Separation-based Joint Decoding in Compressive Sensing

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***Abstract*—We introduce a joint decoding method for compressive sensing that can simultaneously exploit sparsity of individual components of a composite signal. Our method can significantly reduce the total number of variables decoded jointly by separating variables of large magnitudes in one domain and using only these variables to represent the domain. Furthermore, we enhance the separation accuracy by using joint decoding across multiple domains iteratively. This separation-based approach improves the decoding time and quality of the recovered signal. We demonstrate these benefits analytically and by presenting empirical results.**

I. Introduction

Compressive sensing recovers a signal from reduced measurements by exploiting sparsity of the signal (see, e.g., [1], [5], [7], [13]). Given a K-sparse signal x of length N with $0 < K \ll N$, we can decode x from M measurements given by

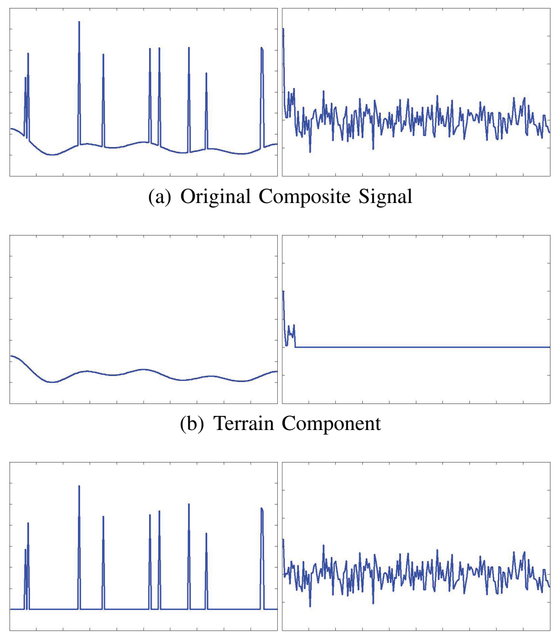
$$y = \Phi x$$

where Φ is an $M \times N$ random measurement matrix. Suppose $M \geq cK\log(N/K)$ for some constant $c > 0$. We can recover x using an optimization procedure such as ℓ_1-minimization [7]. For $K \ll N$, we can use $M \ll N$ resulting a large compression gain.

Compressive sensing works for an input signal x, which has a sparse representation in some known basis. A signal may not exhibit sparsity in one basis because some of its components are sparse in one domain while other components are sparse in another domain. One way to decode such signal is to work with an overcomplete basis. We call this *joint decoding*. As described in Section III, joint decoding recovers variables across multiple domains by simultaneously exploiting sparsity in the components. The idea of joint decoding is known in the compressive sensing literature such as Luo et al. [15].

In this paper we propose *separation-based joint decoding*. This method is useful for a composite signal where one of its components has a more distinguished sparse structure such as a large spike than other signal components. We use a sieving approach to identify the distinguished variables and show that we can reduce the total number of variables in decoding. Furthermore, we demonstrate that iterative joint recovery can refine possible errors induced in the variable sieve step. As a result, our method improves both decoding time and decoding quality.

The approach of this paper is relevant to a variety of applications where the input signal is naturally composite. For example, such a signal could be the summation of measurement results from multiple geographically distributed sensors where each sensor monitors its own nearby region. In a cloud-computing related application, the input signal can be the aggregation of status reports from a large number of servers in a data center [20].



(a) Original Composite Signal

(b) Terrain Component

(c) Spikes Component

Fig. 1. The original composite signal and its two components are displayed in both time domain (left column) and frequency domain (right column). We note from (a) that this signal exhibits sparsity in neither space nor frequency domain. Nevertheless, its two signal components each exhibit their own sparsity; the terrain component is sparse in the frequency domain and the spikes component is sparse in the space domain, as depicted in (b) and (c), respectively.

II. A Composite Signal Example

Fig. 1 depicts an example of a composite signal with two distinct components:

$$S = S_{spikes} + S_{terrain}$$

S is not sparse by itself, but its two components S_{spikes} and $S_{terrain}$ are sparse in two different domains.

Composite signals like S arise naturally in applications where the two components may correspond to:

1) the spikes that represent buildings and vehicles;
2) the terrain that captures a rural scene in the background, which changes smoothly and continuously.

