Secrecy Rate Optimizations for a MISO Secrecy Channel with Multiple Multi-Antenna Eavesdroppers

Zheng Chu, Hong Xing, Martin Johnston, and Stéphane Le Goff

Abstract— **This paper investigates secrecy rate optimization problems for a multiple-input-single-output (MISO) secrecy channel in the presence of multiple multi-antenna eavesdroppers. Specifically, we consider power minimization and secrecy rate maximization problems for this secrecy network. First, we formulate the power minimization problem based on the assumption that the legitimate transmitter has perfect channel state information (CSI) of the legitimate user and the eavesdroppers, where this problem can be reformulated into a second-order cone program (SOCP). In addition, we provide a closed-form solution of transmit beamforming for the scenario of an eavesdropper. Next, we consider robust secrecy rate optimization problems by incorporating two probabilistic channel uncertainties with CSI feedback. By exploiting the *Bernstein-type* inequality and *S-Procedure* to convert the probabilistic secrecy rate constraint into the determined constraint, we formulate this secrecy rate optimization problem into a convex optimization framework. Furthermore, we provide analyses to show the optimal transmit covariance matrix is rank-one for the proposed schemes. Numerical results are provided to validate the performance of these two conservative approximation methods, where it is shown that the *Bernstein-type* inequality based approach outperforms the *S-Procedure* approach in terms of the achievable secrecy rates.**

Index Terms—**MISO system, physical-layer secrecy, secrecy capacity, convex optimization, robust optimization**

I. Introduction

Physical layer security has recently become an emerging technique to complement and significantly improve the communication security of wireless networks. This technique is a fundamentally different paradigm compared to the conventional cryptographic approaches, where secrecy capacity is achieved by exploiting the physical layer properties of the communication system [1]. The concept of physical-layer security was originally developed for wiretap channels in [2], and has recently been recognized as a promising technique to establish secured data transmission between legitimate transceivers in wireless communications [3]–[6]. Multi-antenna techniques have been exploited to secrecy transmission due to their extra spatial degrees of freedom, where the secrecy capacity for multiple-input-multiple-output (MIMO) channels with multiple eavesdroppers has been presented in [7]. The achievable secrecy rates in multi-antenna wiretap channels are constrained by the information rates achieved by the eavesdroppers. In order to improve the secrecy capacity, relays and jamming nodes have been introduced in the secrecy network, which prevents the eavesdroppers from intercepting the messages intended for legitimate users [8]–[11]. In addition, an artificial noise (AN) approach is also a well known technique to confuse eavesdroppers by embedding noise in the transmitted signal [12]–[15]. In [12], an *isotropic* AN scheme has been developed using an orthogonal projection scheme, whereas the spatially selective AN technique is investigated by designing optimal beamformers to confuse the eavesdropper in [14].

In general, it is not always possible to have perfect channel state information (CSI) of the legitimate user and the eavesdropper at the transmitter due to channel estimation and quantization errors. Secrecy rate optimization problems would be more challenging with unknown CSI at the legitimate transmitter. Thus, the robust optimization techniques based on channel uncertainties have been investigated in traditional wireless system (i.e., cognitive radio networks) [16]–[20]. The relationship has been built between the MISO wiretap channel and the MISO cognitive radio (CR) channel [21]. These robust optimization techniques have been extended to secrecy transmission based on the worst-case scheme in recent work [22]–[25]. In [22], an optimal and robust transmit covariance matrix design has been proposed for MISO secrecy channels with multiple multi-antenna eavesdroppers. In [23], a conservative approximation approach at low SNRs has been presented for MIMO wiretap channels, whereas a robust beamforming technique has been developed for MIMO wiretap channels based on an AN approach in [14], [26]. Apart from robust secrecy rate maximization, the robust outage secrecy optimization with only statistical knowledge of channel uncertainties known has been considered in [27], [28]. The robust outage secrecy rate optimization for MIMO wiretap channel has been investigated in [27], where a *Bernstein-type* inequality based Taylor series approximation was presented for the nonconvex outage secrecy rate constraint, while in [28], the outage probability minimization problem of a MISO wiretap channel has been investigated for a target secrecy rate based on the assumption that the only distribution of the eavesdropper's channel error is available at the transmitter.

In this paper, we consider a MISO secrecy network, where a transmitter establishes a secured communication link with a legitimate receiver in the presence of multiple multi-antenna eavesdroppers. For this secrecy network, we solve the following secrecy rate optimization problems:

- *Secrecy rate optimization based on perfect CSI:* We consider a secrecy rate optimization problem based on perfect CSI, in which the transmit power is minimized subject to the secrecy rate constraint. Motivated by [22], we formulate this problem into a second-order cone program (SOCP). In addition, a closed-form solution for a scenario with a single eavesdropper of this power minimization

Zheng Chu, Martin Johnston, and Stéphane Le Goff are with the School of Electric, Electronic Engineering, Newcastle University, Newcastle upon Tyne, NE1 7RU, U.K. (Emails: z.chu@ncl.ac.uk; martin.johnston@ncl.ac.uk; stephane.le-goff@ncl.ac.uk)

Hong Xing is with Center for Telecommunications Research, Kings College London, London, WC2R 2LS, U.K. (Emails: hong.xing@kcl.ac.uk)

The work of Hong Xing was supported in part by China Scholarship Council (CSC).

problem is derived based on Lagrange dual method and Karush-Kuhn-Tucker (KKT) conditions.

- *Secrecy rate optimization based on imperfect CSI:* In the previous optimization framework, we have solved the secrecy rate optimization problem based on the assumption that the transmitter has perfect CSI of the legitimate user and the eavesdroppers. However, in a practical scenario, perfect CSI of the legitimate user and the eavesdroppers might not be available at the legitimate transmitter due to channel estimation and quantization errors. Therefore, we present robust secrecy rate optimization techniques. Unlike the robust secrecy rate optimization in [22], where the imperfect CSI is formulated as a deterministic model with bounded errors, in this paper, more general statistical channel uncertainty models considering imperfect CSI of both the legitimate user and the eavesdroppers are provided due to unavailable error bounds. Based on these channel uncertainty models, we consider the secrecy rate optimization problems (i.e., power minimization and outage secrecy rate maximization) with probabilistic constraints. These optimization problems are not convex in terms of the transmit covariance matrix and the probabilistic secrecy rate constraint. In order to make these problems tractable, we present two conservative approximation approaches (i.e., *Bernstein-type* inequality and *S-Procedure*) to convert these probabilistic constraints into deterministic ones. Based on these approximations, the problems can be reformulated into convex optimization frameworks correspondingly. Furthermore, the optimality of these approaches are proved by investigating the rank-one property of the optimal transmit covariance matrices.

The remainder of the paper is organized as follows. The system model is described in Section II. Secrecy rate optimization problems based on perfect CSI are solved in Section III, whereas the solutions for robust secrecy rate optimization with two types of channel uncertainty models are provided in Section IV. Section V provides simulation results to validate the performance of the proposed algorithms. Finally, Section VI concludes the paper.

A. Notations

We use the upper case boldface letters for matrices and lower case boldface letters for vectors. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose respectively. $\mathrm{Tr}(\cdot)$ and $\mathbb{E}\{\cdot\}$ stand for trace of a matrix and the statistical expectation for random variables. $\mathrm{Vec}(\mathbf{A})$ is the vector obtained by stacking the columns of $\mathbf{A}$ on top of one another and $\otimes$ is the Kronecker product. $\lambda_{max}(*)$ represents the maximum eigenvalue, whereas $v_{\max}(*)$ denotes the eigenvector associated with the maximum eigenvalue. $\mathbf{A} \succeq 0$ indicates that $\mathbf{A}$ is a positive semidefinite matrix. $\mathbf{I}$ and $(\cdot)^{-1}$ denote the identity matrix with appropriate size and the inverse of a matrix respectively. $\|\cdot\|_2$ represents the Euclidean norm of a matrix. $\Re\{\cdot\}$ stands for the real part of a complex number, whereas $|\mathbf{A}|$ denotes the determinant of $\mathbf{A}$. $[x]^+$ represents $\max\{x, 0\}$.

II. System Model

We consider a MISO secrecy channel, where the legitimate transmitter establishes a communications link with the legitimate user equipped with single antenna in the presence of K multi-antenna eavesdroppers. It is assumed that the legitimate transmitter is equipped with N_T transmit antennas, whereas the legitimate receiver and the k-th eavesdropper consist of single and $N_{E,k}$ receive antennas, respectively. The channel coefficients between the legitimate transmitter and the legitimate receiver as well as the k-th eavesdropper are denoted by $\mathbf{h}_s \in \mathbb{C}^{N_T \times 1}$ and $\mathbf{H}_{e,k} \in \mathbb{C}^{N_T \times N_{E,k}}$, respectively. The received signal at the legitimate receiver and the eavesdropper can be written as

$$y_s = \mathbf{h}_s^H \mathbf{x} + \mathrm{n}_s, \; y_e = \mathbf{H}_{e,k}^H \mathbf{x} + \mathbf{n}_{e,k}, \; k = 1, ..., K,$$

where $\mathbf{x} \in \mathbb{C}^{N_T \times 1}$ is the signal intended to the legitimate user. In addition, n_s and $\mathbf{n}_{e,k}$ are zero-mean additive white Gaussian noises with noise variance σ_s^2 and the covariance matrix $\sigma_{e,k}^2 \mathbf{I}$, respectively. The transmit covariance matrix is defined as $\mathbf{Q}_s = \mathbb{E}\left\{\mathbf{x}\mathbf{x}^H\right\}$. The achievable secrecy rate at the k-th legitimate receiver is defined as

$$R_{s,k} = \left[\log(1 + \frac{1}{\sigma_s^2}\mathbf{h}_s^H \mathbf{Q}_s \mathbf{h}_s) - \log\left|\mathbf{I} + \frac{1}{\sigma_{e,k}^2}\mathbf{H}_{e,k}^H \mathbf{Q}_s \mathbf{H}_{e,k}\right|\right]^+, \; \forall k. \quad (1)$$

III. Secrecy Rate Optimization Based on Perfect CSI

In this section, we consider the power minimization problem for a MISO secrecy channel in the presence of K multi-antenna eavesdroppers based on perfect CSI. The power minimization problem for this secrecy network can be formulated as

$$\min_{\mathbf{Q}_s \succeq \mathbf{0}} \mathrm{Tr}\left(\mathbf{Q}_s\right), \; s.t. \; \min_k R_{s,k} \geq R, \; \forall k, \quad (2)$$

where R is the predefined secrecy rate of the legitimate receiver. The problem in (2) is not convex due to the non-convex secrecy rate constraint. Hence, a relaxed problem can be formulated based on the following matrix inequality [22], [29]:

$$|\mathbf{I} + \mathbf{A}| \geq 1 + \mathrm{Tr}(\mathbf{A}), \quad (3)$$

where the equality holds if and only if $\mathrm{rank}(\mathbf{A}) = 1$. Then, the relaxed problem of (2) can be written as

$$\begin{aligned} \min_{\mathbf{Q}_s \succeq \mathbf{0}} \; & \mathrm{Tr}\left(\mathbf{Q}_s\right) \\ s.t. \; & 1 + \frac{1}{\sigma_s^2}\mathbf{h}_s^H \mathbf{Q}_s \mathbf{h}_s \geq 2^R \left[1 + \mathrm{Tr}\left(\frac{1}{\sigma_{e,k}^2}\mathbf{H}_{e,k}^H \mathbf{Q}_s \mathbf{H}_{e,k}\right)\right], \\ & \forall k. \end{aligned} \quad (4)$$

Problem (4) is a semidefinite program (SDP) problem, and the optimal solution to it has been shown to be rank-one [22]. Hence, the optimal solution to the relaxed problem (4) is easily verified to be that of the original problem (2), which confirms the tightness of this relaxation. Accordingly, we consider the following proposition.

Proposition 1: Due to the rank-one solution of the problem in (4), $\mathbf{Q}_s$ can be decomposed as $\mathbf{Q}_s = \mathbf{w}\mathbf{w}^H$, and thus the original power minimization problem for the MISO secrecy channel can be formulated into a SOCP as follows:

$$\min_{\mathbf{w}} \quad \|\mathbf{w}\|_2$$
$$s.t. \quad \begin{bmatrix} \frac{1}{\sigma_s}\mathbf{h}_s^H\mathbf{w} \\ \frac{2^{\frac{R}{2}}}{\sigma_{e,k}}\mathbf{H}_{e,k}^H\mathbf{w} \\ (2^R-1)^{\frac{1}{2}} \end{bmatrix} \succeq_K 0, \quad \forall k. \tag{5}$$

Proof: Please refer to Appendix I. ■

The problem in (5) is a standard convex optimization problem and can be solved by the interior point methods [30].

Corollary 1: For a single eavesdropper scenario, the optimal solution can be derived as

$$\mathbf{w}^* = \sqrt{p^*}\mathbf{v}^*, \quad \mathbf{v}^* = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|_2}, \quad p^* = \lambda^*(2^R-1),$$
$$\lambda^* = \frac{1}{\lambda_{max}(\frac{1}{\sigma_s^2}\mathbf{h}_s\mathbf{h}_s^H - \frac{2^R}{\sigma_e^2}\mathbf{H}_e\mathbf{H}_e^H)}, \tag{6}$$

where $\mathbf{v}_1 = v_{max}(\frac{1}{\sigma_s^2}\mathbf{h}_s\mathbf{h}_s^H - \frac{2^R}{\sigma_e^2}\mathbf{H}_e\mathbf{H}_e^H)$.

