

# Measuring Interest Rate Expectations in Canada

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- *Financial market expectations regarding future changes in the target for the overnight rate of interest are an important source of information for the Bank of Canada.*
- *Although expectations can be derived from the current yield on any short-term fixed-income asset, some assets have proven to be more effective predictors than others.*
- *The implementation of a policy of fixed announcement dates has coincided with the increased predictive powers of these short-term assets.*
- *As a result of this improvement, a relatively simple model of the yield curve can now provide an accurate measure of financial market expectations.*

The decision-making process followed by the Bank of Canada regarding the setting of the target overnight interest rate<sup>1</sup> at a given fixed announcement date (FAD) was outlined in detail in the Summer 2002 issue of the *Bank of Canada Review*.<sup>2</sup> One of the central components of this process is a major briefing by Bank staff to the Governing Council, which incorporates four important pieces of information:

- (i) an analysis of the risks around the economic projection prepared by Bank staff
- (ii) the economic perspective from the regional offices
- (iii) an analysis of information from money and credit data, and
- (iv) financial market expectations regarding policy action.

This article will focus on the last of these items, explaining why the Bank of Canada is interested in financial market expectations of future changes in the policy rate and detailing one method by which those expectations can be quantified.

Interest rate expectations can be assessed using a variety of sources, including expectations implicit in the yields of money market instruments,<sup>3</sup> surveys of private sector forecasters, published reports from investment dealers, and regular interaction with

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1. The target for the overnight interest rate is the Bank's policy rate, which is the key instrument it uses to implement monetary policy.

2. See especially Macklem (2002).

3. Money market instruments are defined as marketable interest-bearing assets with maturities of one year or less, as well as derivative products based on these instruments.

market participants. This article uses the first method: measuring expectations of future levels of the target overnight rate as implied by current money market yields. The theoretical assumptions behind the model used to measure the expectations are explained and tested. Following this, there is a demonstration of the actual derivation of implied expectations.

## Why Measure Interest Rate Expectations?

An accurate assessment of financial market expectations of future changes in the target overnight rate is important for several reasons. At the most fundamental level, the Bank of Canada attempts to influence the rate of inflation by adjusting the one policy instrument it can control directly—the target for the overnight rate of interest. The linkage between the target overnight rate and the rate of inflation follows three key steps. Step one is the effect the overnight rate has on other financial variables (longer-term interest rates, the exchange rate, and other asset prices); step two links these financial variables to aggregate demand, and then to the level of aggregate demand relative to the productive capacity of the Canadian economy (the output gap); step three moves from the output gap and expected inflation to actual inflation (Macklem 2002).

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The financial markets are the mechanism through which the first key step is realized. It is through them that changes in the target overnight rate are transmitted to the other financial variables. Since the effect of a change in the target overnight rate on these variables depends, in part, on the extent to which the change has been anticipated, it is helpful for policy-makers to be aware of the degree to which policy decisions would either constitute a surprise or are well anticipated. Unanticipated changes in the target overnight rate can lead to large changes in other financial variables, resulting in increased volatility and uncertainty.

An accurate measure of market expectations can therefore help policy-makers assess the potential impact of contemplated changes.

Interest rate expectations that are embedded in securities prices can also be a valuable source of information about how market participants view the economy. Observed market prices represent an informal consensus of the future path of interest rates that can provide a point of comparison for the other sources of economic information used by the Bank listed above, such as internal economic forecasts, regional surveys, and money and credit data. Significant differences between the economic views of the Bank of Canada and those of the market that emerge from such comparisons can also highlight issues that the Bank needs to address in future communications.

## The Expectations Hypothesis

The model detailed in this article is based upon the expectations hypothesis, perhaps the best known and most intuitive theory of the term structure of interest rates. According to this hypothesis, longer-term interest rates are determined by the expected value of future short-term interest rates (see Box 1). A long-term interest rate is thus simply the average of expected future short-term rates plus a constant risk premium.<sup>4</sup> If this hypothesis is correct, then the current level of longer-term rates can be used to estimate the expected future values of shorter-term interest rates.

