Analytical Evaluation of Adaptive-Modulation-Based Opportunistic Cognitive Radio in Nakagami-*m* Fading Channels

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Abstract

The performance of adaptive modulation for cognitive radio with opportunistic access is analyzed by considering the effects of spectrum sensing, primary user traffic and time delay for Nakagami-*m* fading channels. Both the adaptive continuous rate scheme and the adaptive discrete rate scheme are considered. Numerical examples are presented to quantify the effects of spectrum sensing, primary user traffic and time delay for different system parameters.

Index Terms

Adaptive modulation, cognitive radio, opportunistic access, primary user traffic.

I. Introduction

Adaptive modulation is effective in increasing the link spectral efficiency for communications over fading channels [1] - [4]. On the other hand, cognitive radio has been proposed as one of the most promising solutions to the problem of "spectrum scarcity" [5]. Applying adaptive modulation to cognitive radio provides much flexibility. Several issues may arise compared with conventional adaptive modulation [1] - [4]. First, although the licensed channel is deemed as unoccupied by spectrum sensing during the first part of the frame, the licensed user of the channel or the primary user may come back at any time in the second part of the frame when the data of the cognitive radios' are being transmitted. In this case, the signal-to-noise ratio (SNR) in the channel may change due to the appearance of the primary user. Second, there is a possibility that spectrum sensing will make a misdetection such that the licensed channel is deemed unoccupied while the primary user is actually transmitting. There is also a possibility that spectrum sensing will make a false alarm such that the licensed channel is deemed occupied while the primary user is actually absent. In this case, extra interferences may be considered for the adaptive modulation of the cognitive radio user. Therefore, it is of great interest to study these effects on adaptive modulation for cognitive radios.

In [6], adaptive modulation in an underlay cognitive radio system was studied to optimize the transmitter rate and power. In [7] and [8], the effect of primary user (PU) traffic on cognitive radio performance was investigated. In [9], spectral efficiency for adaptive modulation in cognitive radio was analyzed and a cross-layer design was proposed but imperfect spectrum sensing and PU traffic were not considered. In [10], optimization for cognitive radio transmission using adaptive modulation was performed by considering an underlay system, not an interweave system used in this paper. In [11], the capacity gain offered by adaptive modulation in cognitive radio was analyzed for an underlay system. All these works either analyze an underlay system or ignore adaptive modulation or ignore PU traffic and imperfect spectrum sensing.

In this letter, we evaluate the effects of spectrum sensing, PU traffic and time delay on the performance of adaptive modulation for an interweave cognitive radio system with opportunistic access to the licensed channel that experiences Nakagami-*m* fading. Both the bit error rate (BER) and the link spectral efficiency (SE) are investigated. The BER evaluation shows the variation of the actual BER from the target BER due to channel mismatch in adpative modulation and therefore, is also useful, as studied previously [1] - [4]. In addition to the adaptive continuous rate (ACR) scheme, the more practical adaptive discrete rate (ADR) scheme is also studied. Numerical results show that spectrum sensing, time delay and PU traffic during the data transmission of the cognitive radio have significant impact on the performance of adaptive modulation, and the degree of impact depends on the SNR of the cognitive radio user, the SNR of the primary user, the channel condition and the PU traffic intensity. Compared with [1], the contribution of our work is to quantify the effects of primary user traffic and spectrum sensing for adaptive modulation in cognitive radios that were not considered in [1].

The rest of the letter is organized as follows. In Section II, the system model is introduced. Section III analyzes the performance of adaptive modulation for cognitive radios. Section IV presents the numerical examples, while conclusions are drawn in Section V.

II. System Model

Consider a cognitive radio system where transmission is performed on a frame-by-frame basis, as shown in Fig. 1. In each frame, the first part is the overhead that is used for spectrum sensing and training. Define H_0 as the hypothesis that the licensed channel is free and H_1 as the hypothesis that the licensed channel is occupied. Assume that the probability of false alarm is given by $P_{fa} = Pr\{H_1|H_0\}$ and the probability of misdetection is given by $P_{md} = Pr\{H_0|H_1\}$. For later use, the *a priori* probabilities of H_0 and H_1 are defined as $Pr\{H_0\}$ and $Pr\{H_1\}$, respectively. The second part of each frame is for data transmission, where adaptive modulation is performed. Assume that there are Q symbols per frame dedicated for data transmission. Each has a symbol interval of T_s. Similar to [1], consider multi-level quadrature amplitude modulation (M-QAM). We use rate adaptation but no power adaptation. The BER of the coherent M-QAM with two-dimensional

Gray coding over an additive white Gaussian noise (AWGN) channel can be approximated as [1, eq. (28)]

$$P_e(M,\gamma_e) \approx 0.2e^{-\frac{3\gamma_e}{2(M-1)}} \tag{1}$$

where γ_e is the signal-to-disturbance ratio (SDR) for the cognitive radio user and M is the constellation size. The disturbance includes noise and interference, if any. It has been a common method to replace the signal-to-noise ratio with the SDR in the calculation of error rate, even when the interference is not Gaussian [12], [13]. Thus, the cognitive transmitter requires knowledge of P_e and γ_e to determine M for rate adaptation from (1). The value of P_e is often predetermined at some target value P_0 and is available at the transmitter. The value of γ_e can be estimated using estimators in [14] by the receiver and then fed back to the transmitter. We assume perfect knowledge of γ_e. The choice of the constellation size is made before the data transmission starts in the second part of each frame. Once the constellation size is chosen, it is fixed until the end of the frame. During the secondary transmission, cognitive radio may suffer from primary user interference caused by sensing error or primary traffic, as well as channel decorrelation due to feedback delay. These effects are analyzed in the paper. However, the constellation size will not be chosen again according to these effects during the secondary transmission.

The PU traffic is assumed to follow an independent and identically distributed on-off process with "0" representing the case when the licensed channel is free and "1" representing the case when the licensed channel is occupied. The holding time of each case is assumed to be exponential with mean parameter λ for "0" and μ for "1". At the beginning of the secondary data transmission, the licensed channel is busy with probability $p_b = \frac{\mu}{\lambda+\mu}$ and free with probability $p_f = \frac{\lambda}{\lambda+\mu}$. The status transition probability matrix is given by [15]

$$\mathbf{P} = \begin{pmatrix} p_{00}(T_s) & p_{01}(T_s) \\ p_{10}(T_s) & p_{11}(T_s) \end{pmatrix} = \frac{1}{\lambda+\mu} \begin{pmatrix} \mu+\lambda e^{-(\lambda+\mu)T_s} & \lambda-\lambda e^{-(\lambda+\mu)T_s} \\ \mu-\mu e^{-(\lambda+\mu)T_s} & \lambda+\mu e^{-(\lambda+\mu)T_s} \end{pmatrix} \tag{2}$$

where each element $p_{s_1 s_2}(T_s)$ represents the probability that the channel is in s_1 when it was in s_2 T_s seconds ago, where $s_1, s_2 = 0,1$. The status change of the primary user only happens once during the data transmission, which is the case when the average frame length of the primary user signal is larger than QT_s. Ideally, the cognitive radio user should utilize the channel only when the primary user is absent to avoid inference. However, due to misdetection, this is not possible. This paper therefore evaluates effects of false alarm and misdetection on the cognitive radio performance. In the evaluation, cognitive radio senses the channel at the beginning of its frame and completes its transmission within its frame but does not estimate the PU duration.