Proof: Please refer to Appendix II. ■

IV. Robust Secrecy Rate Optimization Based on Statistical Channel Uncertainty Models

In the previous section, we have solved the secrecy rate optimization problem based on the assumption that the transmitter has perfect CSI of the legitimate user and the eavesdroppers. However, in practical scenarios, perfect CSI of the legitimate user and the eavesdroppers might not be available at the legitimate transmitter due to channel estimation and quantization errors. Thus, robust secrecy rate optimization has been proposed based on the worst case secrecy rate in [22], [23], where the channel uncertainties were formulated through deterministic models. However, it is not possible that the legitimate transmitter always obtains these deterministic models accurately due to insufficient channel estimations. Therefore, we solve the robust secrecy rate optimization problems with a probabilistic secrecy rate constraint based on two statistical channel uncertainty models in the following subsections:

A. Problem Formulation

In this subsection, we consider two secrecy rate optimization (i.e., power minimization problem and secrecy rate maximization problem) frameworks with a probabilistic secrecy rate constraint. These problems can be formulated as,

$$\min_{\mathbf{Q}_s\succeq\mathbf{0}} \quad \text{Tr}(\mathbf{Q}_s),$$
$$s.t. \quad \Pr\left\{\min_k R_{s,k}\geq R\right\}\geq 1-\rho, \ \forall k, \tag{7a}$$

$$\max_{\mathbf{Q}_s\succeq\mathbf{0}} \quad R,$$
$$s.t. \quad \Pr\left\{\min_k R_{s,k}\geq R\right\}\geq 1-\rho, \ \forall k,$$
$$\text{Tr}(\mathbf{Q}_s) \leq P, \tag{7b}$$

The problems in (7) can be relaxed as

$$\min_{\mathbf{Q}_s\succeq\mathbf{0}} \quad \text{Tr}(\mathbf{Q}_s),$$
$$s.t. \quad \Pr\left\{\log(1+\frac{1}{\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s)-\log\left|\mathbf{I}+\frac{1}{\sigma_{e,k}^2}\mathbf{H}_{e,k}^H\mathbf{Q}_s\mathbf{H}_{e,k}\right|\geq R\right\}$$
$$\geq 1-\rho, \ \forall k, \tag{8a}$$

$$\max_{\mathbf{Q}_s\succeq\mathbf{0}} \quad R,$$
$$s.t. \quad \Pr\left\{\log(1+\frac{1}{\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s)-\log\left|\mathbf{I}+\frac{1}{\sigma_{e,k}^2}\mathbf{H}_{e,k}^H\mathbf{Q}_s\mathbf{H}_{e,k}\right|\geq R\right\}$$
$$\geq 1-\rho, \ \forall k,$$
$$\text{Tr}(\mathbf{Q}_s) \leq P, \tag{8b}$$

where $\rho \in (0,1]$ is the maximum allowable secrecy outage probability for the k-th eavesdropper, and P is the maximum available transmit power.

Remark: For the robust power minimization problem, the transmitter requires a certain amount of transmit power to achieve the predefined secrecy rate within the required outage probability. However, due to insufficient transmit power or due to the extremely worse channel conditions of the main channel than the eavesdropper's, the robust power minimization problem (8a) with a probabilistic secrecy rate constraint might turn out to be infeasible. To overcome this infeasibility issue, we consider the robust secrecy rate maximization problem (8b) subject to the same secrecy outage probability and transmit power constraints. Similar statement has been found in [23]. Alternatively, the physical meaning of (8b) can be interpreted as follows. Under the transmit power constraint, what the maximum secrecy rate R is that can be achieved subject to the (secrecy) outage probability less than 100 ρ % (i.e., 100 ρ %-secrecy outage capacity) [3], [31]. In order to solve (8b), we propose a two-stage algorithm. In the first stage, for any given R that makes (8a) feasible, we solve it to obtain the minimized transmit power. It is easily observed that the optimum value of R in (8b) monotonically increases with the transmit power (i.e., Tr($\mathbf{Q}_s$)). In the second stage, we update R via a bisection method [30], [32]. Hence, without loss of generality, the remaining part of our paper only focuses on solving (8a), which can be reformulated into a tractable problem by employing *Bernstein-type* inequality or *S-Procedure*, though it is non-convex.

B. Channel Uncertainty Models

In this paper, we specifically consider two statistical channel uncertainty models: Partial and full statistical channel uncertainty models.

- *Partial Channel Uncertainty Model*: Here, it is assumed that the legitimate transmitter can receive the channel estimations from the eavesdroppers, however, it has only imperfect CSI of the eavesdropper due to the limitation of the estimation or quantization errors. Accordingly, we can obtain the following channel uncertainty model:

$$\mathbf{H}_{e,k} = \bar{\mathbf{H}}_{e,k} + \mathbf{E}_{e,k}, \forall k,$$

where $\bar{\mathbf{H}}_{e,k} \in \mathbb{C}^{N_T\times N_{E,k}}$ is the estimated CSI of the k-th eavesdropper, and vec($\mathbf{E}_{e,k}$) $\sim \mathcal{CN}(0,\mathbf{R}_{e,k})$ are the corresponding statistical errors, where $\mathbf{R}_{e,k}$ is a positive semidefinite (PSD) matrix ($\succeq \mathbf{0}$).

- *Full Channel Uncertainty Model*: In this case, we consider the channel uncertainty model in which we model both the legitimate receiver and the eavesdroppers as being imperfect. The actual channels between the legitimate transmitter and the legitimate receiver as well as the k-th eavesdropper can be modelled respectively as

$$\mathbf{h}_s = \bar{\mathbf{h}}_s + \mathbf{e}_s,$$
$$\mathbf{H}_{e,k} = \bar{\mathbf{H}}_{e,k} + \mathbf{E}_{e,k}, \forall k,$$

where $\bar{\mathbf{h}}_s \in \mathbb{C}^{N_T\times 1}$, $\bar{\mathbf{H}}_{e,k} \in \mathbb{C}^{N_T\times N_{E,k}}$ are the estimated CSI, and $\mathbf{e}_s \sim \mathcal{CN}(0, \mathbf{R}_s)$, $\text{vec}(\mathbf{E}_{e,k}) \sim \mathcal{CN}(0, \mathbf{R}_{e,k})$ are the corresponding statistical errors. In addition, $\mathbf{R}_s$ and $\mathbf{R}_{e,k}$ are PSD matrices (i.e., $\mathbf{R}_s \succeq \mathbf{0}$, $\mathbf{R}_{e,k} \succeq \mathbf{0}$).

C. Robust Power Minimization Based on Partial Statistical Channel Uncertainty Models

In this subsection, we consider the power minimization problem based on the assumption of imperfect CSI only for the eavesdroppers, where we apply two conservative reformulation approaches utilizing a *Bernstein-type* inequality and *S-Procedure* to transform the probability constraint into a deterministic one. We rewrite the original problem based on partial channel uncertainty model as

$$\min_{\mathbf{Q}_s\succeq\mathbf{0}} \text{Tr}(\mathbf{Q}_s)$$
$$s.t. \Pr\left\{\log(1+\frac{1}{\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s) - \log\left|\mathbf{I}+\frac{1}{\sigma_{e,k}^2}\mathbf{H}_{e,k}^H\mathbf{Q}_s\mathbf{H}_{e,k}\right| \geq R\right\}$$
$$\geq 1-\rho, \mathbf{H}_{e,k} = \bar{\mathbf{H}}_{e,k} + \mathbf{E}_{e,k}, \text{vec}(\mathbf{E}_{e,k}) \sim \mathcal{CN}(0, \mathbf{R}_{e,k}), \ \forall k. \quad (9)$$

The above problem is not convex in terms of the probabilistic constraint of the secrecy rate. By considering the inequality in (3), the secrecy rate probability constraint can be relaxed as follows:

$$\Pr\left\{\text{Tr}(\mathbf{H}_{e,k}^H\mathbf{Q}_s\mathbf{H}_{e,k}) \leq \frac{\sigma_{e,k}^2}{2^R}(1+\frac{1}{\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s) - \sigma_{e,k}^2\right\} \geq 1-\rho,$$
$$\mathbf{H}_{e,k} = \bar{\mathbf{H}}_{e,k} + \mathbf{E}_{e,k}, \ \text{vec}(\mathbf{E}_{e,k}) \sim \mathcal{CN}(0, \mathbf{R}_{e,k}), \ \forall k. \quad (10)$$

The left hand side (LHS) of the constraint in (10) cannot be solved in terms of a closed-form expression. Thus, we consider the following reformulation for this probabilistic constraint. From the following matrix identities,

$$\text{Vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\text{Vec}(\mathbf{X}), \quad (11a)$$
$$\text{Tr}(\mathbf{A}^T\mathbf{B}) = \text{Vec}(\mathbf{A})^T\text{Vec}(\mathbf{B}), \quad (11b)$$
$$(\mathbf{A} \otimes \mathbf{B})^T = \mathbf{A}^T \otimes \mathbf{B}^T. \quad (11c)$$

The constraint in (10) can be expressed as follows:

$$\Pr\left\{\mathbf{e}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{e}_{e,k} + 2\Re\{\mathbf{e}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\} + \bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k} \leq c_k\right\} \geq 1-\rho, \ \forall k, \quad (12)$$

where $c_k = \frac{\sigma_{e,k}^2}{2^R}(1 + \frac{1}{\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s) - \sigma_{e,k}^2$, $\bar{\mathbf{h}}_{e,k} = \text{vec}(\bar{\mathbf{H}}_{e,k})$ and $\mathbf{e}_{e,k} = \text{vec}(\mathbf{E}_{e,k})$. Since $\mathbf{e}_{e,k} \sim \mathcal{CN}(0, \mathbf{R}_{e,k})$, we have the following transformation

$$\mathbf{e}_{e,k} = \mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{v}_{e,k}, \quad (13)$$

where $\mathbf{v}_{e,k} \sim \mathcal{CN}(0, \mathbf{I})$. Thus, the constraint in (12) can be equivalently reformulated as

$$\Pr\left\{\mathbf{v}_{e,k}^H\left[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\right]\mathbf{v}_{e,k} + 2\Re\left(\mathbf{v}_{e,k}^H[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}]\right) + [c_k - \bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}] \geq 0\right\} \geq 1-\rho, \ \forall k. \quad (14)$$

1) Robust Power Minimization Based on Bernstein-Type Inequality:

In order to make this probabilistic constraint more tractable, we consider the following *Bernstein-Type* inequality shown in the following lemma.

Lemma 1 [33]: For any $(\mathbf{A}, \mathbf{u}, \mathrm{c})$, where $\mathbf{A} \in \mathbb{C}^{N\times N}$ is a complex hermitian matrix, $\mathbf{u} \in \mathbb{C}^{N\times 1}$, $\mathbf{x} \sim \mathcal{CN}(0, \mathbf{I}_N)$ and $\rho \in (0, 1]$, the following inequalities hold:

$$\Pr\{\mathbf{x}^H\mathbf{Ax} + 2\Re[\mathbf{x}^H\mathbf{u}] + c \geq 0\} \geq 1-\rho, \quad (15)$$

$$\Leftarrow \begin{cases} \text{Tr}(\mathbf{A}) - \sqrt{-2\ln(\rho)}w + \ln(\rho)y + c \geq 0 \\ \left\|\begin{bmatrix}\text{vec}(\mathbf{A}) \\ \sqrt{2}\mathbf{u}\end{bmatrix}\right\| \leq w \\ y\mathbf{I}_N + \mathbf{A} \succeq \mathbf{0} \end{cases} \quad (16)$$

where w and y are slack variables. The equations in (16) are jointly convex in terms of $\mathbf{A}$, w and y. Based on *Lemma 1*, the constraint in (14) can be reformulated into the following form:

$$\text{Tr}\left[\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\right] + \sqrt{-2\ln(\rho)}w_k - \ln(\rho)y_k$$
$$-\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\text{Tr}[\mathbf{h}_s\mathbf{h}_s^H\mathbf{Q}_s] + \bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k} \leq \sigma_{e,k}^2(\frac{1}{2^R}-1), \quad (17a)$$
$$\left\|\begin{bmatrix}\text{vec}(\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}) \\ \sqrt{2}(\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k})\end{bmatrix}\right\|_2 \leq w_k, \quad (17b)$$
$$y_k\mathbf{I} - \mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}} \succeq \mathbf{0}, y_k \geq 0, \ \forall k. \quad (17c)$$

According to (17), the power minimization in (9) can be equivalently formulated as

$$\min_{\mathbf{Q}_s} \ \text{Tr}(\mathbf{Q}_s), \ \ s.t. \ (17), \ \ \mathbf{Q}_s \succeq \mathbf{0}. \quad (18)$$

The problem in (18) is convex and can be solved efficiently by using the interior-point method [34]. In order to guarantee the optimal solution $\mathbf{Q}_s$ of the problem in (18) is also the optimal solution to the original power minimization problem in (9), we have the following proposition to exploiting the rank-one property of the solution $\mathbf{Q}_s$ under the some conditions:

Proposition 2: Provided that the problem in (9) is feasible, the relaxed problem defined in (18) yields a rank-one solution based on some restricted conditions.