The expectations hypothesis has been subjected to extensive empirical testing, and the results have generally rejected the hypothesis.<sup>5</sup> Longer-term interest rates have not been shown to be particularly useful predictors of future short rates. There are two possible explanations for this poor performance. First, longer-term interest rates have provided accurate measures of market expectations, but the expectations have proved to be inaccurate (expectational errors). And second, the risk premium assigned to longer-term rates by the market is not constant, but varies over time.

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4. The risk premium amalgamates several factors, including the liquidity premium, the term premium, and the credit spread.

5. Shiller (1990) provides a general literature review of ten studies that all reject the hypothesis. Canadian studies that also reject the hypothesis include Hejazi, Lai, and Yang (2000), Gravelle and Morley (1998), and Paquette and Strélski (1998).

## Box 1

### The Expectations Hypothesis

The expectations hypothesis maintains that long-term interest rates are rational estimators of future realized short rates, plus a constant risk premium. For the analysis in this article, the short-term interest rate was defined as the target overnight rate. The hypothesis can then be expressed as follows:

$$Y_t(r) = E \left[ \left[ \prod_{i=0}^r (1 + ON_{t+i}) - 1 \right]^{\frac{1}{r}} \middle| \Omega_t \right] + \alpha_r$$

where  $Y_t(r)$  is the  $r$ -period term rate at time  $t$ ,  $ON_t$  is the target overnight rate at time  $t$ ,  $\Omega_t$  is the information set at time  $t$ , and  $\alpha_r$  represents a constant risk premium which can be unique across different maturities.

There are two versions of the expectations hypothesis. The first, the pure expectations hypothesis, sets  $\alpha_r$  equal to zero and maintains that investor expectations of future short-term interest rates are the sole determinant of longer-term interest rates. The second version, the general expectations hypothesis, weakens this constraint slightly, allowing  $\alpha_r$  to take on

non-zero values. These values, however, must be stable over time.

In order to estimate this relationship using ordinary least squares regression, we can rearrange this relationship to become:

$$ON_{t,t+r} = -\alpha + \beta[Y_t(r)] + \varepsilon$$

where

$$ON_{t,t+r} = \left[ \prod_{i=0}^r (1 + ON_{t+i}) - 1 \right]^{\frac{1}{r}}$$

A dummy variable  $\Theta$  was added to adjust for the emergency intermeeting ease in interest rates that followed the 11 September 2001 terrorist attacks, as it would be impossible to expect financial markets to be able to properly anticipate such an event. If the date of the emergency ease falls between  $t$  and  $t+r$ ,  $\Theta$  is set to one. Otherwise,  $\Theta$  is set to zero. As well, the series was stochastically detrended by subtracting the current level of the overnight rate from both sides of the equation.

Some more recent studies (Longstaff 2000, Sack 2002, Durré, Evjen, and Pilegaard 2003), however, support the expectations hypothesis over shorter time horizons. These studies have examined both European and U.S. short-term assets using a more recent sample period (beginning in the early 1990s) that coincided with the advent of generally increased levels of transparency among central banks. This greater transparency may have reduced expectational errors and possibly allowed the expectations hypothesis to hold over the relatively short time horizons examined. As well, inflation rates became far more stationary over this period than they were in the prior decades. Relatively low, stable rates of inflation may have similarly reduced uncertainty and helped the expectations hypothesis to hold.

Recent changes made by the Bank of Canada to the way it conducts monetary policy, including increased levels of transparency and the adoption of the fixed announcement dates, may have also helped market participants to formulate more accurate expectations. Greater transparency helps to raise market participants' awareness of the Bank of Canada's view of the economy, and the fixed announcement dates have reduced much of the uncertainty over the timing of actual changes in the policy rate. If these changes have indeed helped to reduce expectational errors, then a model based on the expectations hypothesis may now be able to provide a reasonably accurate assessment of market expectations.