III. Performance Analysis

In this section, the performance of adaptive modulation in cognitive radio will be analyzed. Both ACR and ADR schemes will be considered. In the following, we start with ACR.

The average BER for the ACR scheme can be expressed as

$$\begin{aligned} < P_e >_{CRACR1} &= \sum_{k_1=1}^{Q-1} < P_e | arrive\ at\ k_1 >_{CRACR1} \cdot Pr\{arrive\ at\ k_1\} \\ &+ \sum_{k_2=1}^{Q-1} < P_e | depart\ at\ k_2 >_{CRACR1} \cdot Pr\{depart\ at\ k_2\} \end{aligned} \quad (3)$$

where $Pr\{arrive\ at\ k_1\}$ is the probability that the PU arrives at the end of the k_1-th symbol and $Pr\{depart\ at\ k_2\}$ is the probability that the PU departs at the end of the k_2-th symbol. We restrict $1 \leq k_1 \leq Q-1$ so that the PU is present at least for one symbol during data transmission; otherwise if $k_1 = Q$, it gives the same case without PU traffic as studied before. In a Nakagami-m fading channel, the SNR of the cognitive radio user γ is a random variable with

$$f_\gamma(x) = \left(\frac{m}{\gamma_s}\right)^m \frac{x^{m-1}}{\Gamma(m)} e^{-m\frac{x}{\gamma_s}}, x \geq 0 \quad (4)$$

where m is the m-parameter assumed to be an integer, γ_s is the average fading power and $\Gamma(\cdot)$ represents the complete Gamma function defined in [16, eq. (8.310.1)]. Consider the case when the idle channel is correctly detected first. For the first k_1 symbols in the data transmission, their average BER is P_0. For the last $Q-k_1$ data symbols in the data transmission, they suffer from the interference caused by the primary user. Using (1) and the fact that the SDR is $\gamma_e = \frac{\gamma}{1+\gamma_p}$

with γ_p being the PU SNR, their average BER can be derived as $0.2(5P_0)^{\frac{1}{1+\gamma_p}}$. Then, the average BER when the primary user arrives at the end of the k_1-th symbol is

$$< P_e|arrive\ at\ k_1 >_{CRACR1}= \frac{1}{Q}[k_1 \times P_0 + (Q-k_1) \times 0.2(5P_0)^{\frac{1}{1+\gamma_p}}] \tag{5}$$

and the probability that the PU arrives at the end of the k_1-th symbol under H_0 is given by

$$Pr\{arrive\ at\ k_1\} = P\{H_0\}(1-P_{fa})p_f p_{00}(T_s)^{k_1} p_{01}(T_s) p_{11}(T_s)^{Q-k_1} \tag{6}$$

where p_f is the probability of free channel not false alarm. Next, consider the case when the busy channel is misdetected. When the primary user leaves at the end of the k_2-th symbol in the data transmission with $1 \leq k_2 \leq Q-1$, the first k_2 data symbols also suffer from the interference with average BER P_0. The last $Q-k_2$ data symbols do not suffer from the interference. Using (1) and the fact that the SDR is $\gamma_e = \gamma$ in this case, their average BER can be derived as $0.2(5P_0)^{1+\gamma_p}$. Again we restrict $1 \leq k_2 \leq Q-1$ so that there is at least one symbol when the PU is absent to distinguish our work from cases without PU traffic. Thus, the average BER when the PU departs at the end of the k_2-th symbol is given by

$$< P_e|depart\ at\ k_2 >_{CRACR1}= \frac{1}{Q}[k_2 \times P_0 + (Q-k_2) \times 0.2(5P_0)^{1+\gamma_p}] \tag{7}$$

and the probability that the primary user leaves at the end of the k_2-th symbol is given by

$$Pr\{depart\ at\ k_2\} = P\{H_1\}P_{md} p_b p_{11}(T_s)^{k_2} p_{10}(T_s) p_{00}(T_s)^{Q-k_2}. \tag{8}$$

Using (5), (6), (7) and (8) in (3), one has the average BER when the PU randomly leaves or comes during the data transmission period. From (3), the average BER of the ACR scheme in cognitive radio depends on spectrum sensing as well as the primary user traffic during the data transmission. With perfect spectrum sensing, $P_{md} = 0$ and $P_f = 0$ such that the second term in (3) will be zero and the first term in (3) will be maximum, changing the BER.

Next, the average link SE for the ACR scheme is derived. When the primary user randomly arrives in the data transmission period, the average link SE in a Nakagami-m fading channel can be derived as

$$\frac{e^{\frac{2mK_0}{3\gamma_s}}}{\ln 2}\sum_{k=0}^{m-1}\left(\frac{2mK_0}{3\gamma_s}\right)^k\Gamma\left(-k,\frac{2mK_0}{3\gamma_s}\right) \tag{9}$$

where $\Gamma(\cdot,\cdot)$ is the complementary incomplete Gamma function defined in [16, eq. (8.350.2)] and $K_0 = -\ln(5P_0)$. When the primary user randomly leaves in the data transmission period, the average link SE can be derived as

$$\frac{e^{\frac{2mK_0(1+\gamma_p)}{3\gamma_s}}}{\ln 2}\sum_{k=0}^{m-1}\left(\frac{2mK_0(1+\gamma_p)}{3\gamma_s}\right)^k\Gamma\left(-k,\frac{2mK_0(1+\gamma_p)}{3\gamma_s}\right). \tag{10}$$

Using (9) and (10), one has

$$\begin{aligned} < \frac{R}{W} >_{CRACR1} =& Pr\{H_0\}(1-P_{fa})\frac{e^{\frac{2mK_0}{3\gamma_s}}}{\ln 2}\sum_{k=0}^{m-1}\left(\frac{2mK_0}{3\gamma_s}\right)^k\Gamma\left(-k,\frac{2mK_0}{3\gamma_s}\right) \\ &+ Pr\{H_1\}P_{md}\frac{e^{\frac{2mK_0(1+\gamma_p)}{3\gamma_s}}}{\ln 2}\sum_{k=0}^{m-1}\left(\frac{2mK_0(1+\gamma_p)}{3\gamma_s}\right)^k\Gamma\left(-k,\frac{2mK_0(1+\gamma_p)}{3\gamma_s}\right). \end{aligned} \tag{11}$$

The average link SE does not depend on the primary user traffic during the data transmission, as it is determined before data transmission. However, it still depends on spectrum sensing through P_{fa} and P_{md}.

