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Non-Bank Dealing and Liquidity Bifurcation in Fixed-Income Markets

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Abstract

Non-bank financial institutions, such as principal-trading firms and hedge funds, increasingly compete with bank-owned dealers in fixed-income markets. Some market participants worry that if non-bank financial institutions push out established bank dealers, liquidity will become unreliable during times of stress. We model non-bank entry and state-dependent liquidity provision. Non-bank participants improve liquidity more during normal times than in stress, leading to a bifurcation of liquidity. In the cross-section, their entry improves liquidity for large and previously unserved small clients; however, banks may no longer provide reliable liquidity to marginal clients. Central bank lending may limit harmful bifurcation during times of stress if that lending is predictable and at sufficiently favourable terms.

Topics: Economic models, Financial institutions, Financial markets, Market structure and pricing
JEL codes: G10, G20, G21, G23, L10, L13, L14

Résumé

Les institutions financières non bancaires, comme les sociétés de négociation pour compte propre et les fonds de couverture, rivalisent de plus en plus avec les courtiers affiliés aux banques sur les marchés de titres à revenu fixe. Certains participants aux marchés craignent que, si les institutions financières non bancaires repoussent les courtiers affiliés aux banques, la liquidité devienne incertaine en période de tensions. Nous modélisons l'entrée des institutions non bancaires sur ces marchés et l'octroi de liquidités en fonction de la conjoncture. Les participants non bancaires améliorent davantage la liquidité en temps normal qu'en période de turbulences, ce qui crée une disparité entre les niveaux de liquidité. Dans l'échantillon représentatif des investisseurs, l'entrée des participants non bancaires améliore la liquidité pour les clients importants et les petits clients qui n'étaient pas servis auparavant. Toutefois, les banques pourraient ne plus fournir de liquidités fiables aux clients marginaux. Les prêts consentis par les banques centrales pourraient limiter cette disparité nuisible en période de tensions si l'octroi de prêts est prévisible et les conditions d'emprunt assez favorables.

Sujets : Modèles économiques; Institutions financières; Marchés financiers; Structure de marché et fixation des prix
Codes JEL : G10, G20, G21, G23, L10, L13, L14

In modern fixed income markets, expanding competition in liquidity provision has improved trading costs for investors. In corporate bond markets, Hendershott et al. (2021) estimate that the direct impact of increased competition for liquidity provision reduces trading costs by 10 to 20 percent. Li and Schürhoff (2019) provide evidence that improved trading costs through better prices arise when clients are not served only by a small subset of core dealers, further echoing the benefits of intermediary competition. However, incumbent dealers are raising the alarm that the increased competition can have serious unintended consequences.

In government bond markets, banks have historically assumed the primary intermediary role. These bank dealers argue that during times of stress, they provide liquidity to institutional investors who require guaranteed access to liquidity to meet demands of margin calls or redemptions. If bank dealers must now compete with non-bank intermediaries (e.g., principal trading firms and hedge funds), the concern is that increased competition in 'good times' will reduce their capacity to absorb liquidity demands during times of stress—when non-bank intermediaries would withdraw—leading to greater market instability.[1] Hence, focusing on liquidity improvements in stable markets ignores this important role that bank dealers play, and competition from non-bank intermediaries may worsen liquidity during times of stress.

In this paper, we study how financial stress affects competition in fixed income markets with a focus on the impacts to market liquidity. We develop a model of client relationships and competition between an incumbent bank that competes with an entrant non-bank. In our model, a bank that chooses to become a dealer is obligated to act as a dedicated market maker. If they enter, they are obligated to provide liquidity at all times. Thus, they enter if it is profitable to do so ex-ante, in expectation. A non-bank may improve competition but has the ability to provide only state-dependent liquidity. That is, they may enter only in states where it is profitable to do so ex-post. The result is a form of liquidity bifurcation, where

[1] See Bloomberg (2024) for one of many examples of popular coverage on this topic.

liquidity diverges non-linearly between normal and stressed times, in otherwise identical assets.

Our first contribution is to show that this bifurcation occurs not just between states of the world, but also in the cross-section of investors: large investors benefit when non-banks enter the market, whereas smaller investors who were marginal clients of bank dealers may suffer. This contribution speaks to a key existing theoretical result on the increased value of dealer-client relationships during times of stress (Carlin et al. 2007). In our model, because new entrants may disrupt these relationships, marginal clients of large dealers may lose this stress-time value. Our second contribution is to discuss a policy response. We show how standard central bank crisis lending may attenuate harmful liquidity bifurcation. In our model, central banks may prevent harmful bifurcation by lending to bank dealers at sufficiently favourable terms, increasing their expected profits from trading during stressed states and encouraging them to retain marginal clients.

Trading in our model takes place in two venues: a client market and an inter-dealer market. In the client market, a bank dealer and a non-bank dealer may purchase bonds from an investor. The bank represents a traditional dealer who represents a designated market maker, while the non-bank represents a "principal trading firm" that makes markets opportunistically.[2] In the client market, the dealers act as oligopolists, competing in imperfect competition with each other. In the inter-dealer market, the bank and non-bank dealers may trade with an additional inter-dealer broker (IDB) participant in a continuously priced double auction.[3] In our model, the IDB participant proxies for all other dealers not trading directly with the investor in question.