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## Selection of Instruments

The Canadian money market contains a wide variety of short-term marketable instruments. In theory, the market yields for all of these products should provide clues to market expectations. It is likely, however, that some instruments are more suitable than others for use in the expectations model, owing to such factors as liquidity, visibility, a wider base of investor participation, and sufficient history to allow empirical testing. Listed below are the three money market assets that appear to be most suitable in this regard.

**Treasury bills** represent short-term obligations of the Government of Canada. They are issued regularly for 3-month, 6-month, and 1-year terms. These instruments are issued at a discount, pay no coupon, and mature at par value. The secondary market for treasury bills is very liquid, with an average daily trading volume of approximately \$4.24 billion.<sup>6</sup>

**Schedule "I" bankers' acceptances (BAS)** are tradable, short-term corporate obligations that are guaranteed by the accepting banks. While they can be issued for any maturity, bankers' acceptances are typically issued for terms of one, two, three, six, and 12 months, with the majority of the issuance concentrated at three months and under. Bankers' acceptances have recently represented one of the most liquid instruments in the money market, with an average daily trading volume of approximately \$5.3 billion.<sup>7</sup>

**Foreign exchange forward implied rates.** A foreign exchange (FX) forward contract is an agreement between counterparties to exchange two currencies at a set price on a future date. The forward exchange rate

is dependent upon the current spot exchange rate and the interest rate differential between the two currencies for the term of the forward agreement.<sup>8</sup> If we know the forward price of a currency, it is possible to extract the implied-term interest rate differential over the life of the agreement. Given the forward price, the spot price, and the appropriate U.S.-dollar interest rate, it is possible to calculate the implied domestic interest rate for the equivalent term.

These specific assets meet a number of criteria. They are frequently traded liquid instruments that have a large outstanding stock or open interest. As well, prices are readily observable and a historical yield series is available for empirical testing. Other assets, including overnight index swaps, do not yet have a sufficiently large yield history to allow for testing.

## Testing the Hypothesis

The expectations hypothesis was tested over two different time periods, using the three instruments outlined above. Two time periods were used because, before implementing the fixed announcement dates in November 2000, the Bank of Canada could change the target for the overnight rate on any date. Once the Bank adopted the system of fixed announcement dates, it made a commitment to consider changes to the overnight rate on a series of eight pre-announced dates each year. Intermeeting moves, while possible, would only be made under exceptional circumstances.<sup>9</sup> The first (pre-FAD) sample covers the period from July 1996 to October 2000; the second (post-FAD) sample covers the period from November 2000 to March 2003. Splitting the sample allows an examination of whether the expectations hypothesis more accurately represents the behaviour of yields in the money market since the fixed announcement dates were adopted.

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8. In Canada, the spread between the spot and forward rates for term  $t$  is a function of the spread between U.S.-dollar LIBOR and the Canadian-dollar equivalent interest rate over the same term. The British Bankers' Association LIBOR setting is the most widely used benchmark for short-term U.S.-dollar interest rates. LIBOR stands for the London Interbank Offered Rate and is the rate of interest at which banks borrow funds from other banks in the London interbank market.

9. In the press release announcing that it was adopting a system of fixed announcement dates, the Bank stated that it would retain the option of taking action between fixed dates, although it would exercise this option only in the event of extraordinary circumstances. To date, only one intermeeting move has occurred. On 17 September 2001, the Bank lowered the overnight rate by 50 basis points following the 11 September 2001 terrorist attacks.

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6. Volume data for treasury bills are based on 2002 Investment Dealers' Association statistics.

7. Volume data for bankers' acceptances are based on 2002 Investment Dealers' Association statistics.

To test the hypothesis, the regression equation outlined in Box 1 was estimated over the two periods, using ordinary least squares. The results, detailed in Box 2, showed a substantial improvement in the predictive power of all assets in the post-FAD period. We cannot say unequivocally that the implementation of the fixed announcement dates was responsible for this improvement. However, the results support the conclusions reached by Parent (2002–2003) that, since the adoption of the fixed announcement dates, financial markets appear to have a greater focus on domestic economic conditions and a better appreciation for the elements that drive monetary policy. They further suggest that expectational errors in pricing money market assets have likely been reduced.