For the ADR scheme, one has to choose the constellation size based on [1, eq. (30)]

$$M_{CRADR1} = 2^n, \gamma_n < \gamma \leq \gamma_{n+1} \tag{12a}$$

$$M_{CRADR2} = 2^n, \gamma_n < \frac{\gamma}{1+\gamma_p} \leq \gamma_{n+1} \tag{12b}$$

where $n = 1, 2, \cdots, N$ index possible different regions of the effective SNR in the cognitive radio channel, $\gamma_n = [erfc^{-1}(2P_0)]^2$ when $n = 1$, $\gamma_n = +\infty$ when $n = N+1$, $\gamma_n = \frac{2}{3}K_0(2^n - 1)$ when $n = 2, 3, \cdots, N$, and $erfc^{-1}(\cdot)$ is the inverse of the complementary Gaussian error function. Effectively, the SNR has been quantized to different regions with each region corresponding to an integer value of the constellation size. The average BER for ADR scheme is derived as

$$\begin{aligned} < P_e >_{CRADR1} =& \sum_{k_1=1}^{Q-1} < P_e | arrive\ at\ k_1 >_{CRADR1} \cdot Pr\{arrive\ at\ k_1\} \\ &+ \sum_{k_2=1}^{Q-1} < P_e | depart\ at\ k_2 >_{CRADR1} \cdot Pr\{depart\ at\ k_2\} \end{aligned} \tag{13}$$

where $Pr\{arrive\ at\ k_1\}$ and $Pr\{depart\ at\ k_2\}$ are given in (6) and (8), respectively. Using a similar method to [1, eq. (35)], one has

$$< P_e|arrive\ at\ k_1 >_{CRADR1}= \frac{1}{Q\sum_{n=1}^{N} na_n}\frac{0.2}{\Gamma(m)}\left(\frac{m}{\gamma_s}\right)^m \left[\sum_{i=1}^{k_1}\sum_{n=1}^{N}\frac{n\Gamma(m,\frac{m\gamma_n}{\gamma_s}+\frac{3\rho_i m\gamma_n}{3(1-\rho_i)\gamma_s+2m(2^n-1)})-n\Gamma(m,\frac{m\gamma_{n+1}}{\gamma_s}+\frac{3\rho_i m\gamma_{n+1}}{3(1-\rho_i)\gamma_s+2m(2^n-1)})}{(\frac{m}{\gamma_s}+\frac{3\rho_i m}{3(1-\rho_i)\gamma_s+2m(2^n-1)})^m}+\sum_{i=k_1+1}^{Q}\sum_{n=1}^{N}\frac{n\Gamma(m,\frac{m\gamma_n}{\gamma_s}+\frac{3\rho_i m\gamma_n}{3(1-\rho_i)\gamma_s+2m(2^n-1)(1+\gamma_p)})-n\Gamma(m,\frac{m\gamma_{n+1}}{\gamma_s}+\frac{3\rho_i m\gamma_{n+1}}{3(1-\rho_i)\gamma_s+2m(2^n-1)(1+\gamma_p)})}{(\frac{m}{\gamma_s}+\frac{3\rho_i m}{3(1-\rho_i)\gamma_s+2m(2^n-1)(1+\gamma_p)})^m}\right] \quad (14)$$

and

$$< P_e|depart\ at\ k_2 >_{CRADR1}= \frac{1}{Q\sum_{n=1}^{N} nb_n}\frac{0.2}{\Gamma(m)}\left(\frac{m}{\gamma_s}\right)^m \left[\sum_{j=1}^{k_2}\sum_{n=1}^{N}\frac{n\Gamma(m,\frac{m\gamma_n(1+\gamma_p)}{\gamma_s}+\frac{3\rho_j m\gamma_n}{\frac{3(1-\rho_j)\gamma_s}{1+\gamma_p}+2m(2^n-1)})-n\Gamma(m,\frac{m\gamma_{n+1}(1+\gamma_p)}{\gamma_s}+\frac{3\rho_j m\gamma_{n+1}}{\frac{3(1-\rho_j)\gamma_s}{1+\gamma_p}+2m(2^n-1)})}{(\frac{m}{\gamma_s}+\frac{3\rho_j m}{3(1-\rho_j)\gamma_s+2m(2^n-1)(1+\gamma_p)})^m}+\sum_{j=k_2+1}^{Q}\sum_{n=1}^{N}\frac{n\Gamma(m,\frac{m\gamma_n(1+\gamma_p)}{\gamma_s}+\frac{3\rho_j m\gamma_n(1+\gamma_p)}{3(1-\rho_j)\gamma_s+2m(2^n-1)})-n\Gamma(m,\frac{m\gamma_{n+1}(1+\gamma_p)}{\gamma_s}+\frac{3\rho_j m\gamma_{n+1}(1+\gamma_p)}{3(1-\rho_j)\gamma_s+2m(2^n-1)})}{(\frac{m}{\gamma_s}+\frac{3\rho_j m}{3(1-\rho_j)\gamma_s+2m(2^n-1)})^m}\right]. \quad (15)$$

Using these equations, (13) can be calculated.

For the average link SE in Nakagami-m fading channels, one has

$$< \frac{R}{W} >_{CRADR1}= Pr\{H_0\}(1-P_{fa})\sum_{n=1}^{N} na_n + Pr\{H_1\}P_{md}\sum_{n=1}^{N} nb_n. \quad (16)$$

where $a_n = \frac{\Gamma(m,\frac{m\gamma_n}{\gamma_s})-\Gamma(m,\frac{m\gamma_{n+1}}{\gamma_s})}{\Gamma(m)}$ and $b_n = \frac{\Gamma(m,\frac{m\gamma_n(1+\gamma_p)}{\gamma_s})-\Gamma(m,\frac{m\gamma_{n+1}(1+\gamma_p)}{\gamma_s})}{\Gamma(m)}$. For comparison, the average BER for the conventional ACR scheme in a Nakagami-m fading channel is given by P_0 and its average link SE is given by [1, eq. (32)]. The average BER for the conventional ADR scheme is given as [1, eq. (35)] and its average link SE is given by [1, eq. (33)].