The existing joint decoding methods use overcomplete basis to exploit the sparsity in both domains. In practice, however, we can use additional information about the input signal. For instance, we may know that most frequency coefficients will be much smaller than a few dominant coefficients. How do we benefit from this additional information? A weighting-based approach [14] can use the information to improve the decoding quality. In this paper, we show how to use such information to improve *both* the decoding time and quality. The weighting approach and our method are not exclusive.

III. Brief Review of Compressive Sensing and Joint Decoding

Compressive sensing theory states that with high probability we can reconstruct a signal from a relatively small number of measurements when the signal is sparse in a known basis. We say that a signal x is (K, δ)-sparse if at most K entries of x are greater than a small positive value δ. We say x is K-sparse if δ is zero.

An $M \times N$ measurement matrix Φ compresses a length-N input signal x to y, the M measurements from x. In other words, $y = \Phi x$. We decode x by solving ℓ_1-minimization problem using linear programming:

$$\min ||x||_{\ell_1} \quad subject\ to \quad y = \Phi x \tag{1}$$

The *restricted isometry property* (RIP) [7] of Φ ensures the existence of an unique ℓ_1-min solution for the reconstructed signal and allows a corresponding error bound. A matrix W satisfies the RIP with parameters (ϵ, K) if for all x such that $x \neq 0$ and $||x||_{\ell_0} \leq K$ we have

$$\left| \frac{||Wx||^2_{\ell_2}}{||x||^2_{\ell_2}} - 1 \right| \leq \epsilon \tag{2}$$

It has been shown that with high probability an $M \times N$ measurement matrix Φ with randomly chosen entries satisfies the (ϵ, $2K$)-RIP with a small ϵ if $M \geq cK\log(N/K)$ [3].

Suppose that we are given compressed measurements $y = \Phi x$, where Φ is an $M \times N$ measurement matrix that satisfies (ϵ, $2K$)-RIP. Let x_K be the vector that equals x on the K largest elements of x and equals 0 otherwise. Candes et al. [5] and Shalev-Schwartz [9] have shown that the reconstructed signal $x^\star$ based on ℓ_1-minimization satisfies

$$||x^\star - x||_{\ell_2} < 2(1 - \sqrt{2}\epsilon)^{-1} K^{-1/2} ||x_K - x||_{\ell_1} \tag{3}$$

This implies that $x^\star$ is a good approximation of x when x is (K, δ)-sparse for some small δ.

For an orthonormal basis Ψ, the matrix product ($\Phi\Psi$) can also be shown to satisfy (ϵ, $2K$)-RIP with high probability. If we know $x = \Psi s$ for some Ψ where s is sparse, we will be able to decode s by ℓ_1-minimization:

$$\min ||s||_{\ell_1} \quad subject\ to \quad y = (\Phi\Psi)s \tag{4}$$

After we obtain s, we can recover x from $x = \Psi s$.

Joint decoding is an extension to the standard decoding described above. It resembles the use of overcomplete representations in signal processing [17]. Consider, for example, a composite input signal composed of two components as in the spikes-terrain example described earlier. Suppose that the input signal is a length N vector, $x = x_a + x_b$, where signal components x_a and x_b are sparse in Ψ_a and Ψ_b, respectively. That is, $x_a = \Psi_a s_a$ and $x_b = \Psi_b s_b$ for some transforms Ψ_a and Ψ_b. Let $\Psi = \begin{bmatrix} \Psi_a & \Psi_b \end{bmatrix}$; that is, Ψ is an $N \times 2N$ matrix with its left half equal to Ψ_a and its right half equal to Ψ_b. Then

$$\begin{aligned} x &= x_a + x_b = \Psi_a s_a + \Psi_b s_b \\ &= \begin{bmatrix} \Psi_a & \Psi_b \end{bmatrix} \begin{bmatrix} s_a \\ s_b \end{bmatrix} = \Psi s \end{aligned} \tag{5}$$

Thus, we can decode s using $y = (\Phi\Psi)s$, and recover x using $x = \Psi s$.

This process finds x_a and x_b simultaneously in the two domains associated with Ψ_a and Ψ_b, thus we call it joint decoding. Joint decoding uses the same minimization process as that in standard compressive sensing, but it involves an increased number of variables (that is, $2N$ variables rather than original N variables) in the minimization due to the use of the $N \times 2N$ overcomplete basis Ψ. We refer this method the *conventional joint decoding*.