The results also show that, in the post-FAD period, the expectations hypothesis could not be rejected for either the 1- or 3-month terms. According to the hypothesis, the value of the  $\beta$  coefficient produced by the equation should equal one. In the post-FAD period, at least one asset in each maturity produced estimates of  $\beta$  that were not significantly different from one.

Charts 1 and 2 plot the evolution of this coefficient over time, showing that, after a relatively brief adjustment period following the implementation of the fixed announcement dates, the values of  $\beta$  started to converge towards the expected value of one.

All of the money market assets tested showed significant improvements in predictive power after the implementation of the fixed announcement dates. Some assets, however, performed better than others. Bankers' acceptances were best in the 1-month term, while all three assets performed similarly in the 3-month sector. Given the dominance of bankers' acceptances in the 1-month term, and the desirability of maintaining consistency of instruments across the yield curve, it was decided that the expectations model should use bankers' acceptance yields for both 1- and 3-month maturities.<sup>10</sup>

10. Consistency of instruments across the yield curve allows for easier interpretation of the results. It also allows for smoother interpolation between data points.

## Box 2

### Regression Results

The expectations hypothesis was tested by estimating the following regression equation:

$$ON_{t,t+r} - ON_t = -\alpha + \beta_1[Y_t(r) - ON_t] + \beta_2\Theta + \varepsilon$$

The equation was estimated over two time periods. The period from July 1996 to October 2000 represents the pre-FAD sample, and the period November 2000 to September 2002 represents the post-FAD sample. Yields from treasury bills, bankers' acceptances, and foreign exchange forwards were used as the independent variable  $Y_t(r)$ .

According to the expectations hypothesis, the value of  $\beta_1$  that the regression produces should be equal to one. If this is true, then the hypothesis cannot be rejected. One- and 3-month maturities were tested, and the results appear in the table.

### Regression Results<sup>a</sup>

1-month asset	$\beta_1$ <sup>b</sup>	$R^2$	3-month asset	$\beta_1$	$R^2$
Treasury bill pre-FAD	0.04 (0.00)	0.2%	Treasury bill pre-FAD	0.59 (0.00)	19.3%
Treasury bill post-FAD	0.69 (0.00)	67.3% (61.3%)	Treasury bill post-FAD	1.02* (0.71)	85.0% (72.3%)
Bankers' acceptances pre-FAD	0.78 (0.02)	39.5%	Bankers' acceptances pre-FAD	1.11* (0.19)	51.8%
Bankers' acceptances post-FAD	0.99* (0.89)	82.0% (76.1%)	Bankers' acceptances post-FAD	1.03* (0.96)	83.4% (70.5%)
FX implied pre-FAD	0.40 (0.00)	18.1%	FX implied pre-FAD	0.95* (0.59)	44.4%
FX implied post-FAD	0.78 (0.01)	67.1% (60.0%)	FX implied post-FAD	1.02* (0.70)	81.9% (66.3%)

a.  $R^2$  values are not directly comparable between the pre- and post-FAD periods, as the post-FAD sample includes another explanatory variable in the regression (the 11 September 2001 dummy variable). The post-FAD  $R^2$  values in brackets are the values when the dummy variable is excluded from the estimation.

b.  $P$ -values appear in brackets. When the  $p$ -value is greater than 0.10, the observation is marked with an asterisk and the expectations hypothesis cannot be rejected at the 90 per cent confidence level.