1) Effect of Time Delay: In practice, the channel may experience time-varying fading. Assume the Jakes model where the correlation coefficient of two channel gains satisfies $\rho = J_0(2\pi f_D\tau)$, $J_0(\cdot)$ is the zero-order Bessel function of the first kind defined in [16, eq. (8.402)] and f_D is the maximum Doppler shift. Using the same method as in [1], one has the average BER for the ACR scheme as

$$\begin{aligned} < P_e >_{CRACR2} &= P\{H_0\}(1-P_{fa})\sum_{k_1=1}^{Q-1} Pr\{k_1\}[\sum_{i=1}^{k_1} I(\rho_i, K_0) + \sum_{i=k_1+1}^{Q} I(\rho_i, \frac{K_0}{1+\gamma_p})] \\ &+ P\{H_1\}P_{md}\sum_{k_2=1}^{Q-1} Pr\{k_2\}[\sum_{j=1}^{k_2} I(\rho_j, K_0) + \sum_{j=k_2+1}^{Q} I(\rho_j, K_0(1+\gamma_p))] \end{aligned} \quad (17)$$

where $I(x,y) = \frac{0.2(1-x)^m y^m}{Q\Gamma(m)} \int_{u_1(x,y)}^{1} \frac{u^{2m-1}}{(1-u)^{m+1}} e^{-\frac{yu(1-xu)}{1-u}} du$ with $u_1(x,y) = \frac{mT_1}{mT_1+(1-x)y\gamma_s}$ is a notational term used to simplify the expression of (17), ρ_i represents the correlation coefficient between the estimated SNR and the i-th data symbol, $i = 1, 2, \cdots, Q$, $1+\gamma_p$ is in the denominator in the first term because the PU randomly arrives at the k_1-th symbol, similar to (5), and $1+\gamma_p$ is in the numerator in the second term because the PU randomly leaves at the k_2-th symbol, similar to (7). Detailed procedures for derivation can be adapted from [1, eq. (49)]. Similarly, the average BER for the ADR scheme is derived as

$$\begin{aligned} < P_e >_{CRADR2} &= P\{H_0\}(1-P_{fa})\sum_{k_1=1}^{Q-1} Pr\{k_1\} \times P_e(k_1)_{CRADR} \\ &+ P\{H_1\}P_{md}\sum_{k_2=1}^{Q-1} Pr\{k_2\} \times P_e(k_2)_{CRADR} \end{aligned} \quad (18)$$

where in this case

$$\begin{aligned} &P_e(k_1)_{CRADR} = \frac{1}{Q\sum_{n=1}^{N} na_n} \frac{0.2}{\Gamma(m)} \left(\frac{m}{\gamma_s}\right)^m \\ &\left[\sum_{i=1}^{k_1}\sum_{n=1}^{N} \frac{n\Gamma(m, \frac{mT_n}{\gamma_s} + \frac{3\rho_i mT_n}{3(1-\rho_i)\gamma_s+2m(2^n-1)}) - n\Gamma(m, \frac{mT_{n+1}}{\gamma_s} + \frac{3\rho_i mT_{n+1}}{3(1-\rho_i)\gamma_s+2m(2^n-1)})}{(\frac{m}{\gamma_s} + \frac{3\rho_i m}{3(1-\rho_i)\gamma_s+2m(2^n-1)})^m} + \sum_{i=k_1+1}^{Q} \right. \\ &\left. \sum_{n=1}^{N} \frac{n\Gamma(m, \frac{mT_n}{\gamma_s} + \frac{3\rho_i mT_n}{3(1-\rho_i)\gamma_s+2m(2^n-1)(1+\gamma_p)}) - n\Gamma(m, \frac{mT_{n+1}}{\gamma_s} + \frac{3\rho_i mT_{n+1}}{3(1-\rho_i)\gamma_s+2m(2^n-1)(1+\gamma_p)})}{(\frac{m}{\gamma_s} + \frac{3\rho_i m}{3(1-\rho_i)\gamma_s+2m(2^n-1)(1+\gamma_p)})^m}\right] \end{aligned} \quad (19)$$

$$P_e(k_2)_{CRADR} = \frac{1}{Q\sum_{n=1}^{N} nb_n} \frac{0.2}{\Gamma(m)} \left(\frac{m}{\gamma_s}\right)^m \left[\sum_{j=1}^{k_2}\sum_{n=1}^{N} \frac{n\Gamma(m, \frac{mT_n(1+\gamma_p)}{\gamma_s} + \frac{3\rho_j mT_n}{\frac{3(1-\rho_j)\gamma_s}{1+\gamma_p}+2m(2^n-1)}) - n\Gamma(m, \frac{mT_{n+1}(1+\gamma_p)}{\gamma_s} + \frac{3\rho_j mT_{n+1}}{\frac{3(1-\rho_j)\gamma_s}{1+\gamma_p}+2m(2^n-1)})}{(\frac{m}{\gamma_s} + \frac{3\rho_j m}{3(1-\rho_j)\gamma_s+2m(2^n-1)(1+\gamma_p)})^m} + \sum_{j=k_2+1}^{Q} \sum_{n=1}^{N} \frac{n\Gamma(m, \frac{mT_n(1+\gamma_p)}{\gamma_s} + \frac{3\rho_j mT_n(1+\gamma_p)}{3(1-\rho_j)\gamma_s+2m(2^n-1)}) - n\Gamma(m, \frac{mT_{n+1}(1+\gamma_p)}{\gamma_s} + \frac{3\rho_j mT_{n+1}(1+\gamma_p)}{3(1-\rho_j)\gamma_s+2m(2^n-1)})}{(\frac{m}{\gamma_s} + \frac{3\rho_j m}{3(1-\rho_j)\gamma_s+2m(2^n-1)})^m}\right]. \tag{20}$$

The average link SE is the same as before, as the constellation size is determined before data transmission and change of channel condition during the data transmission does not affect it.

2) Effect of Random γ_p: In the following, we consider the case when the primary user signal suffers from Rayleigh fading such that the fading coefficient is Gaussian distributed and the SNR γ_p follows an exponential distribution with parameter $\bar{\gamma}_p$. Then, the average BER and SE for the ACR and ADR schemes can be calculated as

$$\int_0^\infty < u >_v \cdot \frac{1}{\bar{\gamma}_p} e^{-\frac{\gamma_p}{\bar{\gamma}_p}} d\gamma_p \tag{21}$$

where $u = P_e$ or $u = \frac{R}{W}$, $v = CRACR1$ or $v = CRADR1$ and $< u >_v$ is derived as before.

IV. Numerical results and discussion

In this section, numerical examples are presented to examine the effects of spectrum sensing and the primary user traffic on adaptive modulation in cognitive radio systems. In the examination, we set $Pr\{H_0\} = 0.7$ and $Pr\{H_1\} = 0.3$, as most target bands of cognitive radios have a larger vacant probability than an occupied probability. Also, $P_{fa} = 0.1$ and $P_{md} = 0.1$. These are standard parameters proposed in the IEEE 802.22 draft. Other values can also be examined in a similar way. In the cases when γ_p changes so that P_{fa} and P_{md} are also changed, the sample size or detection method used in spectrum sensing can be adjusted to maintain P_{fa} and P_{md}. So our result is general. The target BER is set to $P_0 = 10^{-5}$. Assume that $Q = 10$.

Fig. 2 compares the BER performances of the conventional adaptive modulation with the BER performances of adaptive modulation for cognitive radio in a Nakagami-m fading channel. First, spectrum sensing and primary user traffic degrade the BER performance of adaptive modulation. For example, the BER for the conventional adaptive modulation is at 10^{-5}, while the BER for adaptive modulation in cognitive radio is increased to 1.5×10^{-5}, almost twice as large, which might be considered as significant in some applications. This is caused by the non-zero values of P_{md} and P_f from imperfect spectrum sensing that degrade the performance. Second, the ADR curves for adaptive modulation in cognitive radio are closer to the target BER than the ADR curves for the conventional adaptive modulation. This implies that the ADR scheme in cognitive radio has less room for improvement than the conventional ADR scheme.

