797, 809, 827, 839, ...

These two sequences run fairly even in the number of primes in each group. This "race" is analogous to that of the primes of the two forms $4*k\pm1$. This sequence, numbered eleven, then is when the "lead changes hands" or when plotted on a Cartesian plane the Y=X line is crossed. It is as follows: 3, 11, 83, 103, 919, 967, 1523, 1543, 5641, 5651, 5717, 5741, 9293, 9371, 9403, 9497, 9521, 10501, 10631, 10663, 10733, 10753,

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11489, 11497, (and no others less than 100,000), •••. For all the Primes below one hundred thousand, the lead belongs to the odds over the evens by a 4812 versus 4778 for a margin of just 34. It seems that the "odds" have it, at least at the lower level. The sequence continues:

104947, 105983, 106013, 106087, 106163, 106207, 106397, 106417, 107609, 107713, 107719, 108247, 108677, 108739, 108761, 112337, 112403 112459,

114067, 114269, 114281, 114407, 114689, 114713, 114773, 114967, 114997, 115061, 115079, 115337, 115399, 115429, 115631, 115807, 115853, 115861, 115883, 115901, 116041, 116101, 116933, 116993, 117053, 118709, 118751, 118927, 119359, 119419, 119591, 119617, 119809, 120817, 122449, 122819, 122849, 122887, 123083, 123113, 123203, 123269, 123307, 123593, 123637, 123983, 124133, 124171, 140363, 140411, 140557, 140663, 140681, 147709, 152147, 152189, 152203, 152419, 152729, 152767, 154043, 154061, 154081, 154097, 154373, 154571, 154787, 154823, 154849, 154981, 155003, 155171, 155201, 155299, 155809, 155833, 172399, 172421, 172489, 172709, 172721, 173531, •••.

This twelfth sequence of Primes is when the number of even Cyclics equals the odd Cyclics and this occurs at: 2, 7, 13, 43, 53, 71, 79, 101, 107, 809, 911, 941, 953, 1013, 1049, 1493, 1511, 1531, 5573, 5591, 5639, 5647, 5653, 5693, 5711, 5737, 5849, 9283, 9349, 9397, 9421, 9433, 9463, 9473, 9491, 9511, 10343, 10499, 10627, 10657, 10667, 10729, 10739, 10889, 11483, 11491, 22159,

104933, 104953, 104971, 105023, 105167, 105269, 105341, 105389, 105509, 105527, 105977, 105997, 106019, 106033, 106129, 106181, 106187, 106213, 106391, 106411, 106427, 106441, 106453, 107603, 107621, 107647, 107687, 107699, 107717, 107741, 108233, 108293, 108359, 108413, 108439, 108649, 108727, 108751, 110273, 110291, 112103, 112331, 112397, 112429, 112481, 112507, 112559, 112573, 114043, 114073, 114259, 114277, 114299, 114377, 114649, 114679, 114691, 114769, 114781, 114941, 114973, 115057, 115067, 115099, 115309, 115331, 115363, 115421, 115613, 115663, 115793, 115849, 115859, 115879, 115891, 116027, 116047, 116099, 116107, 116359, 116923, 116929, 116953, 116989, 117043, 117109, 117127, 117731, 118691, 118717, 118747, 118903, 118913, 119267, 119293, 119321, 119417, 119569, 119611, 119627, 119653, 119677, 119689, 119759, 119773, 119797, 120383, 120811, 121169, 122443, 122527, 122789, 122827, 122839, 122861, 122869, 122929, 122963, 123001, 123049, 123077, 123091, 123121, 123191, 123229, 123259, 123289, 123583, 123601, 123631, 123953, 123979, 124001, 124067, 124123, 124139, 124153, 124277, 140333, 140351, 140381, 140407, 140521, 140533, 140551, 140617, 140629, 140659, 140677, 147073, 147139, 147661, 147673, 147703, 147853, 147919, 152123, 152183, 152197, 152417, 152723, 152753, 154027, 154057, 154067, 154079, 154087, 154111, 154159, 154279, 154369, 154487, 154501, 154543, 154573, 154613, 154667, 154681, 154747, 154769, 154789, 154807, 154841, 154943, 154991, 155069, 155153, 155167, 155191, 155291, 155723, 155773, 155801, 155821, 155849, 172093, 172169, 172213, 172373, 172411, 172441, 172673, 172687, 172717, 172741, 173309, 173501, 192191, 192323, 192373, ...

