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Image Denoising by Linear Regression on Non-Local Means Algorithm

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Abstract

Non-Local Means (NL-Means) algorithm which removes the noise from the image have been used in the field widely due to its good performance especially for MR images. Its main idea is using all the pixels which are local and non-local in an image and taking weighted averaging of all values. One negative side of this method is that it considers all pixels in the image without looking at their similarity. This paper proposes a NL-Means algorithm with pixel selection by applying linear regression analysis using root mean squared error (RMSE) value. After regression analysis, RMSE of the neighborhoods is used to exclude non-similar pixels during noise removal. Lastly, obtained results are evaluated by quantitative and qualitative methods.

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1 Introduction

Image denoising is one of the crucial concepts in the image processing field which lets us remove noises from the image. Noise in images is almost inevitable. The reason for noise is either from acquisition or transmission and we have to get rid of these noises for high quality images. The noise can be represented as a function below:

$$v(i) = u(i) + n(i) \tag{1}$$

where v(i) is a noisy image, u(i) is an image without noise and n(i) is the noise added to the u(i). There have been several proposed methods for denoising the images. Some of them are Gaussian smoothing, anisotropic and neighbourhood filterings. All mentioned methods use local averaging commonly by different variations and they are for spatial domain processing. There are also some methods which use frequency domain techniques but it is out of scope in this paper [1–8].

The denoising method should remove the noise from the image without modifying the image itself. If any in-noisy part of the image is modified by the algorithm, this shows the degradation of the method on the image. Gaussian filtering is one of the popular methods in the fields which is used to smooth the image, but it removes some details of the image while doing this [4]. It has a kernel which takes its coefficients from a Gaussian function and it convolves this by the same fixed sized target image patches. It calculates the weighted average and replaces this value by the centered pixel. Gaussian coefficients in the kernel have higher values through the center. Therefore, it is expected to have less noise in flat parts and bigger noise in edges since outer values in the kernel correspond to edges in Gaussian kernel and they will need bigger coefficients. If we give importance to the center coefficients, it will naturally blur the edges when a Gaussian filter is applied. Anisotropic filtering is another preferred method which is used to make better textured images which are seen from some angles. High degradations are also observerd in this method because of the presence of curved edges in textures. Median filters are another method used to remove impulse noise most of the time but it is not enough to remove Rician noise which is a type of artifact caused during acquisition of the MRI image [9].

NL-Means algorithm is a good choice with a good image restoration performance. It looks over the whole picture for denoising the image. Firstly, it chooses a pixel in the image to be denoised and a kernel that covers it. Second step is to specify a search area in which similarity between the current kernel and target kernels in the search area will be calculated. The values which will be looked at are the grey level values between 0 and 255 if it has 8 bit intensity level resolution. Each corresponding pixel in target and current kernel will be subtracted from each other and it will be the distance between two kernels. Then, for the current kernel, all of the distances will be summed and according to the distances of each pair, their weights will be assigned. If the distance between elements of pairs is small, then it will be given a higher weight since their grey levels are close to each other, and vice versa. By this procedure, all elements in a search area will be considered for the current pixel, and closer values will be obtained for noise removal. For the noisy image v, the new calculated intensity NLM(v(i)) of pixel i is a weighted average of the pixel intensities v(j) in the search area I in the NL-Means algorithm.

$$NLM(v(i)) = \sum_{j \in I} w(i,j)v(j) \tag{2}$$

The overall weights of the pixels in the search area must be equal to one and all of the weights of each pixel must have a value between zero and one. The weight of each pixel is calculated by the following formula.

$$w(i,j) = \frac{1}{Z(i)} e^{-\left(\frac{d(i,j)}{h^2}\right)} \tag{3}$$

where d(i,j) is the Gaussian-weighted Euclidean distance, h is degree of filtering and Z(i) is the normalizing constant calculated by sum of all weights in the search area [10].

$$Z(i) = \sum_{j \in I} e^{-\left(\frac{d(i,j)}{h^2}\right)} \tag{4}$$

An optimized version of the NL-Means algorithm was developed by Lu et al. They used median and mean absolute error metrics to select the pixels under a certain threshold [11]. The result of this study was demonstrated in the corresponding paper and variations of the NL-Means algorithm is proven to give good performance by the writers of this paper.

2 Method

In order to obtain better denoised images by using NL-Means algorithm, dissimilar pixels must be excluded since they have no meaning while calculating the new restored value of the corresponding pixel and they also occupy some space. Moreover, this reduces the weights of more similar pixels overall. Similar to Lu et al, an optimized version was developed in this study by selecting certain pixels in the image. However, RMSE value is used for pixel selection in our method as a difference and a novel formula was applied. The main idea behind this method is using the root mean squared error (RMSE) of neighbourhoods which was derived from linear regression. RMSE is used to evaluate how well the prediction of a model was. Firstly, linear regression which is used to find correlation between the input and output variables on a set of data is applied [12]. Then, RMSE values are calculated from the trained model. Regression model has a simple equation if it is a linear regression, which is shown in the below formula.

$$Y = A + BX \tag{5}$$

For the current kernel whose centered pixel will be restored, each target kernel in the search area will be used for linear regression analysis and RMSE value will be obtained from each target kernel. Input will be the range of pixel numbers from 0 to 81 since we use 9*9 kernel. Output will be the grey level values between 0 and 255. Fitted model of one sample kernel can be seen in Figure 1.