Proof: Please refer to Appendix III. ■

2) Robust Power Minimization Based on S-Procedure:

In this subsection, we consider another conservative reformulation for the probabilistic constraint of the robust power minimization based on *S-Procedure*. In order to set the channel uncertainty regions for (14), the following *Lemma* is required:

Lemma 2 [35]: Provided a set $\mathcal{S} \subset \mathbb{C}^{N\times 1}$ with $\Pr\{v \in \mathcal{S}\} \geq 1-\rho$ such that $\forall v \in \mathcal{S}, \mathbf{v}^H\mathbf{Av} + 2\Re\{\mathbf{v}^H\mathbf{u}\} + c \geq 0$, we equivalently obtain

$$\Pr\{\mathbf{v}^H\mathbf{Av} + 2\Re\{\mathbf{v}^H\mathbf{u}\} + c \geq 0\} \geq 1-\rho \quad (19)$$

From *Lemma 2*, given the following deterministic quadratic constraint

$$\mathbf{v}_{e,k}^H[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}]\mathbf{v}_{e,k}+2\Re\{\mathbf{v}_{e,k}^H[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}]\} +(c_k-\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k})\geq 0,\forall k, \tag{20}$$

such that $\mathbf{v}_{e,k}$ belongs to the following set

$$\mathcal{S}=\{\mathbf{v}_{e,k}|\Pr(\mathbf{v}_{e,k}^H\mathbf{v}_{e,k}\leq\gamma_{e,k}^2)\geq 1-\rho\},\forall k. \tag{21}$$

Since $\mathbf{v}_{e,k}\sim\mathcal{CN}(0,\mathbf{I}_{N_{E,k}N_T})$, it can be easily verified that $\|\mathbf{v}_e\|^2$ is a *Chi-square* random variable with degrees of freedom (DoF) $2N_{E,k}N_T$. The probability of the event in (20) with channel uncertainty regions in (21) is $1-\rho$, thus,the channel uncertainty region always holds for $\gamma_{e,k}=\sqrt{\frac{F^{-1}(1-\rho)}{2}}$, where $F^{-1}(a)$ denotes the inverse cumulative distribution function of the *Chi-square* random variable at a. Thus, the probabilistic constraint can be equivalently reformulated into the following inequalities:

$$\begin{aligned}&\mathbf{v}_{e,k}^H[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}]\mathbf{v}_{e,k}+2\Re\{\mathbf{v}_{e,k}^H[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}]\}\\&\quad+(c_k-\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k})\geq 0,\\&-\mathbf{v}_{e,k}^H\mathbf{v}_{e,k}+\gamma_{e,k}^2\geq 0.\end{aligned}$$

In order to incorporate the channel uncertainties in the robust optimization framework, we consider the following lemma:

*Lemma 3 (*S-Procedure*) [36]:* Let $f_k(\mathbf{x}), k=1,2$, be defined as

$$f_k(\mathbf{x})=\mathbf{x}^H\mathbf{A}_k\mathbf{x}+2\Re\left\{\mathbf{b}_k^H\mathbf{x}\right\}+c_k, \tag{22}$$

where $\mathbf{A}_k=\mathbf{A}_k^H\in\mathbb{C}^{n\times n}$, $\mathbf{b}_k\in\mathbb{C}^{n\times 1}$ and $c_k\in\mathbb{R}$. The implication $f_1(\mathbf{x})\geq 0\implies f_2(\mathbf{x})\geq 0$ holds if and only if there exists $\mu\geq 0$ such that

$$\begin{bmatrix}\mathbf{A}_2 & \mathbf{b}_2\\ \mathbf{b}_2^H & c_2\end{bmatrix}-\mu\begin{bmatrix}\mathbf{A}_1 & \mathbf{b}_1\\ \mathbf{b}_1^H & c_1\end{bmatrix}\succeq\mathbf{0}, \tag{23}$$

provided there exists a point $\tilde{\mathbf{x}}$ with $f_1(\tilde{\mathbf{x}})>0$.

By exploiting *S-Procedure*, the power minimization in (9) can be reformulated as follows:

$$\begin{aligned}\min_{\mathbf{Q}_s,\lambda_k}\quad &\mathrm{Tr}(\mathbf{Q}_s)\\ s.t.&\begin{bmatrix}\lambda_k\mathbf{I}-[\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}] & -\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\\ -\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}} & t_k-\lambda_k\gamma_{e,k}^2\end{bmatrix}\succeq\mathbf{0},\\ &\mathbf{Q}_s\succeq\mathbf{0},\lambda_k\geq 0,\forall k,\end{aligned} \tag{24}$$

where $t_k=(\frac{1}{2^R}-1)\sigma_{e,k}^2+\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s-\mathbf{h}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{h}_{e,k}$. The relaxed problem in (24) is a semidefinite programming (SDP) and can be solved efficiently by using convex optimization software [34]. Similarly, we also show that the optimal solution to the relaxed problem in (24) is the solution to the original problem in (9) by using the following lemma:

Proposition 3: Provided that the relaxed problem in (24) is feasible such that $R_{s,k}>0$ $(\forall k)$, the optimal solution of this problem always returns rank-one.

Proof: Please refer to Appendix IV. ■

D. Robust Power Minimization Based on Full Channel Uncertainty Model

In the previous section, we investigated the robust secrecy rate optimization based on the partial channel uncertainty model. Now we study a more challenging model with the imperfect CSI of the legitimate receiver as well as that of the eavesdroppers. Comparing with the previous model, in which the probabilistic constraint of the secrecy rate consists of only one CSI error, it is more difficult to handle the probabilistic constraint of the secrecy rate in terms of the channel estimation errors of both the legitimate receiver and the eavesdroppers. According to this framework, the original robust power minimization problem in (8a) can thus be re-expressed as

$$\begin{aligned}\min_{\mathbf{Q}_s\succeq\mathbf{0}}\quad&\mathrm{Tr}(\mathbf{Q}_s),\\ s.t.&\Pr\left\{\log(1+\frac{1}{\sigma_s^2}\mathbf{h}_s^H\mathbf{Q}_s\mathbf{h}_s)-\log\left|\mathbf{I}+\frac{1}{\sigma_{e,k}^2}\mathbf{H}_{e,k}\mathbf{Q}_s\mathbf{H}_{e,k}^H\right|\geq R\right\}\\&\geq 1-\rho,\forall k.\end{aligned} \tag{25}$$

Based on this full channel uncertainty model, we aim to solve the robust power minimization problem in (25) by converting the probabilistic constraint into a deterministic one by exploiting the *Bernstein-type* inequality and *S-Procedure*.

1) Robust Power Minimization Based on Bernstein-Type Inequality: In this subsection, we employ the *Bernstein-Type* inequality to tackle the secrecy rate probabilistic constraint in (25), which can be modified by using the matrix inequality in (3) and (11) as follows:

$$\begin{aligned}\Pr\Bigg\{&\frac{1}{\sigma_s^2}\left[\mathbf{e}_s^H\mathbf{Q}_s\mathbf{e}_s+2\Re\{\mathbf{e}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s\}+\bar{\mathbf{h}}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s\right]\\&-\frac{2^R}{\sigma_{e,k}^2}\Big[\mathbf{e}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{e}_{e,k}+2\Re\{\mathbf{e}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\}\\&+\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\Big]\geq 2^R-1\Bigg\}\geq 1-\rho,\ \forall k.\end{aligned} \tag{26}$$

The above probability constraint can be written in matrix form as follows:

$$\begin{aligned}&\Pr\Bigg\{[\mathbf{e}_s^H,\mathbf{e}_{e,k}^H]\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{Q}_s & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}(\mathbf{I}\otimes\mathbf{Q}_s)\end{bmatrix}[\mathbf{e}_s^H,\mathbf{e}_{e,k}^H]^H\\&+2\Re\left\{[\mathbf{e}_s^H,\mathbf{e}_{e,k}^H]\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{Q}_s & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}(\mathbf{I}\otimes\mathbf{Q}_s)\end{bmatrix}[\bar{\mathbf{h}}_s^H,\bar{\mathbf{h}}_{e,k}^H]^H\right\}\\&+[\bar{\mathbf{h}}_s^H,\bar{\mathbf{h}}_{e,k}^H]\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{Q}_s & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}(\mathbf{I}\otimes\mathbf{Q}_s)\end{bmatrix}[\bar{\mathbf{h}}_s^H,\bar{\mathbf{h}}_{e,k}^H]^H\geq 2^R-1\Bigg\}\\&\geq 1-\rho,\ \forall k.\end{aligned} \tag{27}$$

In order to transform the above constraint based on the *Bernstein-type* inequality as described in Subsection IV-C, we rewrite the CSI errors of the legitimate receiver and the eavesdropper as $\mathbf{e}_s=\mathbf{R}_s^{\frac{1}{2}}\mathbf{v}_s$, and $\mathbf{e}_{e,k}=\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{v}_{e,k}$, respectively, where $\mathbf{v}_s\sim\mathcal{CN}(0,\mathbf{I}_{N_T})$ and $\mathbf{v}_{e,k}\sim\mathcal{CN}(0,\mathbf{I}_{N_TN_{e,k}})$, and set $\mathbf{v}_k=[\mathbf{v}_s^H,\mathbf{v}_{e,k}^H]^H$, $\forall k$. Thus, this probability constraint can be reformulated as

$$\Pr\left\{\mathbf{v}_k^H\mathbf{A}_k\mathbf{v}_k+2\Re\{\mathbf{v}_k^H\mathbf{u}_k\}+\mathrm{c}_k\geq 0\right\}\geq 1-\rho,\ \forall k, \tag{28}$$

where

$$\mathbf{A}_k=\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}} & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\end{bmatrix},$$

$$\mathbf{u}_k=\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\end{bmatrix}[\bar{\mathbf{h}}_s^H,\bar{\mathbf{h}}_{e,k}^H]^H,$$

$$\mathrm{c}_k=[\bar{\mathbf{h}}_s^H,\bar{\mathbf{h}}_{e,k}^H]\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{Q}_s & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}(\mathbf{I}\otimes\mathbf{Q}_s)\end{bmatrix}[\bar{\mathbf{h}}_s^H,\bar{\mathbf{h}}_{e,k}^H]^H+1-2^R.$$

By applying *Lemma 1*, the constraint in (28) can be expressed as

$$\mathrm{Tr}(\mathbf{A}_k)-\sqrt{-2\ln(\rho)}w_k+\ln(\rho)y_k+\mathrm{c}_k\geq 0, \tag{29a}$$

$$\left\|\begin{bmatrix}\mathrm{vec}(\mathbf{A}_k)\\ \sqrt{2}\mathbf{u}_k\end{bmatrix}\right\|_2\leq w_k, \tag{29b}$$

$$y_k\mathbf{I}+\mathbf{A}_k\succeq\mathbf{0}, y_k\geq 0,\forall k. \tag{29c}$$

Thus, replacing the constraints (26) with (29), the power minimization problem (25) can be equivalently written as

$$\min_{\mathbf{Q}_s\succeq\mathbf{0}}\ \mathrm{Tr}(\mathbf{Q}_s),\quad s.t.\ \ (29),\ \forall k. \tag{30}$$

The problem in (30) is a convex problem, and can be solved by using interior point methods. With more complex structure of the relaxed problem in (30), it is more challenging to directly prove a rank-one solution of $\mathbf{Q}_s$. However, the following proposition guarantees a rank-one solution.

Proposition 4: Provided that the problem in (25) is feasible, the relaxed problem (30) yields a rank-one solution under some restricted conditions.

Proof: Please refer to Appendix V. ■

2) Robust Power Minimization Based on S-Procedure: In this subsection, we consider another conservative reformulation for the probabilistic constraint of the robust power minimization based on *S-Procedure* given a full channel uncertainty model. This optimization problem can be rewritten as

$$\begin{aligned}&\min_{\mathbf{Q}_s\succeq\mathbf{0}}\ \ \mathrm{Tr}(\mathbf{Q}_s)\\ &s.t.\ \Pr\Big\{\frac{1}{\sigma_s^2}(\bar{\mathbf{h}}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s+2\Re\{\mathbf{e}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s\}+\mathbf{e}_s^H\mathbf{Q}_s\mathbf{e}_s)\\ &-\frac{2^R}{\sigma_{e,k}^2}[\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}+2\Re\{\mathbf{e}_{e,k}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\}\\ &+\mathbf{e}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{e}_{e,k}]\geq 2^R-1\Big\}\geq 1-\rho,\ \forall k.\end{aligned} \tag{31}$$

In order to relax the probabilistic constraint in (31) into a deterministic one, we consider $\mathbf{e}_s=\mathbf{R}_s^{\frac{1}{2}}\mathbf{v}_s$ and $\mathbf{e}_{e,k}=\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{v}_{e,k}$, respectively, where $\mathbf{v}_s\sim\mathcal{CN}(0,\mathbf{I}_{N_T})$ and $\mathbf{v}_{e,k}\sim\mathcal{CN}(0,\mathbf{I}_{N_TN_{E,k}})$, and thus (31) can be reformulated as follows:

$$\begin{aligned}&\min_{\mathbf{Q}_s\succeq\mathbf{0}}\ \ \mathrm{Tr}(\mathbf{Q}_s)\\ &s.t.\ \Pr\Big\{\frac{1}{\sigma_s^2}(\mathbf{v}_s^H\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}}\mathbf{v}_s+2\Re\{\mathbf{v}_s^H\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\bar{\mathbf{h}}_s\}+\bar{\mathbf{h}}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s)\\ &-\frac{2^R}{\sigma_{e,k}^2}[\mathbf{v}_{e,k}^H\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{v}_{e,k}+2\Re\{\mathbf{v}_{e,k}^H\mathbf{R}_s^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\}\\ &+\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}]\geq 2^R-1\Big\}\geq 1-\rho,\ \forall k.\end{aligned} \tag{32}$$

From [37], the channel uncertainty regions are equivalently defined as follows:

$$\Rightarrow\mathbb{R}_s=\{\mathbf{v}_s:\mathbf{v}_s^H\mathbf{v}_s\leq\gamma_s^2\},\ \mathbb{R}_{e,k}=\{\mathbf{v}_{e,k}:\mathbf{v}_{e,k}^H\mathbf{v}_{e,k}\leq\gamma_{e,k}^2\}, \tag{33}$$

where $\gamma_s=\sqrt{\frac{F_s^{-1}(1-\rho)}{2}}$ and $\gamma_{e,k}=\sqrt{\frac{F_e^{-1}(1-\rho)}{2}}$; F_s^{-1} and F_e^{-1} are the inverse cumulative density function (CDF) of the *Chi-squared* distributed variables with degrees of freedom (DoF) $2N_T$ and $2N_TN_{E,k}$, respectively. Thus, we can obtain the following derivations,

$$\begin{aligned}&\min_{\mathbf{Q}_s\succeq\mathbf{0}}\mathrm{Tr}(\mathbf{Q}_s)\\ &s.t.\frac{1}{\sigma_s^2}(\mathbf{v}_s^H\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}}\mathbf{v}_s+2\Re\{\mathbf{v}_s^H\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\bar{\mathbf{h}}_s\}+\bar{\mathbf{h}}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s)\\ &-\frac{2^R}{\sigma_{e,k}^2}[\mathbf{v}_{e,k}^H\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{v}_{e,k}+2\Re\{\mathbf{v}_{e,k}^H\mathbf{R}_s^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\}\\ &+\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}]\geq 2^R-1,\\ &\quad\mathbf{v}_s^H\mathbf{v}_s\leq\gamma_s^2,\ \ \mathbf{v}_{e,k}^H\mathbf{v}_{e,k}\leq\gamma_{e,k}^2,\ \forall k.\end{aligned} \tag{34}$$