The thirteenth sequence is the other half of the equation which is represented by the cycles and that is the orders. However, in this case the evens out number the odds generally by a ratio of 2 to 1. Therefore, in this race the line Y=2*X is employed. What follows is when the lead switches and it is as follows: 3, 53, 61, 67, 137, 173, 181, 191, 197, 199, 223, 227, 233, 239, 251, 283, 419, 439, 463, 467, 503, 563, 571, 599, 607, 613, 619, 631, 659, 787, 1069, 5521, 5531, 10163, 10181, 10271, 10301, 11483, 11491, 22133, 22571, 22573, 22739, 22907, 22937, 23311, 23327, 28151, 28211, 28319, 28517, 28631, 28663, 28759,

28789, 28879, 28909, 29959, 30011, 30323, 32789, 32797, 49459, 49667, 49681, 49843, 50461, 50683, 50909, 50923, 51001, 51031, 55073, 55079, 55109, 55117, 55291, 55609, 55621, 55631, 55691, 55763, 55829, 55837, 55933, 56009, 56113, 56123, 56149, 56333, 60649, 60773, 60793, 61643, 61657, 61667, 61681, 61757, 61813, 61843, 61967, 61991, 62137, 62351, 62401, 62483, 62501, 62507, 62539, 62591, 63179, 63197, 63277, 63467, 63487, 63493, 63521, 64483, 64499, 65267, 65287, 65293, 65393, 66919, 67129, 68489, 68539, 76511, 76579, 76597, 77167, 77213, 79621, 79627, 79687, 80239, 80263, 80363, 80449, 80471, 80489, 81283, 81299, 81307, 81343, 81559, 81629, 81637, 81649, 81667, 82219, 82237, 82699, 82763, 82939, 82963, 82997, 83003, 83023, 83641, 84061, 84067, 84121, 84347, 84377, 84437, 84503, 84523, 86183, 87151, 87181, 87187, 87221, 87293, 87313, 87317, 91373, 91397, 91423, 91757, 91781, 91801, 91811, 92111, 92297, 92311, 94291, 94307, 94321, 94427, 94463, 94477, 94541, 94559, 94811, 94999, 98297, 98323, 98411, 98507, 98737,

100403, 100417, 100519, 129539, 129707, 130241, 130307, 130343, 130649, 130657, 130681, 130769, 131321, 132893, 132911, 132961, 133853, 133993, 133999, 134059, 136133, 136193, 137437, 156979, 157649, 159617, 159667, 160091, 160243, 160313, 160319, 163859, 163883, 163927, 163973, 163981, 163987, 175897, 189347, 189353, 189559, 189593, 189599, 189617, 189653, 189713, 190031, 190093, 190283, 190313, 190403, ...

The fourteenth sequence is when they are "neck and neck" and it is

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as follows: 2, 43, 59, 131, 163, 179, 193, 211, 229, 241, 277, 293, 389, 409, 431, 461, 499, 547, 569, 587, 601, 617, 641, 653, 757, 773, 823, 881, 1063, 5527, 5563, 5639, 10159, 10177, 10267, 10289, 11489, 11587, 22123, 22259, 22567, 22637, 22709, 22727, 22751, 22877, 22921, 23053, 23293, 23321, 27437, 27479, 27919, 27947, 28031, 28201, 28283, 28307,

28349, 28477, 28513, 28559, 28603, 28661, 28751, 28771, 28901, 29483,
29917, 29947, 29989, 30319, 32441, 32783, 32801, 49451, 49639, 49669,
49771, 49801, 50051, 50101, 50459, 50627, 50671, 50893, 50929, 50993,
51061, 51131, 55061, 55103, 55127, 55229, 55259, 55619, 55633, 55663,
55681, 55711, 55733, 55823, 55903, 55931, 56003, 56101, 56131, 56239,
56267, 56311, 60623, 60647, 60779, 61651, 61673, 61781, 61837, 61961,
62057, 62099, 62131, 62213, 62383, 62497, 62533, 62563, 63149, 63247,
63317, 63347, 63443, 63473, 63499, 63599, 66489, 65239, 65269, 65309,
65353, 65381, 67073, 67121, 68449, 68483, 68501, 68531, 76507, 76561,
76829, 77153, 77191, 79613, 79669, 79693, 80231, 80251, 80279, 80317,
80347, 80447, 80473, 81203, 81239, 81293, 81331, 81551, 81619, 81647,
82217, 82231, 82261, 82657, 82723, 82781, 82913, 82981, 83009, 83077,
83597, 83701, 84059, 84089, 84349, 84391, 84463, 84499, 84521, 85621,
86179, 86243, 87133, 87179, 87211, 87277, 87299, 91369, 91387, 91411,
91733, 91771, 91807, 91867, 92003, 92189, 92221, 92237, 92269, 92317,
94229, 94273, 94309, 94399, 94433, 94447, 94531, 94547, 94793, 94961,
98207, 98269, 98317, 98327, 98337, 98407, 98479, 98561, 98731,