Note that for the same number of measurements M, the measurement matrix satisfies (ϵ, $2K$)-RIP with a larger K if a smaller N is used in decoding. This implies that $||x_K - x||_{\ell_1}$ is smaller, and consequently the error bound given earlier on $||x^\star - x||_{\ell_2}$ is also smaller. This provides a motivation of our goal — to reduce the number of variables in joint decoding by variables separation.

IV. Separation-based Joint Decoding

Joint decoding uses an overcomplete basis to simultaneously exploit sparsity in multiple domains [8]. As discussed in Section III, the number of variables in decoding is the number of components in all domains. Having more variables leads to increased decoding time and reduced compression rate (M/K) for achieving the same decoding quality [7].

A. Separation and Joint Decoding

We now explain our separation-based method that reduces the number of variables in joint decoding. For the clarity of presentation, we consider a case with only two domains. Following the notations from Section III, we consider a composite input signal of length N, $x = x_a + x_b$ with $x_a = \Psi_a s_a$ and $x_b = \Psi_b s_b$ where s_a and s_b are K_a- and K_b-sparse. The separation-based joint decoding departs from the conventional joint decoding by employing the following two steps:

1) *Separation step.*
We perform decoding for a selected subset of domains or *separation domains*, which are the ones that we anticipate to have stronger components. Then, we identify the leading variables in these domains based on

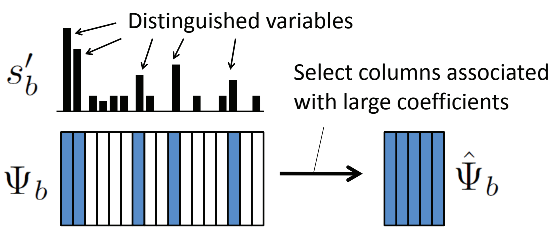


Fig. 2. Using the reconstruction of s'_b to reduce Ψ_b to $\hat{\Psi}_b$.

the reconstruction. Suppose the domain associated with Ψ_b is selected. Then, we reconstruct an approximate solution to s_b by decoding s'_b using

$$y = (\Phi\Psi_b)s'_b \tag{6}$$

Note that

$$\begin{aligned} y &= \Phi(x_a + x_b) \\ &= \Phi(\Psi_a s_a + \Psi_b s_b) \\ &= \Phi\Psi_b(\Psi_b^{-1}\Psi_a s_a + s_b) \\ &= \Phi\Psi_b s'_b \end{aligned}$$

where $s'_b = s_b + \Psi_b^{-1}\Psi_a s_a$. Thus s'_b is an approximate of s_b with an error term $\Psi_b^{-1}\Psi_a s_a$. We sort the elements in the reconstructed s'_b according to their magnitudes, and keep only the largest L elements (see Fig. 2). We call them the *distinguished variables* for the domain associated with Ψ_b, and use $\hat{s}_b$ to denote the set of these variables. The parameter L is so chosen that $\hat{s}_b$ includes the K_b largest nonzero variables in s_b with a good chance.

2) *Joint decoding step.* To compute s_a and rectify the possible errors in s_b computed from the previous step, we perform joint decoding for all domains with a reduced overcomplete basis. For the separation domain, the basis contains only those basis vectors that correspond to the distinguished variables. We decode s_a and $\hat{s}_b$ with the reduced overcomplete basis:

$$y = (\Phi \left[\begin{array}{cc} \Psi_a & \hat{\Psi}_b \end{array} \right]) \left[\begin{array}{c} s_a \\ \hat{s}_b \end{array} \right] \tag{7}$$

where $\hat{\Psi}_b$ consists of a subset of columns of Ψ_b that correspond to the distinguished variables in $\hat{s}_b$, as depicted in Fig. 2.

Our separation-based decoding method has general applicability. For example, we can use separation-based approach to assist the identification of bad measurements [10] in compressive sensing.