The worst-case optimization framework is developed based on the following reformulations:

$$\begin{aligned}&\min_{\mathbf{Q}_s\succeq\mathbf{0}}\mathrm{Tr}(\mathbf{Q}_s)\\ &s.t.\quad t_s-t_{e,k}\geq 2^R-1,\\ &\frac{1}{\sigma_s^2}(\mathbf{v}_s^H\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}}\mathbf{v}_s+2\Re\{\mathbf{v}_s^H\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\bar{\mathbf{h}}_s\}+\bar{\mathbf{h}}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s)\geq t_s,\\ &\frac{2^R}{\sigma_{e,k}^2}[\mathbf{v}_{e,k}^H\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{v}_{e,k}+2\Re\{\mathbf{v}_{e,k}^H\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\}\\ &+\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}]\leq t_{e,k},\\ &\quad\mathbf{v}_s^H\mathbf{v}_s\leq\gamma_s^2,\ \ \ \mathbf{v}_{e,k}^H\mathbf{v}_{e,k}\leq\gamma_{e,k}^2,\ \forall k,\end{aligned} \tag{35}$$

where $t_s>0$ and $t_{e,k}>0$ are slack variables for the achieved rate of the legitimate receiver and the k-th eavesdropper, respectively. By exploiting *S-Procedure* in *Lemma 3*, the problem can be reformulated in (36) on the top of the next page. This reformulated problem is a SDP, and can be solved efficiently by the interior-point method, and the following proposition is provided to show that the optimal solution of (36) is rank-one:

Proposition 5: The optimal solution to problem (36) can be proven to be rank-one provided that problem (25) is feasible.

Proof: Please refer to appendix VI. ■

$$\min_{\mathbf{Q}_s\succeq\mathbf{0}} \mathrm{Tr}(\mathbf{Q}_s)$$
$$s.t.\ t_s - t_{e,k} \geq 2^R - 1, \tag{36a}$$
$$\mathbf{T}_s = \begin{bmatrix} \mu_s\mathbf{I} + \frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}} & \frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\bar{\mathbf{h}}_s \\ \frac{1}{\sigma_s^2}\bar{\mathbf{h}}_s^H\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}} & \frac{1}{\sigma_s^2}\bar{\mathbf{h}}_s^H\mathbf{Q}_s\bar{\mathbf{h}}_s - t_s - \mu_s\gamma_s^2, \end{bmatrix} \succeq \mathbf{0}, \tag{36b}$$
$$\mathbf{T}_{e,k} = \begin{bmatrix} \mu_k\mathbf{I} - \frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}} & -\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k} \\ -\frac{2^R}{\sigma_{e,k}^2}\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}} & t_{e,k} - \frac{2^R}{\sigma_{e,k}^2}\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k} - \mu_{e,k}\gamma_{e,k}^2 \end{bmatrix} \succeq \mathbf{0}, \tag{36c}$$
$$\mu_s \geq 0, \mu_{e,k} \geq 0,\ \forall k. \tag{36d}$$

V. Simulation Results

In this section, we provide simulation results to validate the theoretical results derived in previous sections. In order to evaluate the performance of the proposed scheme, we consider a MISO secrecy channel with multiple multi-antenna eavesdroppers, which consists of one multi-antenna legitimate transmitter, one single-antenna legitimate receiver and three multi-antenna eavesdroppers. In addition, the legitimate transmitter has five antennas (i.e., $N_T = 5$), and each eavesdropper is equipped with three antennas (i.e., $N_{E,k} = 3,\ \forall k$). The transmit power is assumed to be 10 dB unless specified. Moreover, all of the channels are generated using zero-mean circularly symmetric independent and identically distributed (i.i.d) complex Gaussian random variables, and the noise power at the legitimate user and the eavesdroppers are assumed to be one (i.e., $\sigma_s^2 = \sigma_{e,k}^2 = 1$). The outage probabilities are set to be $\rho = 0.05$.

A. Power Minimization Based on Perfect CSI

In this subsection, we provide simulation results to validate the closed form solution derived in (6) for the special case of one legitimate user and one multi-antenna eavesdropper . In addition, the original power minimization problem can be formulated into a SOCP framework. We obtain the required transmit power by solving the SOCP, the SDP and closed-form expression for five different channels as shown in Table I where the target secrecy rate is set to be 2. From this table, it can be observed that these three results are the same, which validate the closed-form solution and the SOCP formulation.

Channels	Closed-form	Convex optimization	
		SOCP	SDP in [22]
Channel 1	1.8081	1.8081	1.8081
Channel 2	1.4943	1.4943	1.4943
Channel 3	1.1292	1.1292	1.1292
Channel 4	0.6896	0.6896	0.6896
Channel 5	1.6659	1.6659	1.6659

TABLE I: The transmit power from closed-form solution and convex optimization framework.

B. Robust Secrecy Rate Optimization with Partial Channel Uncertainties

In this subsection, we evaluate the performance of the proposed robust secrecy rate optimization by exploiting channel uncertainties of the eavesdroppers. Here, the eavesdroppers' CSI error covariance matrices have been assumed to be $\mathbf{R}_{e,k} = \varepsilon_{e,k}^2\mathbf{I}$, where $\varepsilon_{e,k}^2$ represents the channel error variance of the k-th eavesdropper. It is assumed the channel error variance $\varepsilon_{e,k}^2 = 0.01$ or 0.04 unless specified.
Fig. 1 shows the cumulative density function (CDF) of the achieved secrecy rate for the secrecy rate optimization problem, where the target secrecy rate is set to be 1. From this result, one can observe that the *Bernstein-type* inequality based scheme can satisfy the outage constraint, whereas the *S-Procedure* based scheme has a small proportion of the achieved secrecy rates that cannot satisfy the outage constraint since approximately 10 % of the secrecy rates are below the target secrecy rate. Fig. 2 represents the achieved secrecy rate

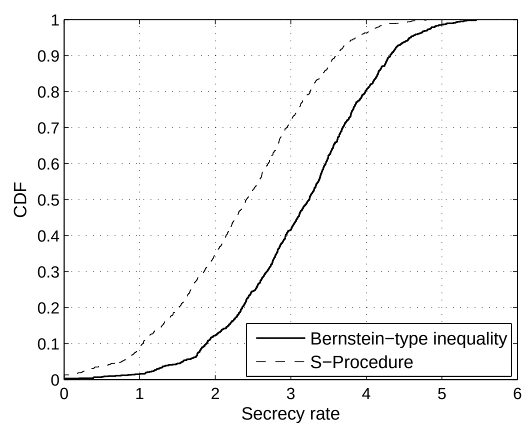

Fig. 1: The CDF of secrecy rate with partial channel uncertainties.

performance of these two robust proposed schemes based on partial channel uncertainties with different transmit powers, where these achieved secrecy rates increase with the transmit power, and the *Bernstein-type* inequality based scheme has a better performance than *S-Procedure* based scheme. The achieved secrecy rate performance of these two robust proposed schemes based on partial channel uncertainties with different error variances (i.e., $\varepsilon_{e,k}^2$) is plotted in Fig. 3. As seen in this result, the achieved secrecy rates of both robust proposed schemes and the worst-case scheme decrease with increasing error variance. Additionally, both robust proposed scheme outperform the worst-case scheme.

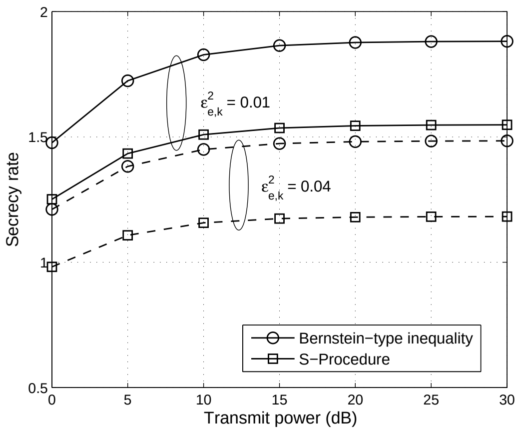

Fig. 2: The secrecy rate with different transmit powers based on partial channel uncertainties.

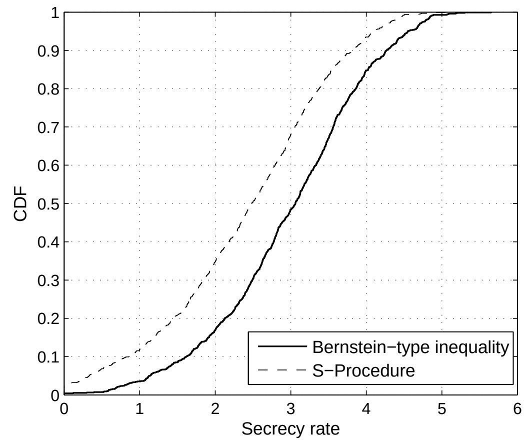

Fig. 4: The CDF of secrecy rate with full channel uncertainties.

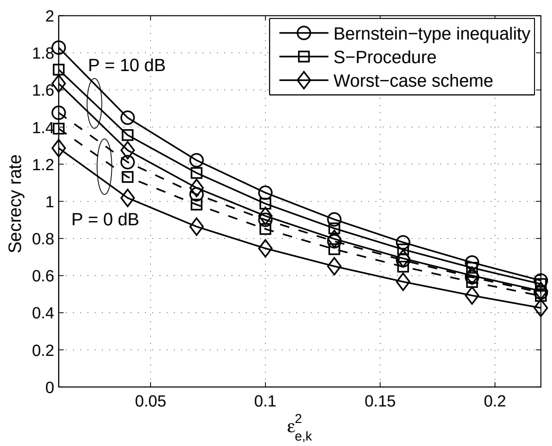

Fig. 3: The secrecy rate with different error variance based on partial channel uncertainties.

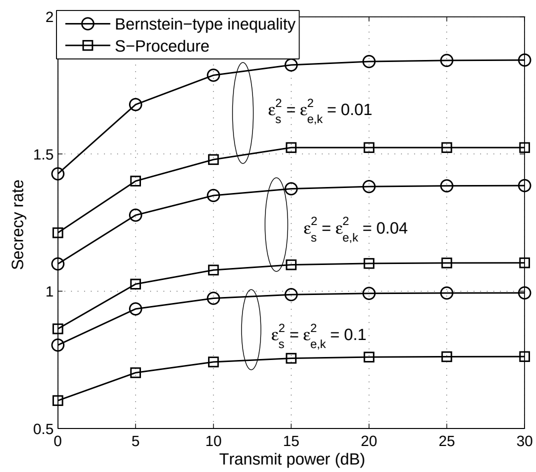

Fig. 5: The secrecy rate with different transmit powers based on full channel uncertainties.

C. Robust Secrecy Rate Optimization with Full Channel Uncertainties

Next, we provide simulation results to evaluate the performance of the robust secrecy rate optimization based on the full channel uncertainty model, where the CSI of both the legitimate user and the eavesdroppers are not available at the legitimate transmitter. The CSI error covariance matrices have been assumed to be $\mathbf{R}_s = \varepsilon_s^2\mathbf{I}$, $\mathbf{R}_{e,k} = \varepsilon_{e,k}^2\mathbf{I}$, where ε_s^2 and $\varepsilon_{e,k}^2$ represent the channel error variances of the legitimate user and the k-th eavesdropper, respectively. Here, we set the channel error variances as $\varepsilon_s^2 = \varepsilon_{e,k}^2 = 0.01$, 0.04 or 0.1.

We show the CDF of the achieved secrecy rate for the secrecy rate optimization problem in Fig. 4, where the target secrecy rate is set to be 1, and the *Bernstein-type* inequality based scheme can satisfy the outage constraint since the approximately 5 % of the achieved secrecy rates are below the target secrecy rate. However, the *S-Procedure* based scheme has approximately 10 % of the achieved secrecy rates and cannot satisfy the outage constraint, which is under the predefined secrecy rate. Fig. 5 represents the achieved secrecy rate performance of these two robust proposed schemes based on full channel uncertainties (i.e., $\varepsilon_s^2 = \varepsilon_{e,k}^2 = 0.01$, 0.04 or 0.1) with different transmit powers, where these achieved secrecy rates increase with transmit power, and the *Bernstein-type* inequality based scheme outperforms that of *S-Procedure*. The achieved secrecy rate performance of these two robust proposed schemes based on full channel uncertainties with different error variances is shown in Fig. 6. As seen in this result, the achieved secrecy rates of the proposed schemes and the worst-case schemes decrease with error variance. In addition, the *Bernstein-type* inequality based scheme outperforms the *S-Procedure* based scheme and the worst-case scheme. Moreover, the achieved secrecy rate versus the number of the eavesdroppers (i.e., K) is shown in Fig. 7. It is observed from this result that the achieved secrecy rate decreases as more eavesdroppers are in the presence. In addition, *Bernstein-type* inequality based scheme outperforms the *S-Procedure* based one in terms of the achieved secrecy rate.