Fig. 3 has the same system settings as Fig. 2, except that it uses $m = 1$ for the Rayleigh fading. In this case, the BER performance of the ACR scheme in cognitive radio is almost identical to that in Fig. 2 when $m = 2$, while the BER performance of the ADR scheme in cognitive radio is worse than that in Fig. 2 when $m = 2$. This suggests that harsh channel condition degrades the BER performance further, which agrees with intuition. One also sees that the gap between conventional adaptive modulation and adaptive modulation in cognitive radio increases when m decreases by comparing Figs. 2 and 3. Fig. 4 has the same system settings as Fig. 2, except that the primary user traffic intensity is changed to $\lambda = \mu = \frac{1}{200T_s}$. One sees that the gap between the conventional adaptive modulation and the adaptive modulation in cognitive radio reduces when the primary user traffic intensity decreases. This is expected, as the chance of having a mismatch between the channel condition used to choose the constellation size and the channel condition the actual data transmission experiences is reduced when the primary user is less active.

Fig. 5 compares the link SE performances of the conventional adaptive modulation with the link SE performances of adaptive modulation for cognitive radio for different system parameters. From Fig. 5, the link SE for adaptive modulation in cognitive radio is smaller than the link SE for conventional adaptive modulation. Therefore, spectrum sensing degrades the SE performance too due to the combined effects of the non-zero values of P_{md} and P_f in (11) and (16). Fig. 6 shows the BER performance of adaptive modulation in cognitive radio for different values of the correlation coefficient. In this figure, the correlation coefficient is assume to be the same for all data symbols for convenience. One sees that the BER increases when the normalized Doppler shift increases. The maximum normalized Doppler shift that could be accommodated in this case is around 2×10^{-2}. The BER degradation can be further reduced by increasing the m parameter in the channel. Other cases can also be discussed to quantify the effect of time delay on channel quality feedback. Fig. 7 shows the BER performance of adaptive modulation in cognitive radio when γ_p is exponentially distributed. Similar observations to those made from Figs. 2 - 4 can be made from Fig. 7.

V. Conclusion

The effects of spectrum sensing, time delay and PU traffic on the BER and SE performances of adaptive modulation for interweave cognitive radios have been evaluated and compared with adaptive modulation in conventional systems with exclusive licenses that do not suffer from spectrum sensing and primary user traffic. The evaluation has shown that spectrum sensing, time delay and PU traffic cause considerable degradation for BER in most cases. Specifically, the non-zero values of P_{fa} and P_{md} due to imperfect spectrum sensing increase the BER, higher PU traffic intensity leads to larger gaps between adaptive modulations for conventional systems and for cognitive radio systems, and a larger time delay causes an increase in BER above certain threshold. It has also shown that the PU traffic and time delay do not affect the SE performance of adaptive modulation but spectrum sensing degrades the SE performance. The non-zero values of P_{fa} and P_{md} due to imperfect spectrum sensing reduces the link SE for cognitive radio systems compared with conventional systems. In practical systems, one may also employ coding and/or hybrid automatic repeat request (H-ARQ). The employment of coding will make adaptation more difficult, as the inverse function of the coded BER needed to calculate the constellation size is often complicated. On the other hand, H-ARQ involves with the MAC layer protocol and although it is effective, it is beyond the scope of this work that focuses on adaptive modulation. Therefore, they are not considered in this work but represent good topics for future works.

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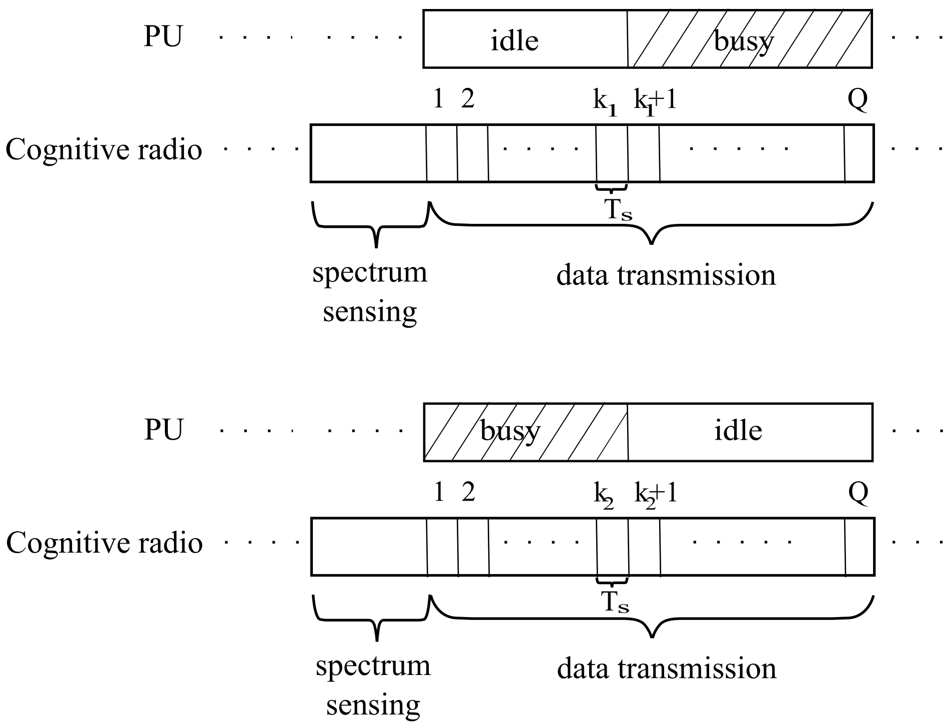

Fig. 1. A diagram of the cognitive radio frame with randomly arriving or departing primary user.