B. Improving Separation by Iteration

If we missed some nonzero variables in s_b in the separation step, then we will be solving

$$y = (\Phi \left[\begin{array}{cc} \Psi_a & \hat{\Psi}_b \end{array} \right]) \left[\begin{array}{c} s'_a \\ \hat{s}_b \end{array} \right] \tag{8}$$

in the joint decoding step, where s'_a is an approximation of s_a distorted by the variables missing from $\hat{s}_b$. We then separate a set of distinguished variables in the reconstructed s_a. We denote them as $\hat{s}_a$, and use the following to decode s''_b:

$$y = (\Phi \left[\begin{array}{cc} \hat{\Psi}_a & \Psi_b \end{array} \right]) \left[\begin{array}{c} \hat{s}_a \\ s''_b \end{array} \right] \tag{9}$$

where $\hat{\Psi}_a$ contains the columns in Ψ_a corresponding to $\hat{s}_a$. By Eq. (9), we have

$$y = \Phi\Psi_b(\Psi_b^{-1}\Psi_a\hat{s}_a + s''_b)$$

Note that

$$y = \Phi\Psi_b(\Psi_b^{-1}\Psi_a s_a + s_b)$$

The above two equations imply that

$$s''_b = s_b + \Psi_b^{-1}\Psi_a(s_a - \hat{s}_a)$$

Compared to s'_b, s''_b has a smaller error term as an approximation of s_b. This means that distinguished variables extracted from s''_b can identify non-zeros in s_b better than s'_b. We have described an iterative method which can refine the separation several times to improve the quality of the identified distinguished variables.

V. Analysis

We claim that reducing variables in joint decoding leads to reduced decoding time and reduced errors. In this section we present our analysis on this claim.

A. Separation Efficiency

During the separation step, we choose the separation domain where there are variables with relatively large magnitudes. In this case, the separation step can separate these variables even when the number of measurements is insufficient for accurate decoding. For the spikes-terrain composite signal considered earlier, we would use the space domain as the separation domain because it is relatively easy to separate out variables with large spikes in the space domain.

A key to the performance of the separation-based approach is L, the number of coefficients we need to keep for the separation domains. L depends on how well the distinguished variables can be separated from other variables in the separation domains. If the decoded result in the separation step is close to the ground truth, then L will be close to K.

As discussed by Kung et al. [10], when the nonzeros floor is sufficiently large (the spikes in the spike-terrain example), L can be close to K. In the simulation experiments in Section VI, we demonstrate that a small L is sufficient for distinguished varaibles identified by separation to include all nonzero variables.

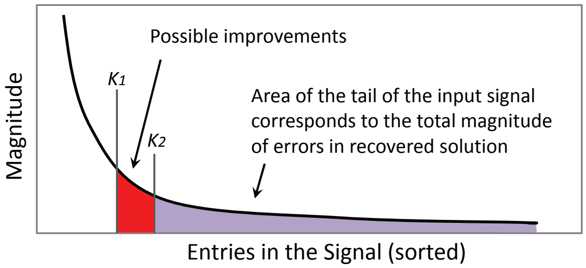


Fig. 3. Impact of increasing K in decoding error.

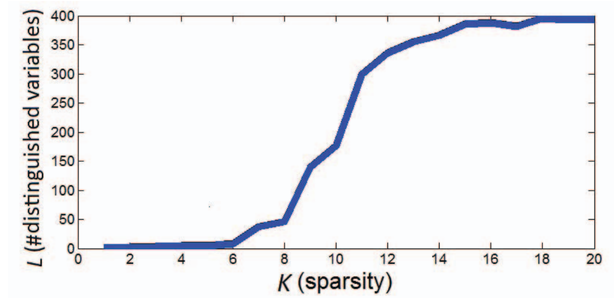


Fig. 4. Number of distinguished varaibles (L) we need to keep so that in the separation step 95% of the cases will capture every non-zero variable. When K is smaller than 6, L is almost the same as K.

B. Decoding Time

For a two-domain case, we reduced the number of variables in joint decoding from $N + N$ to $N + L$, where L is the number of distinguished variables. As explained earlier, for sparse input signal we expect L to be substantially smaller than N.

The decoding time in compressive sensing is the running time of the ℓ_1-minimization. We have observed empirically that the decoding time under ℓ_1-minimization can be as high as O($N^{3.7}$) where N is the total number of variables in the decoding. Since the running time increases rapidly with N, reducing N will have an amplifying effect on reducing decoding time. Suppose that the separation-based method reduces the number of variables from $2N$ to $1.05N$, then the decoding time will be improved by a factor of $2^{3.7}/(1 + 1.05^{3.7})$. This translates to approximately a six-fold speedup.