Chart 1

**Rolling Beta( $\beta$ ) Estimates: 1-Month BAs**

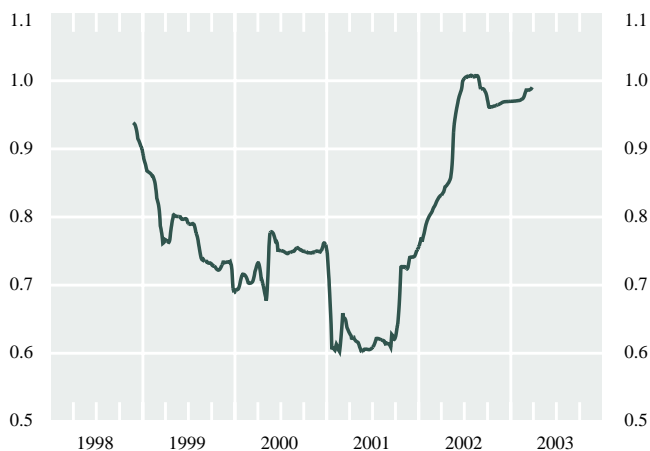
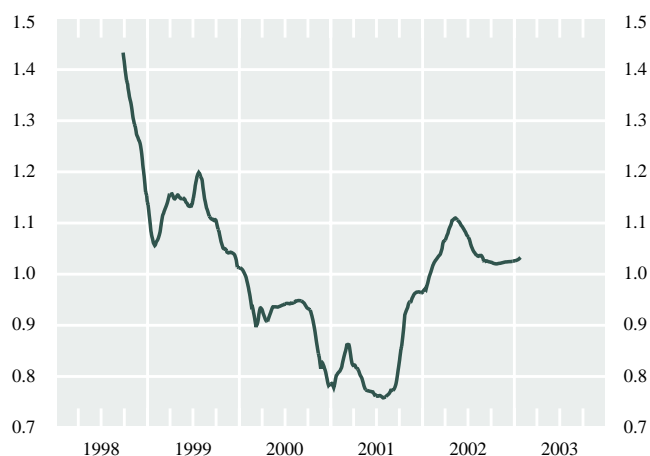


Chart 2

**Rolling Beta( $\beta$ ) Estimates: 3-Month BAs**



The results demonstrate that, while we cannot assign causality, the expectations hypothesis appears to accurately describe the mechanics of the short-term (less than three months to maturity) segment of the Canadian yield curve in the post-FAD period.

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*All of the money market assets tested showed significant improvements in predictive power after the implementation of the fixed announcement dates.*

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### Expectations Beyond Three Months

The assets examined up to this point have all had terms to maturity of three months and under. To measure market expectations beyond this 3-month horizon, we need to use instruments that have longer terms to maturity than those examined thus far. As noted above, considering the desirability of maintaining consistency of instruments across maturities, longer-maturity bankers' acceptances would be the natural choice, given that they are the assets selected for the 1- and 3-month terms. There are problems with this approach, however. While bankers' acceptances are issued with maturities of six and 12 months, these maturity tranches are relatively illiquid.

There is an alternative to using longer-maturity bankers' acceptances that still maintains the consistency of instruments. The 90-day bankers' acceptance futures contract (BAX) traded on the Montreal Exchange represents a notional amount of \$1,000,000 worth of 3-month bankers' acceptances and converges on the 3-month bankers' acceptance rate upon maturity.<sup>11</sup> These contracts are similar to the euro-dollar futures contracts in the United States and represent one of the most liquid and heavily traded instruments in the Canadian money market.<sup>12</sup> These contracts allow for the creation of "synthetic" bankers' acceptances with longer terms to maturity than three months, which can then be used to measure expectations over a longer time horizon.

To determine whether these contracts are suitable for measuring expectations, the first three contracts were tested to see if they are unbiased predictors of future

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11. For a review of the bankers' acceptance futures contract, see Harvey (1996).

12. Average daily volume as of February 2003 was over 12,000 contracts (representing a notional value of \$1.2 billion), and average open interest was approximately 85,000 contracts (Montreal Exchange 2003).

3-month bankers' acceptance rates.<sup>13</sup> The test was restricted to the first three contracts because liquidity drops off quickly beyond this point. The details of the regressions appear in Box 3.

The results support the hypothesis that, over an entire interest rate cycle, the first three BAX contracts were rational estimators of future 3-month bankers' acceptance rates. The results for the first contract were very robust, comparable to those for the 1- and 3-month bankers' acceptances. The results for the second and third contracts were less robust, although still significant, reflecting the increased uncertainty associated with longer-maturity instruments. Nevertheless, these full-period results are strong enough to warrant including the BAX contracts in the expectations model. As these contracts are widely used as indicators of interest rate expectations by market participants, the additional information likely outweighs the continued increase in uncertainty generated by moving further out the time-to-maturity spectrum.