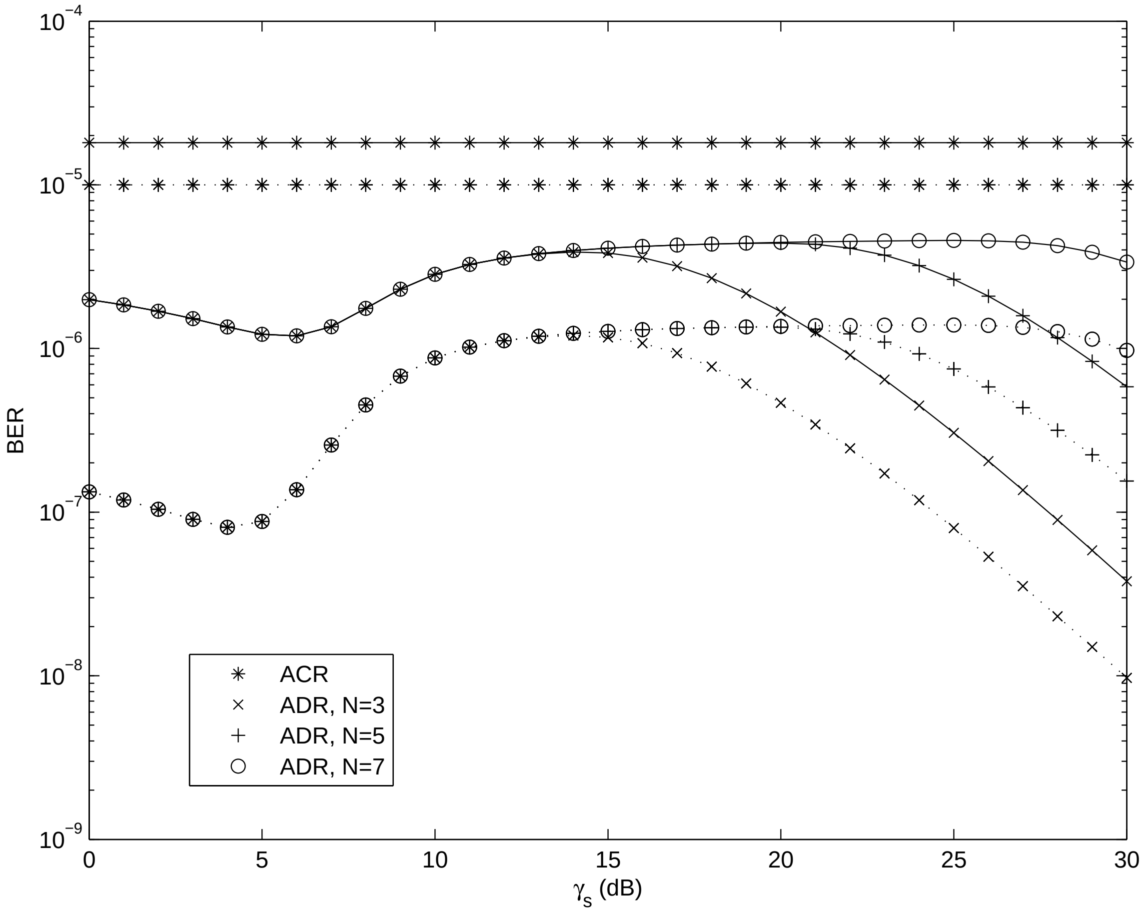

Fig. 2. Comparison of the BER for conventional adaptive modulation (dotted lines) and the BER for adaptive modulation for cognitive radio (solid lines) in Nakagami-m fading channels ($m = 2$) with $\gamma_p = 0\ dB$ and $\lambda = \mu = \frac{1}{100T_s}$.

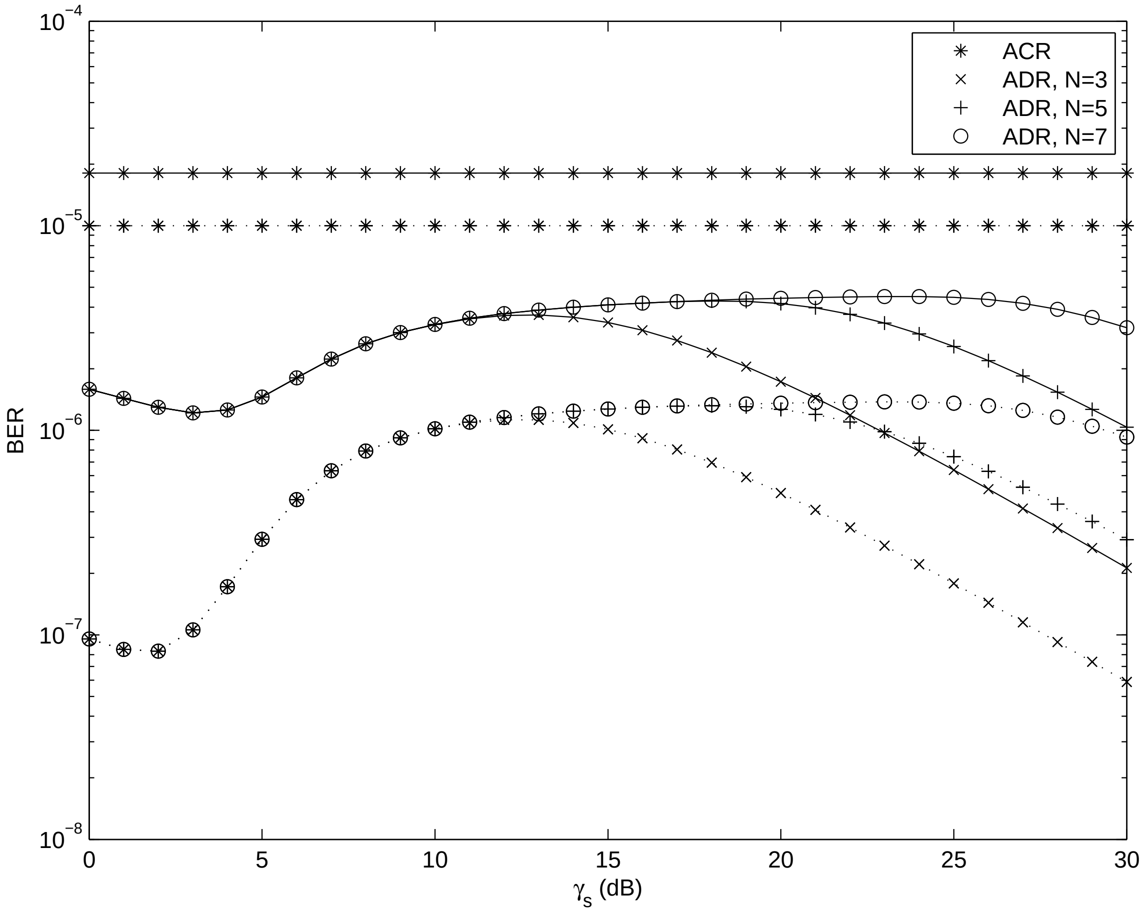

Fig. 3. Comparison of the BER for conventional adaptive modulation (dotted lines) and the BER for adaptive modulation for cognitive radio (solid lines) in Rayleigh fading channels ($m = 1$) with $\gamma_p = 0\,dB$ and $\lambda = \mu = \frac{1}{100T_s}$.