C. Decoding Error

To compress a K-sparse signal, the required number of measurements is: $M = cK\log(N/K)$. Reducing N means that we can handle a larger K without increasing M. If we can eliminate some variables in the decoding process, then the computed $x^\star$ will have a tighter error bound $2(1 - \sqrt{2}\epsilon)^{-1}K^{-1/2}||x_K - x||_{\ell_1}$ as noted in Eq. (3). While the precise amount of error reduction depends on the properties of the signal, in this case we can expect a smaller error in the decoded solution as depicted in Fig. 3.

VI. Experimental Results Based on Numerical Simulation

We use the ℓ_2-distance metric for measuring errors in decoded solutions. We choose a Gaussian random matrix for the measurement matrix Φ and decode compressive measurements using ℓ_1-magic [16].

A. Separation Efficiency

The first experiment shows how we should chose L in the separation step. Since $N + L$ is the number of variables we need to decode in joint decoding, small value of L is better.

We consider a simple scenario. There are K spikes in space domain and K spikes in frequency domain. The magnitude of spikes in space domain is set twice the spikes in frequency domain.

In this experiment we use $N = 400$ and $M = 80$. We are interested in the cases where the conventional joint decoding method can decode reliably, which are when $K < 4$. Fig. 4 shows the number of variables we need to keep (L) so that in the separation step 95% of the cases will capture every non-zero variable. As shown in Fig. 4, a small L is sufficient for $K < 6$. This means that our method is likely to work well when there are clear spikes.

For robust decoding, we can run our method first and fail over to the conventional joint decoding if the decoded result with our method is bad. Assuming $L/N = 5\%$, we achieve the average speedup of $(2N)^{3.7}/(N^{3.7} + (1.05N)^{3.7} + 0.05(2N^{3.7})) = 456\%$.

B. The Spikes-Terrain Composite Signal

In Fig. 5, we compare decoding errors of conventional joint decoding and separation-based joint-decoding for composite signal like the one shown in Fig. 1. The two components of the signal are:

1) 10 randomly placed spikes;
2) smooth changing slope that can be represented with 10 Discrete Cosine Transform (DCT) coefficients. The magnitude of the slope is kept to be under 20% of the spike average.

For this experiment, we first decode all the 200 variables in the space domain and separate out 30 distinguished variables that are likely to include all 10 spikes. In the subsequent joint decoding step, we use only these 30 variables instead of 200 variables to represent the space domain. The total number of variables in the joint decoding step is therefore 230 (200 variables for the frequency domain and 30 for the space domain.) Conventional joint decoding solves 400 variables at once and suffers much higher decoding cost.

Fig. 5 shows that for this type of signal the separation-based approach provides better decoding quality in additional to improved decoding speed.

C. Test on Natural Image

We consider a natural image and its reconstructions as depicted in Fig. 6. The image is down-sampled to 35×60 pixels in grey level. Gradient color is used for displaying purposes. The ships in the image correspond to distinguished variables in the space domain (like those corresponding to the spikes in the spikes-terrain example) whereas the ocean can

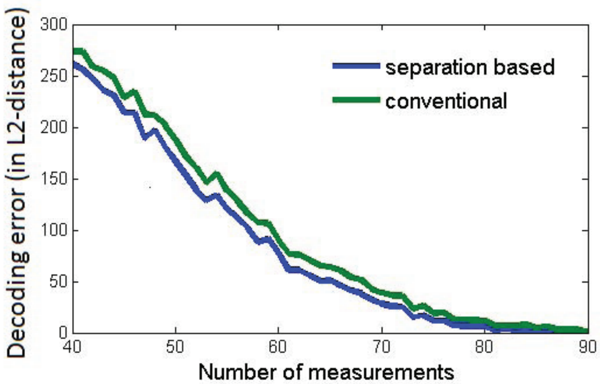
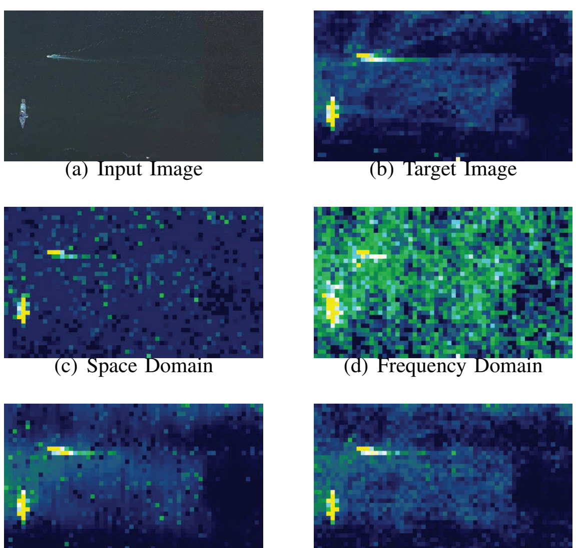