13. The first three contracts are the three quarterly contracts that have the closest settlement dates.

## Deriving Interest Rate Expectations

The expectations hypothesis has been shown to provide a reasonably accurate description of the behaviour of yields in the Canadian money market after the implementation of the fixed announcement dates. Current market yields can therefore be used to derive implicit interest rate expectations. The first step in this derivation is to construct a yield-to-maturity curve from the observed yields of money market instruments, which can then be used to derive implied forward overnight rates.<sup>14</sup>

The observed market yields on bankers' acceptances must be adjusted for the presence of constant risk premiums. Estimated average values for the risk premiums were obtained by setting  $\beta = 1$  and re-estimating the regressions outlined above. Their values appear in Table 1.

14. Forward interest rates are implied break-even rates. They represent the level that future interest rates will have to reach in order to equate returns across assets of different maturities. For example, the 1-month forward rate is the implied rate that will equate the returns from holding two consecutive 1-month assets and holding a 2-month asset.

### Box 3

#### BAX Regression Results

The expectations hypothesis was tested using BAX contracts by estimating the following regression equation:

$$3mBA_m - 3mBA_t = -\alpha + \beta_1[BAX_t - 3mBA_t] + \beta_2\Theta + \varepsilon$$

where  $3mBA_m$  represents the 3-month BA rate at the time of the BAX contract's settlement,  $3mBA_t$  is the current 3-month BA rate at time  $t$ , and  $BAX_t$  is the BAX contract yield at time  $t$ .

According to the expectations hypothesis, the value of  $\beta_1$  that the regression produces should be equal to one. If this is true, then the hypothesis cannot be rejected. The first three contracts were tested and the results appear in the table.

#### Regression Results<sup>a</sup>

Contract	$\beta_1^b$	$R^2$
First BAX	0.98* (0.89)	45.6% (35.7%)
Second BAX	0.82* (0.07)	61.0% (38.5%)
Third BAX	0.87* (0.15)	72.9% (48.1%)

a.  $R^2$  values in brackets represent the values when the 11 September 2001 dummy variable is excluded from the regression.

b.  $P$ -values appear in brackets. When the  $p$ -value is greater than 0.05, the observation is marked with an asterisk and the expectations hypothesis cannot be rejected at the 95 per cent confidence level.

**Table 1**  
**Estimates of Risk Premiums**

Instrument	Estimated risk premiums relative to overnight rate	Estimated risk premiums relative to 3-month BAs	Total risk premiums <sup>a</sup>
1-month BA	4 bps		4 bps
3-month BA	11 bps		11 bps
First BAX	11 bps	0.1 bp per day to settlement of contract	11 bps + 0.1 bp per day to settlement
Second BAX	11 bps	15 bps	26 bps
Third BAX	11 bps	28 bps	39 bps

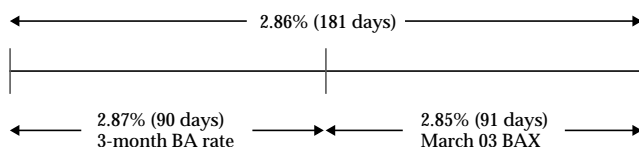
a. The estimated risk premiums for the BAX contracts are relative to the 3-month bankers' acceptance rate at the contract's expiry. As a result, the total risk premium for each contract needs to include the 3-month bankers' acceptance premium.