100391, 100411, 100517, 129533, 129671, 130223, 130279, 130337, 130363,
130651, 130687, 130729, 130787, 131449, 132887, 132929, 132953, 133843,
133877, 133981, 134033, 134053, 136139, 136189, 136559, 136603, 136999,
156971, 157793, 157823, 157841, 159589, 160073, 160087, 160163, 160201,
160231, 160309, 160343, 163853, 163871, 163909, 163979, 175891, 175919,
189311, 189349, 189583, 189613, 189643, 189701, 189961, 190027, 190063,
190159, 190243, 190271, 190301, 190391, ...

This brings up the point that the "race" of the primes of the two forms $4*k\pm1$ is not cited in your Handbook. This sequence, ~~the fifteenth, then~~ is when the "crossovers" take place and is as follows:

3, 26861, 26879, 616841, 617039, 617269, 617471, 617521, 617587, 617689, 617723, 622813, 623387, 623401, 623851, 623933, 624031, 624097, 624191, 624241, 624259, 626929, 626963, 627353, 627391, 627449, 627511, 627733, 627919, 628013, 628427, 628937, 629371, 629429, 629491, 628513, 629767, 630737, 630827, 632813, 632843, 632897, 632923, 633013, 633599,

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633649, 633751, 633797, 633803,

12306137, 12306551, 12308113, 12308587, 12309893, 12309931, 12309961, 12311347, 12311401, 12311443, 12311657, 12311683, 12311837, 12311851, 12311869, 12312043, 12312197, 12312467, 12312497, 12312647, 12312661, 12312787, 12313309, 12314123, 12314321, 12316991, 12317177, 12318107, 12318221, 12318247, 12319753, 12319831, 12321149, 12321187, 12322001, 12322027, 12322217, 12322699, 12323357, 12323383, 12323693, 12324307, 12324353, 12324731, 12324769, 12324839, 12325057, 12325151, 12327857, 12327923, 12327949, 12328079, 12328181, 12328259, 12332989, 12333187, 12334841, 12335003, 12335093, 12335119, 12350017, 12351407, 12360421, 12360643, 12360889, 12360911, 12360937, 12361483, 12361513, 12377311, 12377329, 12377371, 12380293, 12380527, 12382313, 12382367,

951784481, 951784571, 951787157, 951787547, 951787561, 951788179, 951788317, 951789263, 951789989, 951790127, 951790253, 951790327, 951790529, 951790583, 951790633, 951790711, 951791353, 951791527, 951791573, 951791639, 951791669, 951791779, 951791821, 951791887, 951793741, 951793831, 951796141, 951796151, 951796369, 951796567, 951796913, 951796931, 951796981, 951798671, 951798773, 951798803, 951798853, 951798943, 951804209, 951804743, 951804797, 951804859, 951804937, 951804991, 951806341, 951806351, 951806393, 951806963, 951807001, 951807431, 951807473, 951807503, 951807533, 951808719, 951807893, 951807991, 951808673, 951808811, 951809809, 951809939, 951809977, 951811163, 951811801, 951811823, 951811901, 951837659, 951839293, 951839543, 951840521, 951840919, 951841481, 951845347, 951845933, 951846079, 951846433, 951888299, 951890789, 951890839, 951891049, 951891071, 951891181, 951891439, 951891553, 951891667, 951893381, 951893659, 951893809, 951895223, 951895621, 951895751, 951895757, 951895783, 951895937, 951896287, 951896357, 951896591, 951902141, 951902239, 951902333, 951904139, 951904357, 951904391, 951904601, 951904619, 951904669, 951905099, 951905813, 951906323, 951906841, 951907091, 951907097, 951912271, 951912601, 951913507, 951914081, 951914123, 951914741, 951914807, 951914933, 951922219, 951922481, 951922627, 951922681, 951922711, 951922861, 951922967, 951923033, 951923083, 951923153, 951923419, 951923437, 951923447, 951923537, 951923911, 951925033, 951925243, 951925301, 951925483, 951925789, 951932431, 951932521, 951932587, ..., ?, ..., 952223491,

6309280709, ..., ?, ..., 6403150199, 18465126293, ..., ?, ...,

19033524539, ...