VI. Conclusions

In this paper, we have studied different secrecy rate optimization techniques for a MISO secrecy channel. We first formulated the power minimization into a SOCP framework

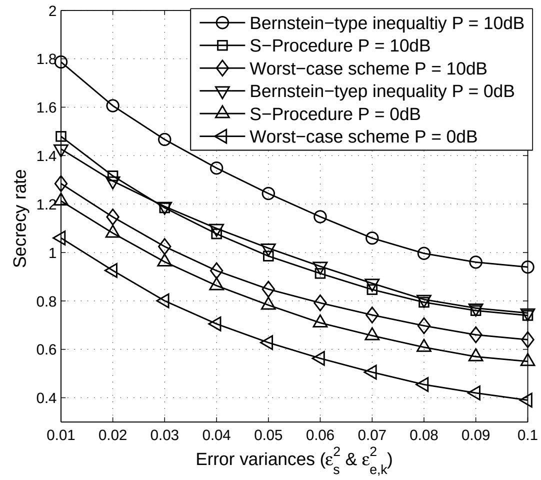

Fig. 6: The secrecy rate with different error variances based on full channel uncertainties.

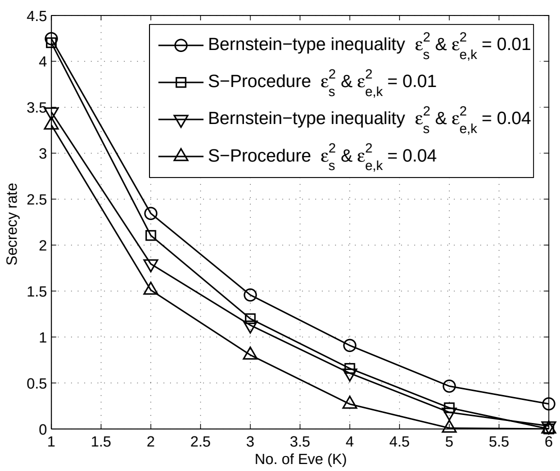

Fig. 7: The secrecy rate with different numbers of the eavesdropper based on full channel uncertainties.

for the case of a single legitimate user and multiple eavesdroppers, and derived a closed-form solution for the scenario with only one eavesdropper. In addition, the robust secrecy rate optimization problems with secrecy rate probabilistic constraints have been presented by incorporating two different statistical channel uncertainties. The original secrecy rate optimization problems were not convex in terms of the probabilistic constraint. In order to make the original problems tractable, we considered two conservative approximation approaches (i.e., *Bernstein-type* inequality and *S-Procedure*) to convert this probabilistic constraint into a deterministic one. Besides, we investigate the rank-one property of the optimal solution for our proposed robust schemes. Simulation results have been provided to validate the performance of the proposed schemes for different scenarios.

ACKNOWLEDGEMENT

The authors would like to sincerely thank the anonymous reviewers for their helpful and insightful comments.

APPENDIX I: PROOF OF PROPOSITION 1

First, due to the rank-one solution of the problem in (4), we can equivalently obtain

$$\begin{aligned}&\min_{\mathbf{w}} \ \|\mathbf{w}\|_2^2\\ &s.t. \ \frac{1+\frac{1}{\sigma_s^2}\mathbf{w}^H\mathbf{h}_s\mathbf{h}_s^H\mathbf{w}}{1+\frac{1}{\sigma_{e,k}^2}\mathbf{w}^H\mathbf{H}_{e,k}\mathbf{H}_{e,k}^H\mathbf{w}} \geq 2^R, \forall k.\end{aligned} \tag{37}$$

Then, the above problem can be modified as follows:

$$\begin{aligned}&\min_{\mathbf{w}} \ \|\mathbf{w}\|_2^2\\ &s.t. \ \frac{2^R}{\sigma_{e,k}^2}\|\mathbf{H}_{e,k}^H\mathbf{w}\|^2+(2^R-1)\leq\frac{1}{\sigma_s^2}|\mathbf{h}_s^H\mathbf{w}|^2, \forall k.\end{aligned} \tag{38}$$

From the following inequality relations

$$\begin{bmatrix} x \\ \mathbf{y} \end{bmatrix} \succeq_K \mathbf{0}, \Leftrightarrow \|\mathbf{y}\|_2 \leq x. \tag{39}$$

The original power minimization problem in (2) can be formulated into a SOCP problem as in (5).
This completes the proof of *Proposition 1*. ■

APPENDIX II: PROOF OF COROLLARY 1

First, we rewrite the problem (37) for only one eavesdropper using the equation $\mathbf{w}=\sqrt{p}\mathbf{v}$ as

$$\begin{aligned}&\min_{p,\mathbf{v}} \ p\mathbf{v}^H\mathbf{v}, \ s.t. \ \frac{\mathbf{v}^H(\mathbf{I}+\frac{p}{\sigma_s^2}\mathbf{h}_s\mathbf{h}_s^H)\mathbf{v}}{\mathbf{v}^H(\mathbf{I}+\frac{p}{\sigma_e^2}\mathbf{H}_e\mathbf{H}_e^H)\mathbf{v}} \geq 2^R,\\ &\qquad\qquad\qquad \mathbf{v}^H\mathbf{v}=1, p\geq 0.\end{aligned} \tag{40}$$

In order to solve this problem, we consider the Lagrange dual problem of (37), which can be written as,

$$\begin{aligned}L(\mathbf{w},\lambda) &= \mathbf{w}^H\mathbf{w}+\lambda 2^R(1+\frac{1}{\sigma_e^2}\mathbf{w}^H\mathbf{H}_e\mathbf{H}_e^H\mathbf{w})\\ &\qquad -\lambda(1+\frac{1}{\sigma_s^2}\mathbf{w}^H\mathbf{h}_s\mathbf{h}_s^H\mathbf{w})\\ &= \mathbf{w}^H\left(\mathbf{I}+\frac{1}{\sigma_e^2}\lambda 2^R\mathbf{H}_e\mathbf{H}_e^H-\frac{1}{\sigma_s^2}\lambda\mathbf{h}_s\mathbf{h}_s^H\right)\mathbf{w}\\ &\qquad +\lambda\left(2^R-1\right),\end{aligned} \tag{41}$$

where λ is non-negative Lagrangian multiplier. The corresponding dual problem is defined as follows:

$$\begin{aligned}&\max_{\lambda} \ \lambda\left(2^R-1\right)\\ &\text{s.t.} \ \mathbf{Z}\triangleq\mathbf{I}+\frac{1}{\sigma_e^2}\lambda 2^R\mathbf{H}_e\mathbf{H}_e^H-\frac{1}{\sigma_s^2}\lambda\mathbf{h}_s\mathbf{h}_s^H\succeq\mathbf{0},\\ &\qquad \lambda\geq 0.\end{aligned} \tag{42}$$

In order to show the strong duality between the problem in (37) and its dual problem, the Hessian matrix of the Lagrangian of the problem in (37) is derived as follows:

$$\nabla_{\mathbf{w}\mathbf{w}^H} = \mathbf{I}+\frac{1}{\sigma_e^2}\lambda 2^R\mathbf{H}_e\mathbf{H}_e^H-\frac{1}{\sigma_s^2}\lambda\mathbf{h}_s\mathbf{h}_s^H. \tag{43}$$

The strong duality holds between the primal problem and its dual problem provided the Hessian is a positive semidefinite matrix [38]. This will be satisfied provided that the original problem in (37) is feasible, which implies that the strong duality holds between the original problem (37) and its dual problem. Thus, we can derive the optimal λ^* from the positive semidefinite constraint in (42) as follows:

$$\lambda^* = \frac{1}{\lambda_{max}(\frac{1}{\sigma_s^2}\mathbf{h}_s\mathbf{h}_s^H - \frac{2^R}{\sigma_e^2}\mathbf{H}_e\mathbf{H}_e^H)}. \quad (44)$$

Note that the above equation can be obtained based on the fact $\text{Tr}(\mathbf{A}) \geq \lambda_{\max}(A)$. Thus, we can obtain the minimum power as

$$p^* = \lambda^*(2^R - 1). \quad (45)$$

In addition, the optimal $\mathbf{w}$ lies in the null space of $\mathbf{Z}$, thus

$$\mathbf{v}_1 = v_{max}(\frac{1}{\sigma_s^2}\mathbf{h}_s\mathbf{h}_s^H - \frac{2^R}{\sigma_e^2}\mathbf{H}_e\mathbf{H}_e^H), \quad \mathbf{v} = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|_2}. \quad (46)$$

This completes the proof of *Corollary 1*. ∎

APPENDIX III: PROOF OF PROPOSITION 2

We first rewrite the relaxed power minimization problem as follows:

$$\begin{aligned}
\min_{\mathbf{Q}_s} \quad & \text{Tr}(\mathbf{Q}_s), \\
s.t. \quad & \text{Tr}\left[\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\right] + \sqrt{-2\ln(\rho)}w_k - \ln(\rho)y_k \\
& -\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\text{Tr}(\mathbf{h}_s\mathbf{h}_s^H\mathbf{Q}_s) + \bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k} \leq \sigma_{e,k}^2(\frac{1}{2^R}-1), \quad (47a) \\
& \left\|\begin{bmatrix} \text{vec}(\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}) \\ \sqrt{2}(\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}) \end{bmatrix}\right\|_2 \leq w_k, \quad (47b) \\
& y_k\mathbf{I} - \mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}} \succeq \mathbf{0}, \quad (47c) \\
& y_k \geq 0, \mathbf{Q}_s \succeq \mathbf{0}, \forall k,
\end{aligned}$$

In order to show the solution to the above problem is rank-one, the SOCP constraint (47b) can be restrictedly given by

$$\begin{aligned}
&\sqrt{\|\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\|_F^2 + 2\|\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\|^2} \\
&\leq \sqrt{\|\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\|_F^2(\|\mathbf{R}_{e,k}^{\frac{1}{2}}\|_F^2 + 2\|\bar{\mathbf{h}}_{e,k}\|^2)} \\
&\leq \sqrt{\text{Tr}[(\mathbf{I}\otimes\mathbf{Q}_s)(\mathbf{I}\otimes\mathbf{Q}_s)^H]}\sqrt{\text{Tr}^2(\mathbf{R}_{e,k}) + 2\text{Tr}(\mathbf{R}_{e,k})\|\bar{\mathbf{h}}_{e,k}\|^2} \\
&\quad \leq w_k, \Rightarrow \text{Tr}[(\mathbf{I}\otimes\mathbf{Q}_s)(\mathbf{I}\otimes\mathbf{Q}_s)^H]l_k^2 \leq w_k^2, \quad (48)
\end{aligned}$$

where $l_k = \sqrt{\text{Tr}^2(\mathbf{R}_{e,k}) + 2\text{Tr}(\mathbf{R}_{e,k})\|\bar{\mathbf{h}}_{e,k}\|^2}$. By exploiting $\text{Tr}[(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D})] = \text{Tr}(\mathbf{AB}\otimes\mathbf{CD})$, $\text{Tr}(\mathbf{A}\otimes\mathbf{B}) = \text{Tr}(\mathbf{A})\text{Tr}(\mathbf{B})$ and $(\mathbf{A}\otimes\mathbf{B})^T = \mathbf{A}^T\otimes\mathbf{B}^T$, we can obtain

$$\begin{aligned}
& l_k^2 N_{E,k}\text{Tr}(\mathbf{Q}_s\mathbf{Q}_s^H) \leq w_k^2, \\
& \Rightarrow \lambda_{max}(\mathbf{Q}_s\mathbf{Q}_s^H) \leq \text{Tr}(\mathbf{Q}_s\mathbf{Q}_s^H) \leq \frac{w_k^2}{l_k^2 N_{E,k}}, \\
& \Rightarrow \mathbf{Q}_s\mathbf{Q}_s^H \preceq t_k^2\mathbf{I}, \quad \Rightarrow \mathbf{S}_k = \begin{bmatrix} t_k\mathbf{I} & \mathbf{Q}_s \\ \mathbf{Q}_s^H & t_k\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad (49)
\end{aligned}$$

where $t_k^2 = \frac{w_k^2}{l_k^2 N_{E,k}}$. Thus, the constraint in (49) can be rewritten into the following linear matrix inequality (LMI):

$$\begin{aligned}
& \begin{bmatrix} t_k\mathbf{I} & \mathbf{Q}_s \\ \mathbf{Q}_s^H & t_k\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \Rightarrow \begin{bmatrix} t_k\mathbf{I} & \mathbf{0} \\ \mathbf{0}^H & t_k\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{Q}_s \\ \mathbf{Q}_s^H & \mathbf{0} \end{bmatrix} \succeq \mathbf{0}, \\
& \begin{bmatrix} t_k\mathbf{I} & \mathbf{0} \\ \mathbf{0}^H & t_k\mathbf{I} \end{bmatrix} \succeq \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}\mathbf{Q}_s\begin{bmatrix} \mathbf{0} & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ -\mathbf{I} \end{bmatrix}\mathbf{Q}_s^H\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}, \\
& \|\mathbf{Q}_s\| \leq t_k. \quad (50)
\end{aligned}$$

In order to further reformulate the above LMI, we consider the following lemma:

Lemma 4: (*Nemirovski lemma*) [39]: For a given set of matrices $\mathbf{A} = \mathbf{A}^H$, $\mathbf{B}$ and $\mathbf{C}$, the following LMI is satisfied:

$$\mathbf{A} \succeq \mathbf{BXC} + \mathbf{C}^H\mathbf{X}^H\mathbf{B}, \|\mathbf{X}\| \leq t, \quad (51)$$

if and only if there exists non-negative real numbers a such that

$$\begin{bmatrix} \mathbf{A} - a\mathbf{C}^H\mathbf{C} & -t\mathbf{B}^H \\ -t\mathbf{B} & a\mathbf{I} \end{bmatrix} \succeq 0. \quad (52)$$

By applying *Lemma 4* to the LMI in (50), we can obtain

$$\mathbf{S}_k = \begin{bmatrix} \begin{bmatrix} t_k\mathbf{I} & \mathbf{0} \\ \mathbf{0} & t_k\mathbf{I} \end{bmatrix} - a_1\begin{bmatrix} \mathbf{0} \\ -\mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{0} & -\mathbf{I} \end{bmatrix} & -t_k\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \\ -t_k\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} & a_1\mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \quad (53)$$