Nextly, the average of these RMSE values will be calculated. If the RMSE value of each target kernel is less than the average RMSE value of the search area of the current kernel, then these pixels are selected for weight calculation and contribute to the results. Other pixels are discarded [11]. The formulation for pixel selection is calculated as follows:

$$w(i,j) = \begin{cases} \frac{1}{Z(i)} e^{-\left(\frac{d(i,j)}{h^2}\right)} & \text{if } \mathrm{RMSE}(i) < \mathrm{RMSE}_{\mathrm{avg}} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

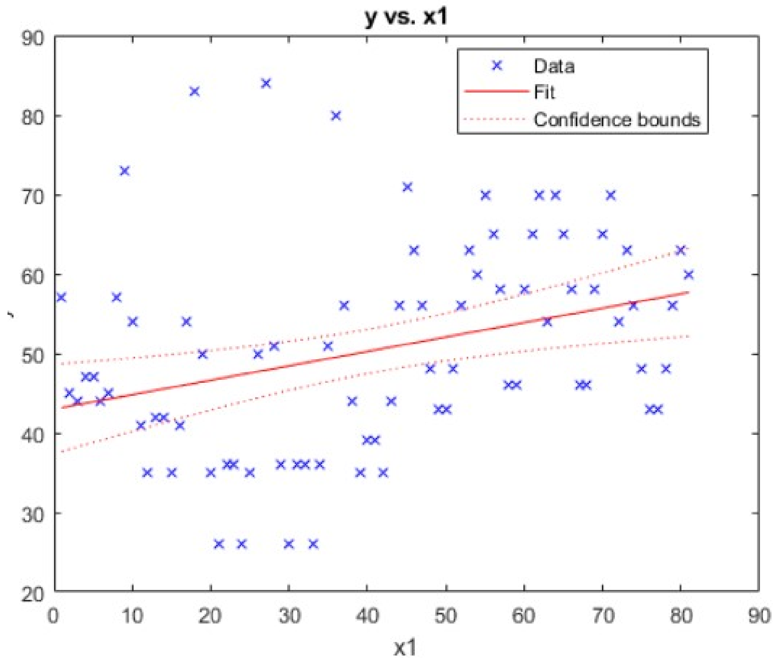

Fig. 1 Regression model of one target kernel

3 Discussion and Results

For practical purposes, search area in the algorithm is taken as 15 height * 15 weight matrix and kernel size as 9*9 matrix. Since regression was applied to a large amount of data, running time of the algorithm was too long. For the image in Figure 2 which has 303*371 pixels, it would take 40 hours to complete. Therefore, size of the image was reduced to 160*101 pixels and it took 6 hours with a powerful machine which has Intel core i7-7700K CPU at 4.20 GHz and 32 GB RAM .

In order to compare our algorithm with other algorithms, two approaches were implied, which are the peak signal to noise ratio (PSNR) and visual quality of the denoised images. PSNR is a technique which is used to measure the quality of a denoised image compared to the original image. It basically compares the maximum pixel value of the original image by the difference between the original image and denoised image. If the output of PSNR is greater, it indicates a better quality denoised image. Table 1 demonstrates the results of our implementation.

Table 1 PSNR RESULTS OF DIFFERENT ALGORITHMS

Method	PSNR
Original Non-Local Means algorithm	30.96
NL-Means algorithm with pixel selection by using RMSE (Our Method)	39.02

Fig. 2 Original uncropped image with Laplacian noise [13]

According to results shown in Table 1, it can be said that our method outperforms the original non-local means algorithm in an image with laplacian noise. Visual differences can also be seen by comparing visual results. Figure 3 represents the results of the implementation.

Fig. 3 Outputs of different denoising methods. Order of the images from left to right, noisy image, the result of our method and the result of original non-local means algorithm

When Figure 3 is analyzed carefully, we can distinguish the difference between the original NL-Means algorithm and our method. Our algorithm smooths the image much better than the original non-local means algorithm. In edges, we have smoother transitions and squared pixel reflections in the noisy image are cleared in our algorithm. The resoın for this result is that we apply linear regression and take the pixels in the target kernel which have less RMSE value than the average RMSE value in the search area of the current kernel. If the color transition of the corresponding target kernel has a linear model, then the RMSE value becomes lower and this target kernel is selected for weight calculation. When we have a laplacian noised image, target kernels with low RMSE values contain edge details which have linear models and these are included in weight calculation. Because of this reason, we can denoise the parts of the image which contain edges well. More importantly, we do not degrade the quality of the image by not modifying the parts of the image which have meaningful information.

Both PSNR value and visual results demonstrate the performance of the algorithms. Someone who measures the goodness of any algorithm should compare both quantitative (PSNR method) and qualitative(visual comparison) analyses for a better comparison. Implementation of the algorithm can be found in the following web address https://github.com/TugayDirek/Image-Denoising-Using-Pixel-Selection-by-RMSE-with-Non-Local-Means-Algorithm. For future works, other algorithms with different inputs can be assessed and better results are tried to be obtained. Running time of our algorithm is also high. Therefore, running time may also be tried to be reduced to obtain the results quickly.

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Declarations

- Conflict of interest
  There is no conflict of interest issue.
- Code availability
  Source code of the algorithm is available at https://github.com/TugayDirek/Image-Denoising-Using-Pixel-Selection-by-RMSE-with-Non-Local-Means-Algorithm.
- Authors' contributions
  T. D. implemented and wrote all parts of the study.
- Ethics approval
  Not applicable.
- Consent to participate
  Not applicable.
- Consent for publication
  Not applicable.
- Availability of data and materials
  Not applicable.
- Funding
  Not applicable.

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