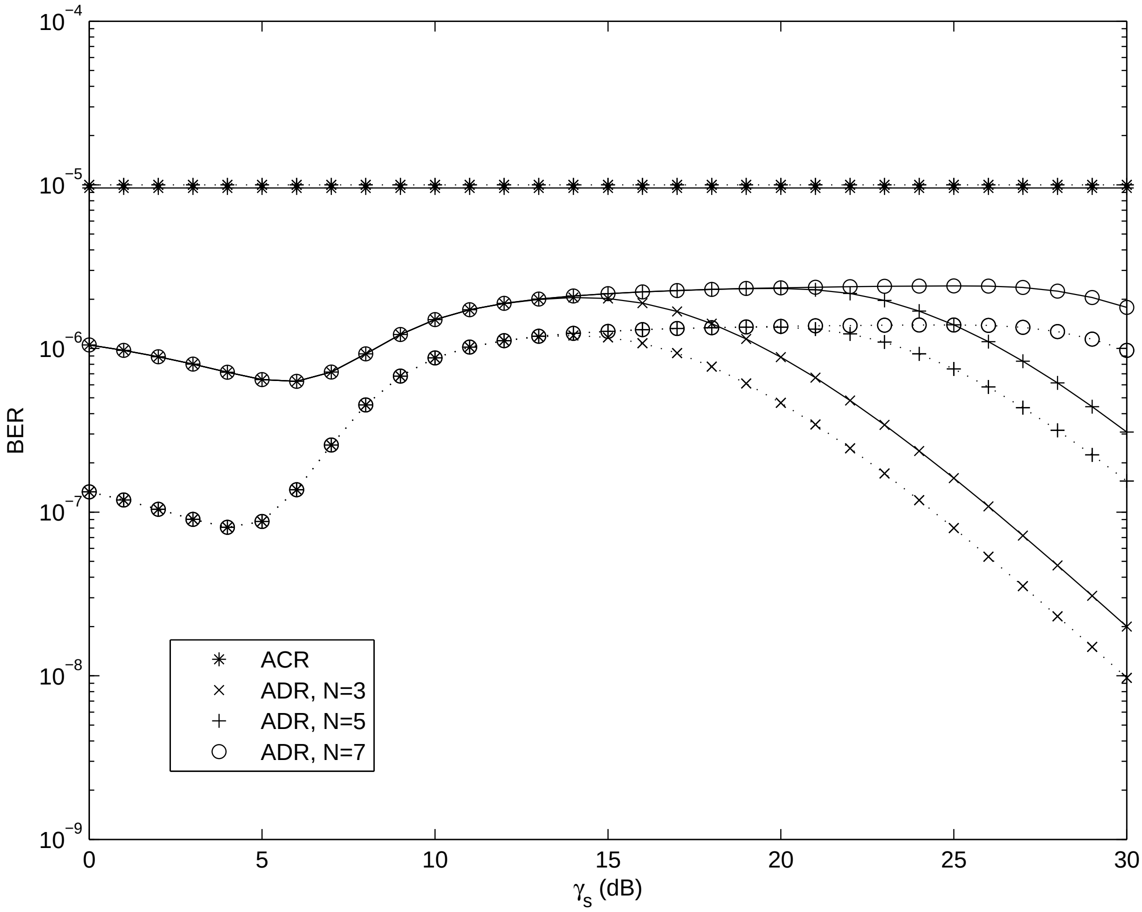

Fig. 4. Comparison of the BER for conventional adaptive modulation (dotted lines) and the BER for adaptive modulation for cognitive radio (solid lines) in Nakagami-m fading channels ($m = 2$) with $\gamma_p = 0\, dB$ and $\lambda = \mu = \frac{1}{200T_s}$.

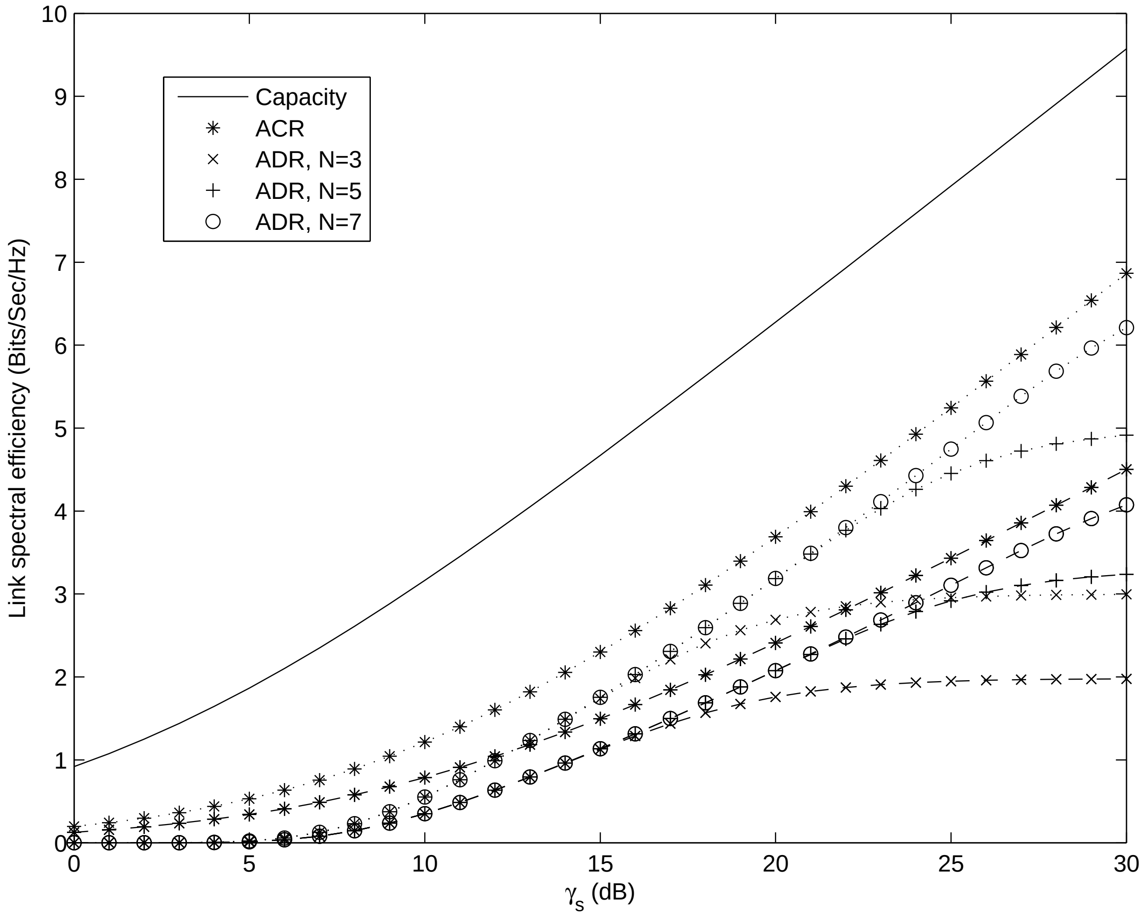

Fig. 5. Comparison of the link SE for conventional adaptive modulation (dotted lines) and the link SE for adaptive modulation for cognitive radio (dashed lines) in Nakagami-m fading channels ($m = 2$) with $\gamma_p = 0\ dB$.