From (53), we claim that constraint (49) can be equivalently rewritten without $\mathbf{Q}_s$. In order to prove rank-one of the power minimization problem, we consider the Lagrangian dual function of (47), which can be expressed in (54) on the top of the next page, where $\mathbf{Z}$, λ_k and $\mathbf{C}_k$ are dual variables associated with $\mathbf{Q}_s$, (47a) and (47c), respectively. In addition, $\mathbf{H}_k^{(n,n)} \in \mathbb{H}_+^{N_T\times N_T}$ and $\mathbf{T}_{e,k}^{(n,n)} \in \mathbb{H}_+^{N_T\times N_T}$ are block submatrices of $\mathbf{R}_{e,k} + \bar{\mathbf{h}}_{e,k}\bar{\mathbf{h}}_{e,k}^H$ and $\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{C}_k\mathbf{R}_{e,k}^{\frac{1}{2}}$, respectively, and can be expressed specifically as follows:

$$\mathbf{R}_{e,k} + \bar{\mathbf{h}}_{e,k}\bar{\mathbf{h}}_{e,k}^H = \begin{bmatrix} \mathbf{H}_k^{(1,1)} & \cdots & \mathbf{H}_k^{(1,N_{E,k})} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_k^{(N_{E,k},1)} & \cdots & \mathbf{H}_k^{(N_{E,k},N_{E,k})} \end{bmatrix} \quad (55)$$

and

$$\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{C}_k\mathbf{R}_{e,k}^{\frac{1}{2}} = \begin{bmatrix} \mathbf{T}_{e,k}^{(1,1)} & \cdots & \mathbf{T}_{e,k}^{(1,N_{E,k})} \\ \vdots & \ddots & \vdots \\ \mathbf{T}_{e,k}^{(N_{E,k},1)} & \cdots & \mathbf{T}_{e,k}^{(N_{E,k},N_{E,k})} \end{bmatrix} \quad (56)$$

We consider parts of the KKT conditions related to the proof

$$\frac{\partial\mathcal{L}}{\partial\mathbf{Q}_s} = 0, \quad (57a)$$

$$\mathbf{Z}\mathbf{Q}_s = \mathbf{0}, \quad (57b)$$

$$\mathbf{Q}_s \succeq \mathbf{0}, \; \mathbf{Z} \succeq \mathbf{0}, \; \lambda_k \geq 0, \; \mathbf{C}_k \succeq \mathbf{0}, \forall k. \quad (57c)$$

According to the KKT condition in (57a), we have

$$\mathbf{I} - \mathbf{Z} + \sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\lambda_k\mathbf{H}_k^{(n,n)} - t\mathbf{h}_s\mathbf{h}_s^H + \sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{T}_{e,k}^{(n,n)} = \mathbf{0}, (58)$$

where $t = \sum_{k=1}^{K}\frac{\lambda_k\sigma_{e,k}^2}{2^R\sigma_s^2}$. Postmultiplying the two sides of (58) by $\mathbf{Q}_s$, and based on (57b), we have

$$\left(\mathbf{I} + \sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\lambda_k\mathbf{H}_k^{(n,n)} + \sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{T}_{e,k}^{(n,n)}\right)\mathbf{Q}_s = t\mathbf{h}_s\mathbf{h}_s^H\mathbf{Q}_s, (59)$$

From (59), we claim that there is at least one λ_k, $\forall k$ such that $\lambda_k > 0$, which is shown by contradiction. If all $\lambda_k = 0$

$$L(\mathbf{Q}_s,\mathbf{Z},\lambda_k,\mathbf{C}_k)=\mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Z}\mathbf{Q}_s)+\sum_{k=1}^{K}\lambda_k\bigg[\mathrm{Tr}[(\mathbf{R}_{e,k}+\bar{\mathbf{h}}_{e,k}\bar{\mathbf{h}}_{e,k}^H)(\mathbf{I}\otimes\mathbf{Q}_s)]-\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\mathrm{Tr}(\mathbf{h}_s\mathbf{h}_s^H\mathbf{Q}_s)$$
$$+\sqrt{-2\ln(\rho)}w_k-\ln(\rho)y_k-\sigma_{e,k}^2(\frac{1}{2^R}-1)\bigg]-\sum_{k=1}^{K}\mathrm{Tr}\bigg[\mathbf{C}_k\bigg(y_k\mathbf{I}-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\bigg)\bigg]$$
$$=\mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Z}\mathbf{Q}_s)+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\lambda_k\mathrm{Tr}(\mathbf{H}_k^{(n,n)}\mathbf{Q}_s)-\sum_{k=1}^{K}\frac{\lambda_k\sigma_{e,k}^2}{2^R\sigma_s^2}\mathrm{Tr}(\mathbf{h}_s\mathbf{h}_s^H\mathbf{Q}_s)+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathrm{Tr}[\mathbf{T}_{e,k}^{(n,n)}\mathbf{Q}_s]. \quad (54)$$

for $\forall k$, then $t=0\Rightarrow\left(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{T}_{e,k}^{(n,n)}\right)\mathbf{Q}_s=\mathbf{0}$ (c.f. (59)) such that $\mathbf{Q}_s=\mathbf{0}$ due to $\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{T}_{e,k}^{(n,n)}\succ\mathbf{0}$, which implies that the legitimate transmitter does not send any information to the legitimate receiver. Thus, we claim that there exists at least one $\lambda_k>0$ such that $t>0$ holds. According to (59), the following relation of rank holds:

$$\mathrm{rank}(\mathbf{Q}_s)=\mathrm{rank}\bigg[\bigg(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\lambda_k\mathbf{H}_k^{(n,n)}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{T}_{e,k}^{(n,n)}\bigg)\mathbf{Q}_s\bigg]$$
$$=\mathrm{rank}(t\mathbf{h}_s\mathbf{h}_s^H\mathbf{Q}_s)\le\min\{\mathrm{rank}(t\mathbf{h}_s\mathbf{h}_s^H),\mathrm{rank}(\mathbf{Q}_s)\}\le 1. \quad (60)$$

This completes the proof of *Proposition 2*. ■

Appendix IV: Proof of Proposition 3

Here, we provide the proof for the rank-one solution of the power minimization problem in (24). The first step is to re-express the Lagrangian function of (24) as follows:

$$L(\mathbf{Q}_s,\mathbf{Z},\mathbf{Y}_k)=\mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Z}\mathbf{Q}_s)-\sum_{k=1}^{K}\mathrm{Tr}(\mathbf{Y}_k\mathbf{A}_k), \quad (61)$$

where

$$\mathbf{A}_k=\begin{bmatrix}\lambda_k\mathbf{I}+[-\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}] & -\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\bar{\mathbf{h}}_{e,k}\\ -\bar{\mathbf{h}}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}} & t_k-\lambda_k\gamma_{e,k}^2\end{bmatrix},$$

in addition, $\mathbf{Z}$ and $\mathbf{Y}_k$ are the dual variables associated with $\mathbf{Q}_s$ and $\mathbf{A}_k$, respectively. Then, we rewrite $\mathbf{A}_k$ for the convenience of notations.

$$\mathbf{A}_k=\begin{bmatrix}\lambda_k\mathbf{I} & \mathbf{0}\\ \mathbf{0} & (\frac{1}{2^R}-1)\sigma_{e,k}^2-\lambda_k\gamma_{e,k}^2\end{bmatrix}+\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}^H\mathbf{Q}_s\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}$$
$$-\begin{bmatrix}\mathbf{R}_{e,k}^{\frac{1}{2}} & \bar{\mathbf{h}}_{e,k}\end{bmatrix}^H(\mathbf{I}\otimes\mathbf{Q}_s)\begin{bmatrix}\mathbf{R}_{e,k}^{\frac{1}{2}} & \bar{\mathbf{h}}_{e,k}\end{bmatrix}. \quad (62)$$

From (62), the Lagrangian dual function can be rewritten as (63), where $\mathbf{S}_k^{(n,n)}\in\mathbb{H}_+^{N_T}$ is a submatrix of $\begin{bmatrix}\mathbf{R}_{e,k}^{\frac{1}{2}} & \bar{\mathbf{h}}_{e,k}\end{bmatrix}\mathbf{Y}_k\begin{bmatrix}\mathbf{R}_{e,k}^{\frac{1}{2}} & \bar{\mathbf{h}}_{e,k}\end{bmatrix}^H$ similar to Appendix III. Next, we consider KKT conditions, which can be obtained as

$$\frac{\partial L}{\partial\mathbf{Q}_s}=\mathbf{I}-\mathbf{Z}-\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}\mathbf{T}\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}^H+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_k^{(n,n)}=\mathbf{0},$$
$$\Rightarrow\mathbf{I}-\mathbf{Z}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_k^{(n,n)}=\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}\mathbf{T}\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}^H, \quad (64)$$

where $\mathbf{T}=\sum_{k=1}^{K}\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\mathbf{Y}_k$. Multiplying $\mathbf{Q}_s$ by the two sides of (64), we have

$$\bigg(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_k^{(n,n)}\bigg)\mathbf{Q}_s=\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}\mathbf{T}\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}^H\mathbf{Q}_s, \quad (65)$$

From the above equality, we show that $\mathbf{T}\neq\mathbf{0}$ by contradiction. If $\mathbf{T}=\mathbf{0}$, then we have $\left(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_k^{(n,n)}\right)\mathbf{Q}_s=\mathbf{0}$. such that $\mathbf{Q}_s=\mathbf{0}$ due to $\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_k^{(n,n)}\succ\mathbf{0}$, which violates $\mathbf{Q}_s\neq\mathbf{0}$ due to $R>0$. Thus, we claim $\mathbf{T}\succ\mathbf{0}$, and we obtain the rank-one relations:

$$\mathrm{rank}(\mathbf{Q}_s)=\mathrm{rank}\bigg(\bigg(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_k^{(n,n)}\bigg)\mathbf{Q}_s\bigg)$$
$$=\mathrm{rank}\bigg(\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}\mathbf{T}\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix}^H\mathbf{Q}_s\bigg)$$
$$\le\mathrm{rank}(\begin{bmatrix}\mathbf{0} & \mathbf{h}_s\end{bmatrix})\le 1, \quad (66)$$

This completes the proof of *Proposition 3*. ■

Appendix V: Proof of Proposition 4

In order to prove the rank-one solution of (29), we first transform this problem into the following form

$$\min_{\mathbf{Q}_s}\mathrm{Tr}(\mathbf{Q}_s)$$
$$s.t.\frac{1}{\sigma_s^2}[\mathrm{Tr}(\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\mathbf{Q}_s)+\mathrm{Tr}(\mathbf{R}_s\mathbf{Q}_s)]$$
$$-\frac{2^R}{\sigma_{e,k}^2}\mathrm{Tr}[(\bar{\mathbf{h}}_{e,k}\bar{\mathbf{h}}_{e,k}^H+\mathbf{R}_{e,k})(\mathbf{I}\otimes\mathbf{Q}_s)]+a_k\ge 0,$$
$$\begin{bmatrix}w_k\mathbf{I} & \mathbf{f}_k\\ \mathbf{f}_k^H & w_k\end{bmatrix}\succeq\mathbf{0},\quad y_k\mathbf{I}_{N_T}+\frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}}\succeq\mathbf{0}, \quad (67a)$$
$$y_k\mathbf{I}_{N_TN_{E,k}}-\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\succeq\mathbf{0}, \quad (67b)$$

where $a_k=1-2^R-\sqrt{-2\ln\rho}w_k+\ln\rho y_k$, and

$$\mathbf{f}_k=\begin{bmatrix}\mathrm{vec}\left(\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s\mathbf{R}_s^{\frac{1}{2}} & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\end{bmatrix}\right)\\ \sqrt{2}\begin{bmatrix}\frac{1}{\sigma_s^2}\mathbf{R}_s^{\frac{1}{2}}\mathbf{Q}_s & \mathbf{0}\\ \mathbf{0} & -\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\end{bmatrix}\begin{bmatrix}\bar{\mathbf{h}}_s^H & \bar{\mathbf{h}}_{e,k}^H\end{bmatrix}\end{bmatrix}. \quad (68)$$

The first SDP constraints in (67a) can also be restrictedly modified by using the similar approach as shown in Appendix III, whilst the Hermitian matrix in the second SDP constraint

$$
\begin{aligned}
L(\mathbf{Q}_s,\mathbf{Z},\mathbf{Y}_k) &= \mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Z}\mathbf{Q}_s)+\sum_{k=1}^{K}\mathrm{Tr}\bigg(\mathbf{Y}_k\left[\mathbf{R}_{e,k}^{\frac{1}{2}}\ \ \bar{\mathbf{h}}_{e,k}\right]^H(\mathbf{I}\otimes\mathbf{Q}_s)\left[\mathbf{R}_{e,k}^{\frac{1}{2}}\ \ \bar{\mathbf{h}}_{e,k}\right]\bigg)\\
&-\sum_{k=1}^{K}\mathrm{Tr}\bigg(\mathbf{Y}_k\begin{bmatrix}\lambda_k\mathbf{I} & \mathbf{0}\\ \mathbf{0} & (\frac{1}{2^R}-1)\sigma_{e,k}^2-\lambda_k\gamma_{e,k}^2\end{bmatrix}\bigg)-\sum_{k=1}^{K}\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\mathrm{Tr}\bigg(\mathbf{Y}_k\left[\mathbf{0}\ \ \mathbf{h}_s\right]^H\mathbf{Q}_s\left[\mathbf{0}\ \ \mathbf{h}_s\right]\bigg)\\
&=\mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Z}\mathbf{Q}_s)+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathrm{Tr}\Big(\mathbf{S}_k^{(n,n)}\mathbf{Q}_s\Big)-\sum_{k=1}^{K}\mathrm{Tr}\bigg(\mathbf{Y}_k\begin{bmatrix}\lambda_k\mathbf{I} & \mathbf{0}\\ \mathbf{0} & (\frac{1}{2^R}-1)\sigma_{e,k}^2-\lambda_k\gamma_{e,k}^2\end{bmatrix}\bigg)\\
&-\sum_{k=1}^{K}\frac{\sigma_{e,k}^2}{2^R\sigma_s^2}\mathrm{Tr}\bigg(\mathbf{Y}_k\left[\mathbf{0}\ \ \mathbf{h}_s\right]^H\mathbf{Q}_s\left[\mathbf{0}\ \ \mathbf{h}_s\right]\bigg), \qquad (63)
\end{aligned}
$$