Once the observed market yields have been adjusted by the estimated average risk premiums, a combination of 1- and 3-month bankers' acceptance rates and the first three BAX contracts can be used to construct a yield-to-maturity curve extending to almost one year, which is generated by "rolling" together a series of bankers' acceptances and BAX futures contracts. The following example demonstrates the process, using price data from 17 December 2002, where:

3-month bankers' acceptance rate = 2.87%

first BAX (17 March 2003) rate = 2.85%

A 6-month bankers' acceptance could have been replicated by purchasing a 3-month bankers' acceptance and the first BAX contract, which settled on 17 March 2003. The first contract yielded 2.85 per cent, which effectively guaranteed a rate of 2.85 per cent on a 3-month bankers' acceptance on 17 March. The strategy is illustrated in the following timeline: the 6-month (181-day) rate is replicated by purchasing a 3-month bankers' acceptance and rolling the investment over into another 3-month bankers' acceptance at a guaranteed rate of 2.85 per cent on 17 March 2003. The effective rate from this strategy is 2.86 per cent. This process can be extended to include the next two contracts, generating spot yields for terms up to one year.



The above example uses market yields that have not been adjusted for the presence of risk premiums. To measure market expectations, it would first be necessary to adjust the observed yields by the estimated risk premiums, then to construct a yield-to-maturity curve using the method outlined above. Once this curve is constructed, it can be used to derive implied forward yields for various start dates. These forward rates are calculated by solving for the implied rate that equates the returns realized by holding a longer-term asset or by rolling over a series of daily investments at the overnight rate. Forward rates represent break-even rates: the level that future overnight rates would have to reach in order for the two investment strategies to produce equal returns.

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*Current market yields can therefore be used to derive implicit interest rate expectations.*

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Using this methodology, the yield-to-maturity curve can be used to calculate the implied future overnight rate for any specific date in the horizon under examination. Since we have restricted the yield curve to instruments maturing within approximately one year, it is only possible to calculate forward rates for 12 months. This period, however, covers the horizon that is of most interest to policy-makers. As we extend beyond one year, market expectations become increasingly uncertain.

## Measuring Expectations: An Example

In the following example, market expectations for the target overnight rate are derived using observed money market yields from 8 May 2002, when market expectations were anticipating gradual increases in the overnight rate. The actual money market yields are outlined in Table 2.

Following the process of rolling together a combination of 3-month bankers' acceptances and BAX contracts that was outlined above, these adjusted yields are then used to create the yield-to-maturity curve that appears in Table 3.

This final adjusted yield-to-maturity curve can then be used to extract the implied forward overnight rates for



**Table 2**

**8 May 2002 Yields**

Instrument	Maturity	Yield	Estimated risk premium	Adjusted yield
Overnight rate	9 May 2002	2.25%	0	2.25%
1-month BA	10 June 2002	2.34%	4 bps	2.30%
3-month BA	12 Aug. 2002	2.55%	11 bps	2.44%
First BAX	17 June to 17 Sept. 2002 <sup>a</sup>	2.73%	15 bps	2.58%
Second BAX	17 Sept. to 16 Dec. 2002	3.20%	26 bps	2.94%
Third BAX	16 Dec. 2002 to 17 Mar. 2003	3.78%	39 bps	3.39%

a. The first BAX contract on 8 May was the June 02 contract. This contract settled on 17 June 2002, meaning the yield on the futures contract represented the future expected 3-month yield between 17 June and 17 September.

**Table 3**

**8 May 2002 Yield-to-Maturity Curve**

Term	Yield to maturity
1-day – 9 May 2002	2.25%
1-month – 8 June 2002	2.29%
3-month – 8 August 2002	2.43%
6-month – 8 November 2002	2.60% <sup>a</sup>
9-month – 8 February 2003	2.80%

a. The 6- and 9-month spot yields were calculated by rolling together a 3-month bankers' acceptance and a series of BAX contracts (as described in the previous section).

upcoming fixed announcement dates. The implied forward rate  $f$  can be extracted from the yield-to-maturity curve using the following equation:

$$f = \frac{(1 + z_{(a+b)})^{(a+b)}}{(1 + z_a)^a}$$

where  $z$  is the yield-to-maturity for a specific term,  $a$  is the start time for the forward rate (i.e., the time to the FAD date in question), and  $b$  is the term-to-maturity of the forward rate (i.e., the 1-day rate).