Fig. 5. Comparing methods in decoding error for the spikes-terrain example.

be sparsely represented in the frequency domain. The image is not sparse in the frequency domain due to the presence of the ships, nor in the space domain because of the presence of the ocean background.



(a) Input Image (b) Target Image (c) Space Domain (d) Frequency Domain (e) Joint-decoding (f) Separation-based

Fig. 6. Decoding quality comparison: (a) A natural photograph. (b) A corresponding image to be compressed and recovered: down-sampled, gray scale version of the input image. The gradient color is just for displaying purposes. There are a total of 2100 pixels. 700 measurements are in the compressions to be compared: (c) Decoding by exploiting sparsity only in the space domain. (d) Decoding by exploiting sparsity only in the frequency domain using 2-dimensional Discrete Cosine Transform (DCT). (e) Conventional joint decoding, exploiting sparsity in both the space and frequency domains. (f) Separation-based joint decoding, also exploiting sparsity in both domains. Recovered images of (e) and (f) have better quality than those of (c) and (d). The method of (f) is about six times faster than (e) due to the use of reduced number of variables.

As shown in Fig. 6 (d), standard decoding of compressive sensing in the frequency domain does not fully exploit the sparsity of the signal; the ships are visible, but their boundaries are fuzzy. Moreover, the intensity has changed for most parts of the image (the colors are different while the silhouette is still visible). In Fig. 6 (c) we see that decoding in the space domain gives us a clear view of the ships. However, details in the background are lost almost completely.

As depicted in Fig. 6 (e), conventional joint decoding produces a much better result. While it attempts to exploit sparsity fully in both domains, however, it incurs a significantly higher decoding cost. By reducing the number of variables, separation-based joint decoding is approximately six times faster than the conventional joint decoding while yielding an image of comparable quality (Fig. 6 (f)).

VII. Application to Cases Involving Three or More Domains

For the simplicity of presentation, we have focused on an example with only two domains. The basic idea extends to cases involving three or more domains in a straightforward manner. Consider, for example, a pattern matching scenario. A compressed image under compressive sensing comprises some number of parameterized objects such as lines, circles [21], or more complicated objects such as trained targets [18]. Each of these objects would correspond to a different domain. We can separate some of the more pronounced objects to reduce the total number of variables in joint decoding.

In applications where multiple measurements are combined, the signal we wish to recover can have many components, and the total number of variables can be prohibitively large. Separation can help mitigate these problems in a progressive way [19].

VIII. Conclusion

Joint decoding uses overcomplete basis to exploit sparsity of individual components of a composite signal. However, the conventional joint decoding will increase the number of variables to decode by a factor equal to the number of involved domains. As a result, the conventional joint decoding will suffer sharp increase in decoding time. In this paper we have shown that the separation-based joint decoding can substantially reduce the number of variables decoded while simultaneously improving the quality of decoded solution.

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REFERENCES

[1] Emmanuel J. Candes, Terence Tao, *Decoding by linear programming*, in Transactions on Information Theory, vol. 51(12) (2005), 4203V4215.

[2] Emmanuel J. Candes, J. K. Romberg, T. Tao, *Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information*, in Information Theory, IEEE Transactions on, vol. 52, no. 2 (2006), 489V509.

[3] Emmanuel J. Candes, *An Introduction to Compressive Sampling*, in IEEE Signal Processing Magazine (March 2008) (2008). URL: `http://dsp.rice.edu/sites/dsp.rice.edu/files/cs/CSintro.pdf`. Cited in §III.