---

is evidently positive definite as a result of its structure. Then, we consider the Lagrange dual function of the above problem,

$$
\begin{aligned}
&\mathcal{L}(\mathbf{Q}_s,\mathbf{Z},\lambda_k,\mathbf{B}_k,\mathbf{C}_k)=\mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Z}\mathbf{Q}_s)\\
&-\sum_{k=1}^{K}\lambda_k\bigg(\frac{1}{\sigma_s^2}[\mathrm{Tr}(\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\mathbf{Q}_s)+\mathrm{Tr}(\mathbf{R}_s\mathbf{Q}_s)]\\
&-\frac{2^R}{\sigma_{e,k}^2}\mathrm{Tr}[(\bar{\mathbf{h}}_{e,k}\bar{\mathbf{h}}_{e,k}^H+\mathbf{R}_{e,k})(\mathbf{I}\otimes\mathbf{Q}_s)]+a_k\bigg)\\
&-\sum_{k=1}^{K}\mathrm{Tr}\bigg[\mathbf{C}_k\bigg(y_k\mathbf{I}_{N_TN_{E,k}}-\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_{e,k}^{\frac{1}{2}}(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{R}_{e,k}^{\frac{1}{2}}\bigg)\bigg], \qquad (69)
\end{aligned}
$$

According to the relevant KKT condition,

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\mathbf{Q}_s} &= \mathbf{I}-\sum_{k=1}^{K}\frac{\lambda_k}{\sigma_s^2}\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H-\sum_{k=1}^{K}\frac{\lambda_k}{\sigma_s^2}\mathbf{R}_s+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\frac{\lambda_k2^R}{\sigma_{e,k}^2}\mathbf{H}_k^{(n,n)}\\
&+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\frac{2^R}{\sigma_{e,k}^2}\mathbf{R}_k^{(n,n)}-\mathbf{Z}=0, \qquad (70)
\end{aligned}
$$

where $\mathbf{H}_k^{(n,n)}\in\mathbb{H}_+^{N_T}$ is a block submatrix of $\mathbf{h}_{e,k}\mathbf{h}_{e,k}^H+\mathbf{R}_{e,k}$, and $\mathbf{R}_k^{(n,n)}\in\mathbb{H}_+^{N_T}$ is a block submatrix of $\mathbf{R}_{e,k}^{\frac{1}{2}}\mathbf{C}_k\mathbf{R}_{e,k}^{\frac{1}{2}}$. Then, setting

$$
\mathbf{T}=\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\frac{2^R}{\sigma_{e,k}^2}\Big(\lambda_k\mathbf{H}_k^{(n,n)}+\mathbf{R}_k^{(n,n)}\Big)-\bigg(\sum_{k=1}^{K}\frac{\lambda_k}{\sigma_s^2}\bigg)\mathbf{R}_s, \qquad (71)
$$

we can obtain

$$
\mathbf{Z}=\mathbf{T}-\bigg(\sum_{k=1}^{K}\frac{\lambda_k}{\sigma_s^2}\bigg)\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H. \qquad (72)
$$

From (71), it is easily verified that $\mathbf{T}\succ\mathbf{0}$ when $\lambda_k=0$, however, $\lambda_k\neq0$, as discussed in Appendix III. Thus, we only focus on the case of $\lambda_k>0$.
Setting $v=\sum_{k=1}^{K}\frac{\lambda_k}{\sigma_s^2}>0$, one can easily observe that $\mathbf{T}\succeq\mathbf{0}$ and $\mathrm{rank}(v\mathbf{h}_s\mathbf{h}_s^H)=1$ from (72). Letting $\mathrm{rank}(\mathbf{T})=r_{\mathbf{T}}$, we consider the following assumption:
if $\mathbf{T}\succ\mathbf{0}$, then this implies $r_{\mathbf{T}}=N_T$, according to [40, Lemma 5], $\mathrm{rank}(\mathbf{Z})\geq N_T-1$. We can claim $\mathrm{rank}(\mathbf{Z})\neq N_T$ due to $\mathbf{Q}_s\neq\mathbf{0}$. Thus, $\mathrm{rank}(\mathbf{Z})=N_T-1$ only when $\mathrm{rank}(\mathbf{Q}_s)=1$ due to the KKT condition $\mathbf{Z}\mathbf{Q}_s=\mathbf{0}$. Therefore, the remaining part is to show that $\mathbf{T}\succ\mathbf{0}$. By exploiting [40, Appendix D], we can conclude that $\mathbf{T}\succ\mathbf{0}$ such that $\mathrm{rank}(\mathbf{Q}_s)=1$.
This completes the proof of *Proposition 4*. ■

APPENDIX VI: PROOF OF PROPOSITION 5

In order to show the rank-one solution of the problem in (36), we rewrite $\mathbf{T}_s$ and $\mathbf{T}_{e,k}$ as follows:

$$
\mathbf{T}_s=\mathbf{\Xi}_s+\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s, \qquad (73a)
$$

$$
\mathbf{T}_{e,k}=\mathbf{\Xi}_{e,k}-\mathbf{V}_{e,k}^H(\mathbf{I}\otimes\mathbf{Q}_s)\mathbf{V}_{e,k}, \qquad (73b)
$$

where

$$
\mathbf{\Xi}_s=\begin{bmatrix}\mu_s\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -t_s-\mu_s\gamma_s^2,\end{bmatrix},\ \mathbf{V}_s=\frac{1}{\sigma_s}\left[\mathbf{R}_s^{\frac{1}{2}}\ \ \bar{\mathbf{h}}_s\right],
$$

$$
\mathbf{\Xi}_{e,k}=\begin{bmatrix}\mu_k\mathbf{I} & \mathbf{0}\\ \mathbf{0} & t_{e,k}-\mu_{e,k}\gamma_{e,k}^2\end{bmatrix},\ \mathbf{V}_{e,k}=\frac{2^{\frac{R}{2}}}{\sigma_{e,k}}\left[\mathbf{R}_{e,k}^{\frac{1}{2}}\ \ \bar{\mathbf{h}}_{e,k}\right].
$$

Then, we consider the Lagrange dual function of (36) by replacing (36b) and (36c) with (73a) and (73b), respectively.

$$
\begin{aligned}
&\mathcal{L}(\mathbf{Q}_s,\mathbf{Z},\mathbf{A}_s,\mathbf{A}_{e,k},\nu_k,\lambda_s,\lambda_{e,k})=\mathrm{Tr}(\mathbf{Q}_s)-\mathrm{Tr}(\mathbf{Q}_s\mathbf{Z})\\
&-\mathrm{Tr}(\mathbf{T}_s\mathbf{A}_s)-\sum_{k=1}^{K}\mathrm{Tr}(\mathbf{T}_{e,k}\mathbf{A}_{e,k})-\sum_{k=1}^{K}\nu_k(t_s-t_{e,k}-2^R+1)\\
&-\lambda_s\mu_s-\sum_{k=1}^{K}\lambda_{e,k}\mu_{e,k}, \qquad (74)
\end{aligned}
$$

where $\mathbf{Z}$, $\mathbf{A}_s$, $\mathbf{A}_{e,k}$, ν_k, λ_s and $\lambda_{e,k}$ are dual variables associated with $\mathbf{Q}_s$, $\mathbf{T}_s$, $\mathbf{T}_{e,k}$, μ_s, $\mu_{e,k}$, and (36a), respectively. We consider the relevant KKT conditions as follows:

$$
\frac{\partial\mathcal{L}}{\partial\mathbf{Q}_s}=\mathbf{0}, \qquad (75a)
$$

$$
\mathbf{Q_s}\mathbf{Z}=\mathbf{0}, \qquad (75b)
$$

$$
\mathbf{T}_s\mathbf{A}_s=\mathbf{0}, \qquad (75c)
$$

$$
\mathbf{A}_s\succeq\mathbf{0},\mathbf{A}_{e,k}\succeq\mathbf{0},\mathbf{Q}_s\succeq\mathbf{0},\lambda_s\geq0. \qquad (75d)
$$

From (75a), we have

$$
\frac{\partial\mathcal{L}}{\partial\mathbf{Q}_s}=\mathbf{I}-\mathbf{Z}-\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_{e,k}^{(n,n)}=0, \qquad (76)
$$

where $\mathbf{S}_{e,k}^{(n,n)}\in\mathbb{H}_+^{N_T}$ is a block submatrix of $\mathbf{V}_{e,k}\mathbf{A}_{e,k}\mathbf{V}_{e,k}^H$.

$$
\mathbf{V}_{e,k}\mathbf{A}_{e,k}\mathbf{V}_{e,k}^H=\begin{bmatrix}\mathbf{S}_{e,k}^{(1,1)} & \cdots & \mathbf{S}_{e,k}^{(1,N_{E,k})}\\ \vdots & \ddots & \vdots\\ \mathbf{S}_{e,k}^{(N_{E,k},1)} & \cdots & \mathbf{S}_{e,k}^{(N_{E,k},N_{E,k})}\end{bmatrix}. \qquad (77)
$$

By pre-multiplying $\mathbf{Q}_s$ by both sides of (76), we obtain

$$\mathbf{Q}_s\left(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_{e,k}^{(n,n)}\right)=\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H \tag{78}$$

From the above equality, one can observe the following rank relations,

$$\begin{aligned}\text{rank}(\mathbf{Q}_s) &= \text{rank}\left[\mathbf{Q}_s\left(\mathbf{I}+\sum_{k=1}^{K}\sum_{n=1}^{N_{E,k}}\mathbf{S}_{e,k}^{(n,n)}\right)\right]\\ &= \text{rank}\left(\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H\right).\end{aligned} \tag{79}$$

In order to prove $\text{rank}(\mathbf{Q}_s) \leq 1$, we need to show that $\text{rank}(\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H) \leq 1$ holds. Due to (75c), we post-multiply $\mathbf{V}_s^H$ by the two sides of this KKT condition,

$$\mathbf{\Xi}_s\mathbf{A}_s\mathbf{V}_s^H+\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H=\mathbf{0}. \tag{80}$$

As a result of the following equalities,

$$\begin{aligned}&\frac{1}{\sigma_s}\left[\mathbf{R}_s^{\frac{1}{2}}\;\;\mathbf{0}\right]\mathbf{\Xi}_s=\mu_s\left(\mathbf{V}_s-\frac{1}{\sigma_s}\left[\;\mathbf{0}\;\;\bar{\mathbf{h}}_s\;\right]\right),\\ &\frac{1}{\sigma_s}\left[\mathbf{R}_s^{\frac{1}{2}}\;\;\mathbf{0}\;\right]\mathbf{V}_s^H=\frac{1}{\sigma_s^2}\mathbf{R}_s.\end{aligned}$$

By pre-multiplying both sides of (80) by $\frac{1}{\sigma_s}\left[\;\mathbf{R}_s^{\frac{1}{2}}\;\;\;\mathbf{0}\;\right]$, we obtain

$$\begin{aligned}&\mu_s\left(\mathbf{V}_s-\frac{1}{\sigma_s}\left[\mathbf{0}\;\;\bar{\mathbf{h}}_s\right]\right)\mathbf{A}_s\mathbf{V}_s^H+\frac{1}{\sigma_s^2}\mathbf{R}_s\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H=\mathbf{0},\\ &\Rightarrow\left(\mu_s\mathbf{I}+\frac{1}{\sigma_s^2}\mathbf{R}_s\mathbf{Q}_s\right)\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H=\frac{\mu_s}{\sigma_s}\left[\mathbf{0}\;\;\bar{\mathbf{h}}_s\right]\mathbf{A}_s\mathbf{V}_s^H.\end{aligned} \tag{81}$$

Now, we provide the following two scenarios for the equality (81). First, we discuss the scenario when $\mu_s = 0$. From (73a), we have

$$\mathbf{T}_s=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -t_s\end{bmatrix}+\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s. \tag{82}$$

Assuming that $\text{rank}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s) = r_s$, it thus straightforwardly follows from (82) that

$$\begin{aligned}&\text{rank}(\mathbf{T}_s) \geq \text{rank}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s) - \text{rank}\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & t_s\end{bmatrix}=r_s-1,\\ &\Rightarrow\text{rank}(\text{null}(\mathbf{T}_s))\leq N_T+1-(r_s-1).\end{aligned} \tag{83}$$

Assuming that there exists at least one ξ that lies in the null space of $\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s$ such that $\mathbf{Q}_s^{\frac{1}{2}}\mathbf{V}_s\xi = \mathbf{0}$. This assumption holds true, since $\text{null}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s)$ is non-empty, due to $\text{rank}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s) < (N_T + 1)$. We pre-multiply ξ^H and postmultiply ξ on both sides of (82),

$$\xi^H\mathbf{T}_s\xi = \xi^H\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -t_s\end{bmatrix}\xi \geq 0. \tag{84}$$

It is easily verified that $\xi^H\mathbf{T}_s\xi = 0$ due to $t_s > 0$ and therefore,

$$\begin{aligned}&\forall\xi \in \text{null}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s) \Rightarrow \xi \in \text{null}(\mathbf{T}_s),\\ &\qquad\Rightarrow \text{null}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s) \subseteq \text{null}(\mathbf{T}_s).\end{aligned} \tag{85}$$

According to (85),

$$\begin{aligned}&\text{rank}(\text{null}(\mathbf{V}_s^H\mathbf{Q}_s\mathbf{V}_s)) \leq \text{rank}(\text{null}(\mathbf{T}_s)),\\ &\qquad\Rightarrow \text{rank}(\text{null}(\mathbf{T}_s)) \geq N_T + 1 - r_s.\end{aligned} \tag{86}$$