Discrete races can to some point have a tie. In the race of the primes of the form 4*k±1; ie, $\pi_{4,1}(p) = \pi_{4,3}(p)$. Here is the sixteenth sequence when P equals: 2, 5, 17, 41, 461, 26833, 26849, 26863, 26881, 26893, 26921, 616769, 616793, 616829, 616843, 616871, 617027, 617257,

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617363, 617387, 617411, 617447, 617467, 617473, 617509, 617531, 617579,

617681, 617707, 617719, 618437, 618521, 618593, 618637, 622793, 623171,

623303, 623327, 623351, 623383, 623393, 623431, 623839, 623893, 623929,

623947, 623963, 623983, 624007, 624037, 624049, 624089, 624119, 624139,
624163, 624233, 624251, 626921, 626947, 626959, 627041, 627349, 627383,
627433, 627479, 627491, 627541, 627721, 627811, 627859, 627911, 627973,
628423, 628921, 628939, 629339, 629351, 629381, 629401, 629417, 629483,
629509, 629747, 629773, 630713, 630733, 630823, 630841, 632221, 632561,
632609, 632677, 632717, 632777, 632839, 632881, 632911, 633001, 633091,
633151, 633187, 633427, 633467, 633583, 633613, 663629, 633739, 633757,
633781, 633793, 633799, 633877,

12306061, 12306139, 12306227, 12306247, 12306499, 12306523, 12308069,
12308089, 12308563, 12308579, 12309889, 12309907, 12309953, 12310031,
12310223, 12311311, 12311339, 12311389, 12311419, 12311501, 12311513,
12311653, 12311671, 12311821, 12311839, 12311857, 12312031, 12312101,
12312109, 12312173, 12312193, 12312439, 12312463, 12312493, 12312511,
12312571, 12312631, 12312649, 12312683, 12312767, 12313297, 12313319,
12314083, 12314153, 12314173, 12314293, 12314327, 12314507, 12316847,
12316859, 12316943, 12316987, 12317161, 12318091, 12318109, 12318193,
12318209, 12318227, 12319729, 12319763, 12319787, 12319799, 12319841,
12320017, 12320117, 12320141, 12320201, 12321061, 12321121, 12321137,
12321151, 12321163, 12321193, 12321989, 12322019, 12322169, 12322213,
12322223, 12322243, 12322643, 12322691, 12323237, 12323293, 12323321,
12323371, 12323621, 12323681, 12323711, 12324223, 12324287, 12324341,
12324691, 12324733, 12324761, 12324799, 12324827, 12324841, 12324889,
12324937, 12325037, 12325063, 12325127, 12325147, 12327529, 12327569,
12327617, 12327629, 12327781, 12327841, 12327899, 12327911, 12327937,
12328007, 12328051, 12328133, 12328157, 12328177, 12328243, 12328273,
12329957, 12330061, 12332953, 12332981, 12333127, 12333179, 12333193,
12334837, 12334867, 12334991, 12335021, 12335033, 12335077, 12335111,
12349721, 12349801, 12349993, 12351379, 12360353, 12360401, 12360631,
12360653, 12360877, 12360899, 12360917, 12361411, 12361463, 12361493,
12361651, 12361927, 12377291, 12377317, 12377363, 12377441, 12380281,
12380507, 12380519, 12380597, 12380617, 12380629, 12380653, 12382157,
12382241, 12382289, 12382327, 12382429, 12382637, 12382649, 12382709,
12382757, 12382781, 12423457, 12423613, 12424001, ...

This also brings up the "race" of the primes of the two forms $6*\mathbf{k}\pm 1$. This sequence then is as follows: 5, and no other reversal point less than one trillion.

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Also the reversals of the primes of the two forms $3*\mathbf{k}\pm 1$ and it is as follows: 2, 608981813029, 608981813357, 608981813707, 608981813717,

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608981819119, 608981819273, 608981819437, 608981820869, 608981836423, 608981836481, 608981838529, 608981838617, 608981839633, 608981839727, 608981839891, 608981840939, 608981841109, 608981841659, 608981841733, 608981844953, 608981845009, 608981847101, 608981847343, 608981847371, 608981847469, 608981847611, 608981849659, 608981849747, 608981849803, 608981850161, 608981850511, 608981851421, 608981865589, 608981865599, 608981866177, 608981866739, 608981866837, 608981866931, 608981867227, 608981867261, 608981867287, ..., ?, ..., 610968213797, ...