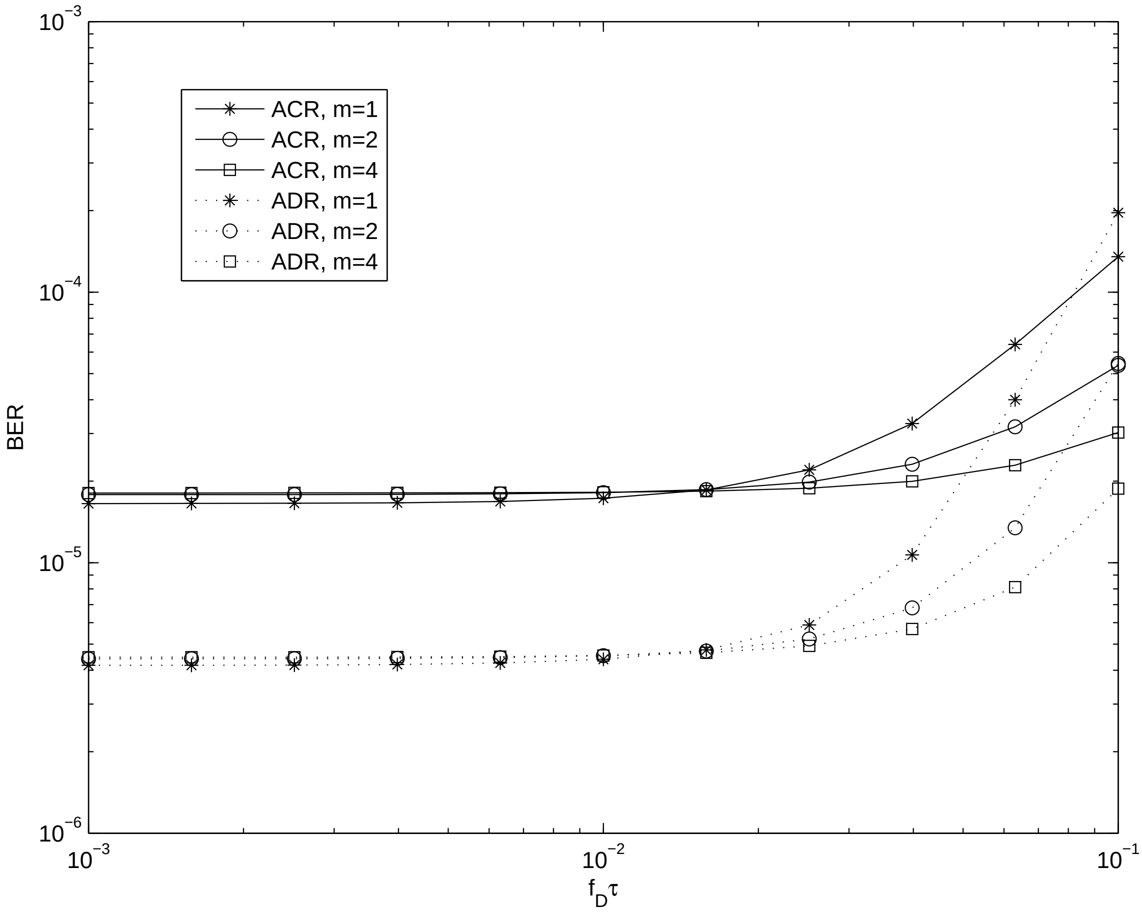

Fig. 6. The BER for adaptive modulation in cognitive radio vs. the normalized Doppler shift for the ACR scheme and the ADR scheme when $N = 5$, $\gamma_s = 20\,dB$, $\gamma_p = 0\,dB$ and $\lambda = \mu = \frac{1}{100T_s}$.

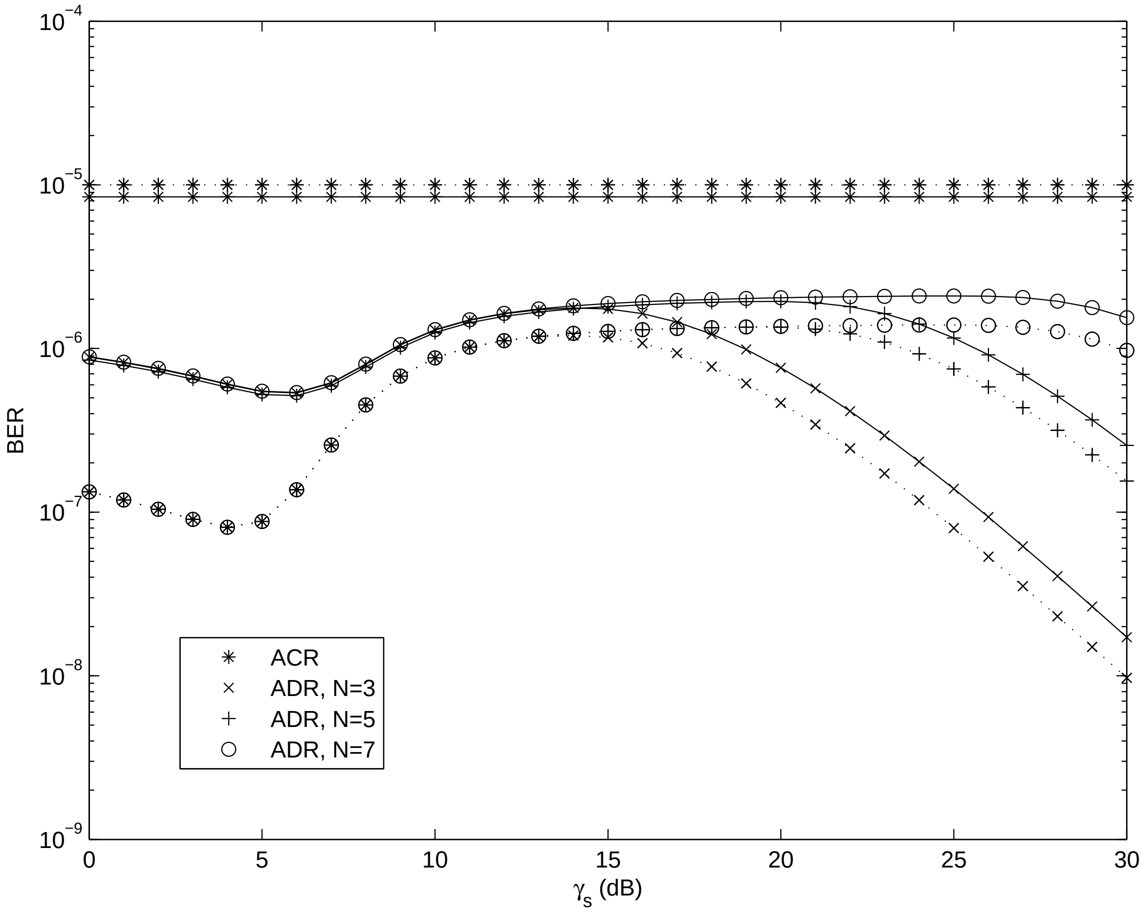

Fig. 7. Comparison of the BER for conventional adaptive modulation (dotted lines) and the BER for adaptive modulation for cognitive radio (solid lines) in Nakagami-m fading channels ($m=2$) with exponentially distributed γ_p at $\overline{\gamma}_p = 5\,dB$ and $\lambda = \mu = \frac{1}{100T_s}$.