Combining (83) with (86), we have

$$N_T+1-r_s\leq\text{rank}(\text{null}(\mathbf{T}_s))\leq N_T+1-(r_s-1). \tag{87}$$

Since $\mathbf{T}_s\mathbf{A}_s = \mathbf{0}$,

$$N_T+1-r_s\leq\text{rank}(\mathbf{A}_s)\leq N_T+1-(r_s-1). \tag{88}$$

Accordingly, $\mathbf{A}_s$ is of the following structure:

$$\mathbf{A}_s=\sum_{i=1}^{N_T+1-r_s}\alpha_i\xi_i\xi_i^H+\beta\eta\eta^H,\;(\alpha_i>0,\forall i,\;\beta\geq 0). \tag{89}$$

If $\beta = 0$, then

$$\begin{aligned}\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H &= \mathbf{Q}_s^{\frac{1}{2}}\mathbf{Q}_s^{\frac{1}{2}}\mathbf{V}_s\left(\sum_{i=1}^{N_T+1-r_s}\alpha_i\xi_i\xi_i^H\right)\mathbf{V}_s^H\\ &= \mathbf{Q}_s^{\frac{1}{2}}\sum_{i=1}^{N_T+1-r_s}\alpha_i\left(\mathbf{Q}_s^{\frac{1}{2}}\mathbf{V}_s\xi_i\xi_i^H\mathbf{V}_s^H\right) = \mathbf{0}.\end{aligned} \tag{90}$$

Together with (79), we obtain $\text{rank}(\mathbf{Q}_s) = 0$, which contradicts to the optimality of the problem (8a). Therefore, we have $\beta > 0$ and

$$\begin{aligned}\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H &= \mathbf{Q}_s^{\frac{1}{2}}\mathbf{Q}_s^{\frac{1}{2}}\mathbf{V}_s^H\left(\sum_{i=1}^{N_T+1-r_s}\alpha_i\xi_i\xi_i^H+\beta\eta\eta^H\right)\mathbf{V}_s\\ &= \mathbf{Q}_s^{\frac{1}{2}}\left(\mathbf{0}+\beta\mathbf{Q}_s^{\frac{1}{2}}\mathbf{V}_s\eta\eta^H\mathbf{V}_s^H\right)\\ &= \beta\mathbf{Q}_s\mathbf{V}_s\eta\eta^H\mathbf{V}_s^H.\end{aligned} \tag{91}$$

One can easily observe from (91) that $\text{rank}(\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H) \leq \text{rank}(\eta\eta^H) = 1$.
Moreover, we provide the proof for the case of $\mu_s > 0$, since $\mu_s\mathbf{I} + \frac{1}{\sigma_s^2}\mathbf{R}_s\mathbf{Q}_s$ is of full-rank, according to (81),

$$\begin{aligned}&\text{rank}(\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H)\\ &\quad=\text{rank}\left[\frac{\mu_s}{\sigma_s}\left(\mu_s\mathbf{I}+\frac{1}{\sigma_s^2}\mathbf{R}_s\mathbf{Q}_s\right)^{-1}\left[\mathbf{0}\;\;\bar{\mathbf{h}}_s\right]\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^H\right]\\ &\quad\leq\text{rank}\left(\left[\mathbf{0}\;\;\bar{\mathbf{h}}_s\right]\right)\leq 1,\Rightarrow\text{rank}(\mathbf{Q}_s\mathbf{V}_s\mathbf{A}_s\mathbf{V}_s^{\mathbf{H}})\leq 1,\end{aligned} \tag{92}$$

which completes the proof of *Proposition 5*. ■

References

[1] X. Zhou, L. Song, and Y. Zhang, *Physical Layer Security in Wireless Communications*. CRC Press, 2013.

[2] A. D. Wyner, "The wire-tap channel," *Bell Syst. Tech. Journ.*, vol. 54, pp. 1355–1387, Jan. 1975.

[3] Y. Liang, H. Poor, and S. Shamai, "Secure communication over fading channels," *IEEE Trans. Inform Theory*, vol. 54, pp. 2470 –2492, Jun. 2008.

[4] M. Bloch and J. Barros, *Physical-Layer Security: From Information Theory to Security Engineering*. Cambridge University Press, 2011.

[5] Y. Liang, H. V. Poor, and S. Shamai (Shitz), "Information theoretic security," *Found. Trends Commun. Inf. Theory*, vol. 5, pp. 355–580, Apr. 2009.

[6] Y. Liang, H. V. Poor, and L. Ying, "Secure communications over wireless broadcast networks: Stability and utility maximization," *IEEE Trans. Inform. Forens. Security*, vol. 6, no. 3, pp. 682–692, Sep. 2011.

[7] A. Khisti and G. W. Wornell, "Secure transmission with multiple antennas II: The MIMOME wiretap channel," *IEEE Trans. Inform. Theory*, vol. 56, no. 11, pp. 5515–5532, Nov. 2010.

[8] L. Dong, Z. Han, A. P. Petropulu, and H. V. Poor, "Improving wireless physical layer security via cooperating relays," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1875–1888, Mar. 2010.

[9] G. Zheng, L.-C. Choo, and K.-K. Wong, "Optimal cooperative jamming to enhance physical layer security using relays," *IEEE Trans. Signal Process.*, vol. 59, no. 3, pp. 1317–1322, Mar. 2011.

[10] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, "Opportunistic relaying for secrecy communications: Cooperative jamming vs relay chatting," *IEEE Trans. Wireless Commun.*, vol. 29, no. 10, pp. 2067–2076, Jun. 2011.

[11] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, "On the application of cooperative transmission to secrecy communications," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 2, pp. 359–368, Feb. 2012.

[12] S. Goel and R. Negi, "Guaranteeing secrecy using artificial noise," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2180–2189, Jun. 2008.

[13] W.-C. Liao, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "QoS-based transmit beamforming in the presence of eavesdroppers: An optimized artificial-noise-aided approach," *IEEE Trans. Signal Process.*, vol. 59, no. 3, pp. 1202–1216, Mar. 2011.

[14] Q. Li and W.-K. Ma, "Spatially selective artificial-noise aided transmit optimization for MISO multi-eves secrecy rate maximization," *IEEE Trans. Signal Process.*, vol. 61, no. 10, pp. 2704–2717, May, 2013.

[15] H.-M. Wang, T. Zheng, and X.-G. Xia, "Secure MISO wiretap channels with multiantenna passive eavesdropper: Artificial noise vs. artificial fast fading," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 94–106, Jan. 2015.

[16] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for multi-user cognitive radios using worst-case performance optimization," in *Proc. Asilomar Conf. Sign., Syst. and Comp., Pacific Grove, CA*, pp. 378–382, Oct. 2008.

[17] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "A robust beamforming based interference control technique and its performance for cognitive radios," in *Proc. Int. Symp. Commun. and Inform. Techno. (ISCIT) 2008*, pp. 9–13, Oct. 2008.

[18] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for cognitive radios using worst-case performance optimization," in *Proc. ISITA 2008, Auckland, New Zealand*, pp. 1–5, Dec. 2008.

[19] G. Zheng, K.-K. Wong, and T.-S. Ng, "Energy-efficient multiuser SIMO: achieving probabilistic robustness with Gaussian channel uncertainty," *IEEE Trans. Commun.,*, vol. 57, no. 6, pp. 1866–1878, Jun. 2009.

[20] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4871–4881, Dec. 2009.

[21] L. Zhang, Y.-C. Liang, Y. Pei, and R. Zhang, "Robust beamforming design: From cognitive radio MISO channels to secrecy MISO channels," in *IEEE GLOBECOM 2009*, pp. 1–5, Nov 2009.

[22] Q. Li and W.-K. Ma, "Optimal and robust transmit designs for MISO channel secrecy by semidefinite programming," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3799–3812, Aug. 2011.

[23] K. Cumanan, Z. Ding, B. Sharif, G. Tian, and K. Leung, "Secrecy rate optimizations for a MIMO secrecy channel with a multiple-antenna eavesdropper," *IEEE Trans. Vehicular Technol.*, vol. 63, no. 4, pp. 1678–1690, May, 2014.

[24] Z. Chu, K. Cumanan, Z. Ding, M. Johnston, and S. Le Goff, "Secrecy rate optimizations for a MIMO secrecy channel with a cooperative jammer," *IEEE Trans. Vehicular Technol.*, vol. 64, no. 5, pp. 1833–1847, May 2015.

[25] H. Xing, K. Wong, Z. Chu, and A. Nallanathan, "Robust MIMO harvest-and-jam helpers and relaying for secret communications," *submitted to possible publication*, 2015. Available at: http://arxiv.org/abs/1502.07066.

[26] A. Mukherjee and L. A. Swindlehurst, "Robust beamforming for security in MIMO wiretap channels with imperfect CSI," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 351–361, Jan. 2011.

[27] Z. Chu, K. Cumanan, Z. Ding, M. Johnston, and S. Le Goff, "Robust outage secrecy rate optimizations for a MIMO secrecy channel," *IEEE Wireless Commun. Lett.*, vol. 4, no. 1, pp. 86–89, Feb. 2015.

[28] S. Gerbracht, C. Scheunert, and E. Jorswieck, "Secrecy outage in MISO systems with partial channel information," *IEEE Trans. Inf. Forensics Security*, vol. 7, no. 2, pp. 704–716, Apr. 2012.

[29] R. A. Horn and C. R. Johnson, *Matrix Analysis*. New York: Cambridge University Press, 1985.

[30] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[31] O. Gungor, J. Tan, C. Koksal, H. El-Gamal, and N. Shroff, "Secrecy outage capacity of fading channels," *IEEE Trans. Inf. Theory,*, vol. 59, no. 9, pp. 5379–5397, Sept., 2013.

[32] S. Ma, M. Hong, E. Song, X. Wang, and D. Sun, "Outage constrained robust secure transmission for MISO wiretap channels," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, pp. 5558–5570, Oct. 2014.

[33] I. Bechar, "A Bernstein-type inequality for stochastic processes of quadratic forms of gaussian variables," *Available[online]: http://arxiv.org/abs/0909.3595.*, 2009.

[34] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming." *Optimization Methods and Software*, Apr. 2012. http://stanford.edu/~boyd/cvx.

[35] K.-Y. Wang, A.-C. So, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "Outage constrained robust transmit optimization for multiuser MISO downlinks: Tractable approximations by conic optimization," *IEEE Trans., Signal Process.*, vol. 62, no. 21, pp. 5690–5705, Nov. 2014.

[36] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia PA: SIAM Studies in Applied Mathematics, 1994.

[37] A. Pascual-Iserte, D. Palomar, A. Perez-Neira, and M. Lagunas, "A robust maximin approach for MIMO communications with imperfect channel state information based on convex optimization," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 346–360, Jan. 2006.

[38] W. Ai and S. Zhang, "Strong duality for the CDT problem: A necessary and sufficient condition," *SIAM J. Optim.*, vol. 19, pp. 1735–1756, Feb. 2009.

[39] Y. Eldar, A. Ben-Tal, and A. Nemirovski, "Robust mean-squared error estimation in the presence of model uncertainties," *IEEE Trans. on, Signal Process.*, vol. 53, no. 1, pp. 168–181, Jan. 2005.

[40] M. Khandaker and K. Wong, "Masked beamforming in the presence of energy-harvesting eavesdroppers," *IEEE Trans. Inf. Forensics Security*, vol. 10, no. 1, pp. 40–54, Jan. 2015.

Zheng Chu received the B.Eng. degree from School of Electric and Information Engineering, Zhengzhou University of Light Industry (ZZULI), in 2007, and the M.Sc. Degree from School of Electric and Electronic Engineering, North China Electrical Power University (NCEPU), in 2011.

He also obtained M.Sc. Degree (with distinction) from School of Electrical and Electronic Engineering, Newcastle University, U.K., in 2012. Currently, he is pursuing towards the Ph.D. degree in Communications, Sensors, Signal & Information Processing (ComS^2IP) Group, School of Electrical and Electronic Engineering, Newcastle University, UK.

His research interests include physical layer security, wireless cooperative networks, wireless power transfer, convex optimization techniques and game theory.

Hong Xing received her B.Eng. degree in electronic sciences and technologies, and B.A. degree in English both from Zhejiang University (ZJU), Hangzhou, China in 2011. She is currently working towards her Ph.D. degree in the Centre for Telecommunications Research, Department of Informatics, King's College London.

From November 2012 to March 2014, she was a visiting Ph.D. student with the Department of Electrical and Computer Engineering, National University of Singapore. Her research interests include physical-layer security, cooperative communications and energy harvesting-enabled wireless communications.

Martin Johnston received his BSc (Hons) degree in Physics with Electronics from Birmingham University, UK, in 1999, his MSc degree in Electronic Engineering from Staffordshire University, UK, in 2001 and his PhD degree in 2006 from Newcastle University, UK. From 2006 - 2014 he worked as a Research Associate at the School of Electrical and Electronic Engineering in Newcastle University and he is currently working there as a Lecturer. His research interests include the design of advanced error-correcting schemes and low-complexity decoding algorithms for wireless communications and data storage devices.

Stphane Y. Le Goff received the BSc, MSc, and PhD degrees, all in electrical engineering, from the University of Western Brittany, Brest, Brittany, France, in 1990, 1991, and 1995, respectively. From 1995 to 1997, he worked as a teacher at the Institut Suprieur dElectronique de Bretagne, Brest. During 1999-2003, Dr. Le Goff was an Assistant Professor at the Emirates Telecommunications Corporation (Etisalat) College of Engineering, UAE. In 2003, he joined the Department of Physics and Electronics at the University of Waikato, Hamilton, New Zealand as a Senior Lecturer in Electronics. Since 2005, he has been a Lecturer in the School of Electrical and Electronic Engineering at Newcastle University, UK. Dr. Le Goff also held visiting positions at the Eastern Mediterranean University, Cyprus, during the Academic Year 1998-1999 and at the Sultan Qaboos University, Oman, from September to December 2004. His research interests include information theory, channel coding, and wireless communication systems.