What follows is to establish the relationship, if any, between $\pi_{3,1}(X)$ and $\pi_{6,1}(X)$ & $\pi_{3,2}(X)$ and $\pi_{6,5}(X)$. The first Prime number being 2 is ≡ 2 mod 3 but is not included in the count of Primes modulus 6. The second Prime number being three is not included in either count. Beginning with the third Prime number, that being five, all Primes are of the form 3*k±1 and 6*k±1. Furthermore; any Prime ≡ 1 mod 3 must be ≡ 1 mod 6 and vice versa. The same holds true for Primes ≡ 2 mod 3 and ≡ 5 mod 6, or if you like, ≡ -1 mod 3 and ≡ -1 mod 6. Now to establish the relationships stated at the beginning of this paragraph. For all Primes $\pi_{3,1}(X) = \pi_{6,1}(X)$ and $\pi_{3,2}(X) = \pi_{6,5}(X) +1$. Remember P_1 = two. From this relationship, I have a hard time reconciling the two previous sequences. If indeed 608981813029 is the reversal point for π_3 with the Modulus one taking the lead for the first time, backtracking will demonstrate that 608981812919 is a reversal point for π_6. Is it the first one? I'm not sure, but 608 billion plus is less than one trillion! Additionally no relationship can be stated for various Primes mod 4 and those of mod 3 or 6.

These previous two series on the "Prime Races" are not very lively so let us move the abscissa or bias the race towards the 1 ≡ mod **k**. But by what mathematical excuse can we employ other than just a desire to make things interesting? Dr. Knuth to the rescue with his convention of letting 1 be the zeroth prime. Now here is what we have when the two sides in the mod 6 "race" are "neck and neck": 5, 11, 17, 23, 31, 41, 47, 67, 73, 83, 97, 103, 109, 127, 157, 167, 211, 227, 233, 241, 379, 439, 1801, 1867, 1873, 1879, •••. And here is what we have when the two sides in the mod 3 "race" are "neck and neck": 2, 3, 7, 13, 19, 37, 43, 79, 163, 223, 229, •••. Both of these series were stopped at Primes near 25 million. In view of the lack of entries in these two series, I suggest that they not be used, but I presented them to show their characteristics.

Beginning in the middle of page 33, Stan Wagon discusses "[t]he prime number theorem has an extension that explains the growth of the sequence of primes in the congruence classes modulo some integer. Let $\pi_n(X,m)$ be the number of primes $p \le X$ such that $p \equiv m$ (mod n). Then the famous theorem of Dirichlet on primes in arithmetic progressions guarantees that each congruence class contains infinitely many primes (provided gcd(m,n) = 1); that is, each function $\pi_n(X,m)$ approaches infinity as X approaches infinity. Moreover, when n is prime, then the p-1 classes are uniformly distributed." Therefore; the following series are when a particular congruence class takes the lead and it is represented by the prime number which puts it into the lead.

The seventeenth series is a collection of $\pi_n(X,m)$ for n = 5, 7, 8, 9, 10, 11 and 12.

More, 2 to enter A7353

For n =5, the series is as follows: 2, 83, 137, 293, 337, 443, 487, 523, 557, 743, 797, 1213, 1277, 1523, 1657, 1733, 1867, 1973, 2027, 2063, 2797, 2833, 2887, 4733, 5227, 5323, 5437, 5503, 5527, 5623, 5897, 5923, 6007, 6133, 6317, 6353, 6427, 6563, 6607, 6703, 7187, 7283, 7307, 7393, 7477, 8963, 9257, 9323, 9397, 9413, 10037, 10133, 11717, 11863, 11887, 11953, 12007, 12263, 12527, 12743, 13457, 13963, 13997, 14173, 14297, 14303, 14387, 14653, 14887, 16903, 16937, 16963, 17117, 20143, 21487, 21503, 21587, 21713, 21737, 22133, 22807, 23003, 23027, 25913, 26417, 26863, 27367, 31193, 31277, 32633, 36187, 36353, 37567, 37643, 37957, ••• .

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For n =7, the series is as follows: 2, 17, 131, 227, 733, 829, 929, 997, 1097, 1123, 1237, 1277, 1447, 1487, 1531, 1627, 1811, 1907, 1993, 2141, 2203, 2267, 2441, 2677, 2707, 3209, 3299, 3433, 3547, 3853, 4003, 4021, 4507, 4679, 4787, 4931, 5081, 5113, 7537, 7577, 7649, 7759, 7817, 8039, 8461, 8543, 8867, 13037, 14327, 14731, 14929, 15263, 15461, 18371, 18913, 18959, 20011, 20051, 20551, 21157, 21481, 21517, 21557, 21577, 21601, 21661, 21727, 21767, 22109, 22189, 23021, 23057, 23371, 23887, 24071, 24097, 24169, 31237, 31267, 31321, 31379, 31531, 32653, 32707, 33521, 33547, 33577, 33617, 33647, 34687, 35983, 45821, 46049, 46769, ••• .