It is these implied forward overnight rates that provide estimates for the expected level of the target overnight rate at a given date. This expected level can then further be transformed into probabilities ( $P$ ) that

the overnight rate will change from one level ( $ON_t$ ) to another ( $ON_{t+1}$ ). The formula for calculating the probability  $P$  is as follows:

$$P = \frac{(f - ON_t)}{(ON_{t+1} - ON_t)}$$

where  $f$  is the implied forward overnight rate at the time of the specific fixed announcement date.

Some caveats to these probability calculations should be noted. The implied overnight rate gives the market's current expectation of what the level of the target overnight rate will be on a given date. It does not give any information about the path that rates could take to reach that level. The probability calculation used assumes that the overnight rate can take one of only two possible discrete values at the next fixed announcement date. This is clearly an oversimplification, as there are some non-zero probabilities that the overnight rate could assume a wider range of possible values. While the results in Table 4 suggest that the market had priced in an increase to the overnight rate of 25 basis points with 100 per cent certainty, it is also possible that expectations were split, with 50 per cent of participants expecting no move and the other 50 per cent expecting an increase of 50 basis points. Market convention, however, is to base probabilities on discrete intervals of 25 basis points, as historically, the Bank has limited its changes to the target overnight rate to increments of 25 (e.g., 25, 50, or 75 basis points).

In this example, the 9-month horizon spans six fixed announcement dates. The implied overnight rates following each of these six dates and the associated probabilities are shown in Table 4.

**Table 4**

**Implied Overnight Rates as of 8 May 2002**

Fixed announcement date	Implied overnight rate	Probability of rate change
4 June 2002	2.50%	100% of an increase to 2.50%
16 July 2002	2.54%	16% of a further increase to 2.75%
4 September 2002	2.70%	80% of an increase to 2.75%
16 October 2002	2.86%	44% of a further increase to 3.00%
3 December 2002	3.03%	12% of a further increase to 3.25%
21 January 2003	3.35%	40% of a further increase from 3.25 to 3.50%

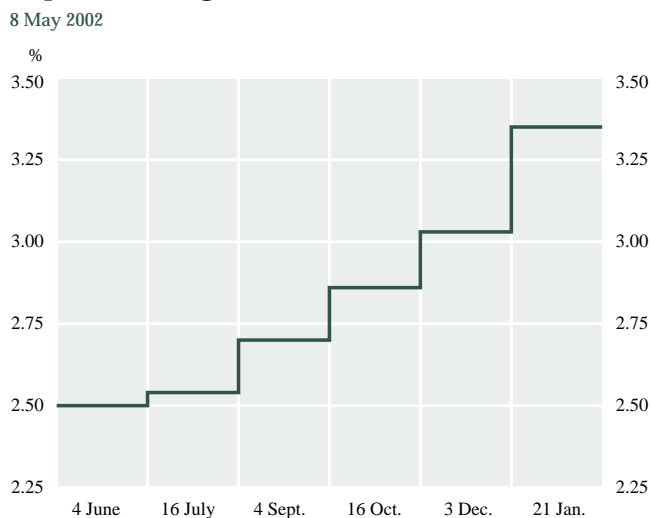
The implied overnight rate is the 1-day forward rate for the fixed announcement date in question. As Table 4 shows, on 8 May 2002, the money market had fully priced in an increase in the overnight rate from 2.25 per cent to 2.50 per cent for the fixed announcement date of 4 June. Progressively more tightening in the rate was priced in for subsequent fixed announcement dates, reaching 110 basis points by the fixed announcement date of 21 January 2003. The expected path of the target overnight rate is illustrated in Chart 3.

## Conclusion

The results of this analysis support the conclusion that, while no causal link has been proven, the shift to the system of fixed announcement dates has coincided with the substantially increased predictive powers of the short-term assets examined. As well, the expectations hypothesis now appears to be a reasonably accurate representation of the mechanics of the short end of the Canadian yield curve. As a result, a relatively simple model based on the expectations hypothesis provides accurate measures of market expectations.

The results obtained from this quantitative model can then be combined with other, more qualitative, assess-

**Chart 3**  
**Implied Overnight Rates**



ments of market expectations, including survey results and general market commentary, and the resulting market expectations can then be communicated to Governing Council as part of the major briefing.

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