[4] RG Baraniuk, *Compressive Sensing*, in IEEE Signal Processing Magazine (July 2007) (2007). URL: `http://dsp.rice.edu/files/cs/baraniukCSlecture07.pdf`.

[5] Emmanuel J. Candes, Terence Tao, *Near-Optimal Signal Recovery From Random Projections: Universal Encoding Strategies?*, in IEEE Transactions on Information Theory 52(12) (2006), 5406-5425. Cited in §III.

[6] David L. Donoho, *Compressed sensing*, in IEEE Transactions on Information Theory 52(4) (2006), 1289-1306.

[7] E. Candes, *The restricted isometry property and its implications for compressed sensing*, in Comptes Rendus Mathematique, Vol. 346, No. 9-10. (May 2008) (2008), 589-592. Cited in §I, §III, §IV.

[8] Thomas Blumensath, Mike E. Davies, *Compressed Sensing and Source Separation*, in ICA 2007 (2007), 341-348. Cited in §IV.

[9] Shai Shalev-Shwartz, *Compressed Sensing: Basic results and self contained proofs*, in manuscript (2009). URL: `www.cs.huji.ac.il/~shais/compressedSensing.pdf`. Cited in §III.

[10] H. T. Kung, Tsung-Han Lin, Dario Vlah, *Identifying Bad Measurements Via Separation in Compressive Sensing*, in Proceedings of the IEEE International Workshop on Security in Computers, Networking and Communications (SCNC 2011), April 2011 (2010). Cited in §IV-A, §V-A.

[11] Gabriel Rilling, Mike Davies, Bernard Mulgrew, *Compressed sensing based compression of SAR raw data*, in SPARS'09 - Signal Processing with Adaptive Sparse Structured Representations (2009). URL: `http://www.scientificcommons.org/54794892`.

[12] Yaakov Tsaig, David L. Donoho, *Extensions of compressed sensing*, in Signal Processing, Volume 86, Issue 3, Sparse Approximations in Signal and Image Processing, March 2006 (2006), 549-571.

[13] David L. Donoho, Michael Elad, Vladimir Temlyakov, *Stable recovery of sparse overcomplete representations in the presence of noise*, in IEEE Transactions on Information Theory,Volume: 52 Issue: 1, Jan 2006 (2006), 6-18.

[14] Emmanuel Candès, Michael Wakin, Stephen Boyd, *Enhancing Sparsity by Reweighted ℓ_1 Minimization*, in Journal of Fourier Analysis and Applications (2008), 877-905. Cited in §II.

[15] Chong Luo, Feng Wu, Jun Sun, Chang Wen Chen, *Compressive data gathering for large-scale wireless sensor networks*, in MobiCom '09 Proceedings of the 15th annual international conference on Mobile computing and networking (2009). Cited in §I.

[16] Emmanuel Cand'es, Justin Romber, *$l1$-magic* (2005). URL: `http://www.acm.caltech.edu/l1magic/`. Cited in §VI.

[17] Te-Won Lee, Michael S. Lewicki, Mark Girolami, Terrence J. Sejnowski, *Blind Source Separation of More Sources Than Mixtures Using Overcomplete Representations*, in IEEE SIGNAL PROCESSING LETTERS, VOL. 6, NO. 4, APRIL 1999 (1999), 87-90. Cited in §III.

[18] John Wright, Allen Y. Yang, Arvind Ganesh, S. Shankar Sastry, Yi Ma, *Robust Face Recognition via Sparse Representation*, in IEEE Transactions on Pattern Analysis and Machine Intelligence (2009), 210-227. Cited in §VII.

[19] Hsieh-Chung Chen, H. T. Kung, Dario Vlah, Bruce Sutery, *Measurement Combining and Progressive Reconstruction in Compressive Sensing*, in submission. Cited in §VII.

[20] H. T. Kung, Chit-Kwan Ling, Dario Vlah, *CloudSense: Continuous Fine-Grain Cloud Monitoring*, in 3rd USENIX Workshop on Hot Topics in Cloud Computing (HotCloud 2011) (2011). Cited in §I.

[21] Ali Cafer Gurbuz, James H. McClellan, Justin K. Romberg, Waymond R. Scott, *Compressive sensing of parameterized shapes in images*, in ICASSP 2008 (2008), 1949-1952. Cited in §VII.