For n =8, the series is as follows: 2, 11, 37, 83, 197, 227, 271, 293, 347, 373, 487, 547, 853, 907, 1069, 1447, 1733, 1831, 1929, 2027, 2053, 2131, 2237, 2251, 2309, 2719, 2749, 3019, 3061, 3083, 3733, 3779, 3877, 3931, 4919, 5179, 5303, 5347, 5407, 6661, 6911, 6949, 6967, 7459, 7789, 11527, 11621, 11887, 12109, 12143, 12157, 13183, 13309, 13339, 13397, 13591, 13613, 14087, 14293, 14327, 14389, 14423, 14741, 14767, 14821, 14887, 14947, 14983, 15077, 15139, 15173, 15227, 15583, 16811, 16831, 17011, 17231, 17683, 17911, 17939, 18199, 18427, 20341, 20507, 20533, 20627, 24077, 24091, 24197, 24659, 24749, 24971, 25037, 25171, 25237, ••• .

For n =9, the series is as follows: 2, 167, 191, 419, 461, 563, 587, 617, 677, 761, 857, 881, 929, 1427, 1451, 1607, 1667, 1777, 1823, 1867, 1913, 2351, 2399, 2459, 4127, 4583, 5039, 5087, 5171, 7283, 7349, 7517, 7547, 76437691, 7901, 8681, 8837, 8933, 11243, 11903, 11927, Mora, I agree

18329, 18371, 19913, 19937, 20201, 20369, 23603, 23627, 23981, 24509, 24767, 24943, 24971, 25087, 25169, 25247, 25357, 25391, 26393, 26417, 27743, 27767, 29759, 29837, 30029, 30269, 31249, 31991, 37307, 37571, 38149, 38219, 38891, 38921, 57329, 57809, 58229, 58601, 58679, 58907, 61547, 62129, 63149, 63299, 63689, 63929, 64013, 64037, 64373, 64451, 64553, ...

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For n =10, the series is as follows: 2, 13, 19, 6173, 6299, 6353, 6389, 16057, 16369, 16427, 16883, 17167, 17203, 17257, 18169, 18517, 18899, 20353, 20369, 20593, 20639, 20693, 20809, 22037, 22109, 22153, 22189, 22343, 22369, 22543, 22679, 23003, 23039, 25147, 25189, 27043, 27329, 27407, 27809, 27827, 28439, 28477, 29009, 29027, 30169, 30197, 30269, 30367, 30539, 30637, 31039, 31397, 32443, 32497, 32573, 35149, 35393, 35509, 35543, 35899, 35933, 36217, 36343, 37987, 38113, 38237, 38653, 38767, 38803, 48109, 48533, 48799, 50053, 52067, 52223, 52757, 55109, 55127, 55399, 58067, 58109, 58757, 59159, 59197, 59239, 59417, 62773, 63067, 63113, 63197, 65203, 65287, 65323, 65497, 66943, 67607, 67733, 68597, 68683, 68777, 68863, 68899, 68947, 69029, 69203, 69857, 70019, 74897, 74959, 75437, 75679, 75797, 76079, 76757, 77929, 78367, 79319, 79367, 79579, 79817, 81929, 81967, 82189, 82217, 82559, 83177, 86729, 86837, 86959, 87427, 87649, 87917, 88169, 88237, 88289, 88337, 88379, 88427, 88499, 88667, 88799, 88817, 89849, 89897, 89909, 90197, 90379, 90407, 90499, 90887, 91199, 91457, 91499, 91703, 92957, 93133, 93377, 93553, 93637, 93703, 93887, 93913, 94007, 94273, 94327, 94543, 96857, 96973, 97157, 97523, 97787, 97843, 98017, 100733, 100787, 101603, 101987, 102023, 102397, 102793, 102877, 102953, 103087, 103123, 103237, 103613, 107867, ...

I have run this series out to 25 million and at no time does a prime ≡ 1 mod 10 have the lead. However; this group is not far behind as demonstrated by the following statistics. At the Prime number 24,999,987; which is the 1565927th prime, those primes congruent to one total 391,329, congruent to three total 391,535, congruent to seven total 391,633, and those congruent to nine total 391,429, for a grand total of 1,565,926 prime numbers. Taking into account the Primes 2 and 5, and the two figures are reconciled, save one. Although ≡ 1 mod 10 has never had the lead in this series, it has never been far behind and often has not been in last place.

For n =11, the series is as follows: 2, 73, 101, 149, 233, 359, 431, 509, 563, 1051, 1091, 1151, 1259, 1459, 1553, 1811, 2609, 2713, 2741, 4363, 4507, 4561, 4919, 5023, 5189, 6761, 7321, 7433, 7717, 7829, 8039, 8081, 8951, 9043, 9203, 9337, 9851, 9931, 10181, 10457, 11437,

11491, 13099, 19841, 19919, 21379, 21767, 21863, 23531, 23623, 32381, 32423, 35089, 37573, 37663, 38299, 48131, 48397, 48593, 48677, 49057, 49451, 49741, 50069, 50159, 50221, 50951, 50993, 51479, 51631, 51659, 51941, 52289, 52489, 52883, 52973, 53591, 53633, 54277, 54403, 54541, 54779, 57163, 57331, 57493, 57829, 57881, 59693, 59729, 60859, 60961, 61261, ••• .

For n =12, the series is as follows: 2, 17, 79, 101, 163, 197, 211, 263, 281, 379, 401, 443, 461, 479, 631, 677, 739, 809, 907, 953, 1087, 1109, 1171, 1193, 1543, 1607, 1721, 1759, 2063, 2203, 2417, 2543, 2633, 2711, 2731, 2753, 3203, 3221, 3323, 3607, 3803, 3847, 3863, 3943, 4397,

4603, 4889, 4999, 5309, 5527, 6869, 6883, 7307, 7853, 8231, 8537, 9103, 9257, 9883, 10211, 11131, 11171, 11299, 11423, 11551, 12239, 12473, 12491, 13411, 13463, 13841, 13967, 14009, 14423, 14669, 14771, 15497, 15551, 15581, 15727, 15761, 15971, 16361, 16787, 17623, 18959, 19231, 19379, 19507, 19997, 20047, 20249, 20599, 20873, 20959, 22937, 23099, 23201, ••• .

Primes with even period lengths or order: 7, 11, 13, 17, 19, 23, 29, 47, 59, 61, 73, 89, 97, 101, 103, 109, 113, 127, 131, 137, 139, 149, 157, 167, 179, 181, 193, 197, 211, 223, 229, 233, 241, 251, 257, 263, 269, 281, 293, 313, 331, 337, 349, 353, 367, 373, 379, 383, 389, 401, 409, 419, 421, 433, 449, 457, 461, 463, 487, 491, 499, 503, 509, 521, 541, 557, 569, ••• .

Primes with odd period lengths or order: 3, 31, 37, 41, 43, 53, 67, 71, 79, 83, 107, 151, 163, 173, 191, 199, 227, 239, 271, 277, 283, 307, 311, 317, 347, 359, 397, 431, 439, 443, 467, 479, 523, 547, 563, 587, 599, 613, 631, 643, 683, 719, 733, 751, 757, 773, 787, 797, 827, 839, 853, 883, 907, 911, 919, 947, 991, 1013, 1031, 1039, 1093, 1123, 1151, 1163, 1187, • • •.

Please notice that at least a cursory review of the two above series would give some empirical credibility to William Shanks' theory "that there are twice as many primes with even period lengths as there are with odd period lengths." The sixty-fifth entry in the odd list is 521, the 98th prime, and in the even list is 1187, the 181st prime. This ratio of one-third to two-thirds has nothing to do with the Artin constant cited earlier. In keeping with the above "races," what follows is the prime reversals: 3, 11, 83, 103, 919, 967, 1523, 1543, 5641, 5651, 5717, 5741, 9293, 9371, 9403, 9497, 9521, 10501, 10631, 10663, 10733, 10753, 11489, 11497, 10447, 105983, 106013, 106087, 106163, 106207, 106397, 106417, 107609, 107713, 107719, 108247, 108677, 108739, 108761, 112337, 112403, 112459, 114067, 114269, 114281, 114407, 114689,

114713, 114773, 114967, 114997, 115061, 115079, 115337, 115399, 115429,
115631, 115807, 115853, 115861, 115883, 115901, 116041, 116101, 116933,
116993, 117053, 118709, 118751, 118927, 119359, 119419, 119591, 119617,
119809, 120817, 122449, 122819, 122849, 122887, 123083, 123113, 123203,
123269, 123307, 123593, 123637, 123983, 124133, 124171, 140363, 140411,
140557, 140663, 140681, 147709, 152147, 152189, 152203, 152419, 152729,
152767, 154043, 154061, 154081, 154097, 154373, 154571, 154787, 154823,
154849, 154981, 155003, 155171, 155201, 155299, 155809, 155833, 172399,
172421, 172489, 172709, 172721, 173531, • • •.

And the sequence in this race when the two sides are equal: 2, 7, 13, 43, 53, 71, 79, 101, 107, 809, 911, 941, 953, 1013, 1049, 1493, 1511, 1531, 5573, 5591, 5639, 5647, 5653, 5693, 5711, 5737, 5849, 9283, 9349, 9397, 9421, 9433, 9463, 9473, 9491, 9511, 10343, 10499, 10627, 10657, 10667, 10729, 10739, 10889, 11483, 11491, 22159,

104933, 104953, 104971, 105023, 105167, 105296, 105341, 105389, 105509,
105527, 105977, 105997, 106019, 106033, 106129, 106181, 106187, 106213,
106391, 106411, 106427, 106441, 106453, 107603, 107621, 107647, 107687,

107699, 107717, 107741, 108233, 108293, 108359, 108413, 108439, 108649, 108727, 108751, 110273, 110291, 112103, 112331, 112397, 112429, 112481, 112507, 112559, 112573, 114043, 114073, 114259, 114277, 114299, 114377, 114649, 114679, 114691, 114769, 114781, 114941, 114973, 115057, 115067, 115099, 115309, 115331, 115363, 115421, 115613, 115663, 115793, 115849, 115859, 115879, 115891, 116027, 116047, 116099, 116107, 116359, 116923, 116929, 116953, 116989, 117043, 117109, 117127, 117731, 118691, 118717, 118747, 118903, 118913, 119267, 119293, 119321, 119417, 119569, 119611, 119627, 119653, 119677, 119689, 119759, 119773, 119797, 120383, 120811, 121169, 122443, 122527, 122789, 122827, 122839, 122861, 122869, 122929, 122963, 123001, 123049, 123077, 123091, 123121, 123191, 123229, 123259, 123289, 123583, 123601, 123631, 123953, 123979, 124001, 124067, 124123, 124139, 124153, 124277, 140333, 140351, 140381, 140407, 140521, 140533, 140551, 140617, 140629, 140659, 140677, 147073, 147139, 147661, 147673, 147703, 147853, 147919, 152123, 152183, 152197, 152417, 152723, 152753, 154027, 154057, 154067, 154079, 154087, 154111, 154159, 154279, 154369, 154487, 154501, 154543, 154573, 154613, 154667, 154681, 154747, 154769, 154789, 154807, 154841, 154943, 154991, 155069, 155153, 155167, 155191, 155291, 155723, 155773, 155801, 155821, 155849, 172093, 172169, 172213, 172373, 172411, 172441, 172673, 172687, 172717, 172741, 173309, 173501, 192191, 192323, 192373, • • •.

On page 79, Mr. Beiler begins a sequence and on page 100, Mr. Yates continues it. This sequence is of those Primes and its square that are primitive divisors of the same Repunit and it is as follows: 3, 487, 56598313 (and no others less that 2^{28}), • • •. Stated another way, these numbers are the only ones less than 2^{28} that have the same cyclic number.

By relaxing the criteria slightly to "List by ranking of the Repunits, those primes that are factors more than once." This sequence then is as follows: 3, 3, 11, 3, 3, 7, 11, 3, 3, 3, 11, 3, 13, 3, 7, 11, 3, 3, • • •. These occur for Repunits of orders: 9, 18, 22, 27, 36, 42, 44, 45, 54, 63, 66, 72, 78, 81, 84, 88, 90, 99, • • •.

The final sequence (trivial?) represents those denominates which in base 10 terminate; ie, numbers consisting only of some powers of two and/or five: 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80, 100, 125, 128, 160, 200, 250, 256, 320, 400, 500, 512, 625, 640, 800, 1000, 1024, 1250, 1280, 1600, 2000, 2048, 2500, 2560, 3125, 3200, 4000, 4096, 5000, 5120, 6250, 6400, 8000, 8192, 10000, 10240, 12500, 12800, 15625, 16000, 16384, 20000, 20480, 25000, 25600, 31250, 32000, 32768, 40000, 40960, 50000, 51200, 62500, 64000, 65536, 78125, 80000, 81920, 100000, 102400, 125000, 128000, ••• .

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Sequentially yours,

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