Trade in our model is motivated through inventory frictions. The investor pays an inventory cost on the bonds which they initially hold, motivating them to sell. Dealers and the

[2]While principal trading firms have not traditionally been registered as dealers, we refer to them as such within the model, as they engage in dealer-like activities.

[3]In practice, principal trading firms now account for an important share of trading in inter-dealer markets. (G30 Working Group on Treasury Market Liquidity 2021). As early as 2015, 8 of the top 10 firms by volume on U.S. inter-dealer market Brokertec were non-banks (Risk.net 2015).

IDB participant must pay a cost proportional to any bonds which they buy, which we refer to as a funding cost. We treat this funding cost as stochastic, representing either normal or stressed funding conditions. During stressed conditions, the dealers and IDB participants bear a higher cost of inventory, disincentivizing the purchase of bonds from the investor.

Before a dealer can trade with the investor, they must pay to establish a relationship. This relationship cost can be seen as a combination of the human resource, regulatory, compliance and other costs involved in establishing and servicing a client account. A bank dealer is treated as an incumbent designated market maker, giving them the first opportunity to take the investor as a client. They must make this choice before the non-bank dealer enters and before the realized funding cost is known. Thus, if the bank enters, it must make markets in both states and may only select entry across client types. A non-bank dealer is treated as an opportunistic entrant who may selectively make markets. Before paying the cost to establish a relationship with the investor, the non-bank may view the investor's relationship with the bank dealer as well as the realized funding cost. Thus, they may select entry based off client type, state of the world and whether they are unserved by existing dealers.

We show that the entry of non-banks can create a bifurcation of liquidity across states of the world in otherwise safe assets. In our model, that means that for some investors, trading volume diverges non-linearly between normal times and stress times when non-bank dealers have the ability to enter the market. We argue that bifurcation can be harmful, or beneficial, depending on the context. The entry of non-banks during normal times can incentivize bank dealers not to enter the market, as expected profits over the combined states would be negative. If the same non-banks do not serve investors during stress, a harmful bifurcation occurs. We show that this harmful bifurcation exists when the difference in funding costs between normal and stress states is large. Alternatively, a non-bank may enter and compete with the existing bank during some states of the world. In this case, liquidity may bifurcate, but the investor remains (weakly) better off during both normal and stress times.

Our model predicts that whether liquidity bifurcation is beneficial or harmful differs in the cross-section of investors. Investors who trade large volumes benefit from the inclusion of non-bank dealers: non-banks enter to serve these clients, while banks still profit from serving them. Similarly, small investors may also benefit, if they are served by non-banks after having not previously been profitable for bank dealers. The investors who suffer from bifurcation are those who were marginal clients of bank dealers before the entry of non-banks. If non-banks compete for these marginal clients during the more profitable times, it can be optimal for banks to no longer serve them at all. As a result, they may only receive service in beneficial states of the world and receive no service during times of stress.

An alternative interpretation of our model is in the context of primary issuance. In practice, bank dealer exit from primary markets may spur the entry to non-banks, which may participate with less regularity (Allen et al. 2024). Interpreted in this manner, the entry of non-banks into primary markets would result in a bifurcation in liquidity in the cross-section of issuances. Larger issuances, such as shorter duration, on-the-run bonds may benefit when non-banks enter primary markets. At the same time, smaller issuances such as inflation-protected debt or sub-national debt may suffer during times of stress if marginal bank dealers leave the market. In that sense, our model advises that the entry of non-banks into primary markets may benefit the largest, most liquid issuers, at the expense of others during times of stress.

Next, we turn to questions of policy. We explore the role of central banks during times of market stress, in the presence of non-bank dealers. One concern is if non-banks take increasing roles as intermediaries, central bank actions to provide liquidity in times of stress will be less effective. Indeed, some central banks have either opened or considered opening crisis facilities for non-banks. For example, in a 2023 speech, the Bank of England's Andrew Hauser outlined the case for expanding central bank lending directly to non-banks (Hauser 2023). In our model, the central bank acts during stress, replacing the high funding cost with a lower one. This can represent a classic central bank lending relationship, where banks can engage in repo trades, or other borrowing, with the central bank to fund their inventories.

We show that, in the presence of non-bank dealers, central bank lending to banks has a lower proportional benefit to investors. In our model, this proportional benefit is represented by the amount traded to investors when the central bank acts during stressed times, compared to the alternative where it does not. This lower impact is driven by two factors: first, non-banks may not receive direct central bank support, and second, their entry into the market decreases the relative inventory cost of all intermediaries, even absent the central bank. This is not to say that the central bank causes liquidity to worsen when non-banks are present; instead, since they may have already improved liquidity absent a central bank action, the impact of the central bank action itself is less drastic. Of particular interest, the central bank may be able to prevent or limit harmful liquidity bifurcation, as it increases bank profits during stress times, increasing their total ex-ante profit expectations and incentivizing entry for a broader set of clients.

0.1 Related Literature

Our model explains the implications of a major change in modern fixed income markets: non-banks that play roles which are increasingly similar to traditional dealers (Eren and Wooldridge 2021). This role has been acknowledged by regulators, such as the U.S. Securities and Exchange Commission (SEC), who adopted a series of rules to expand the central clearing of U.S. Treasury transactions (U.S. Securities and Exchange Commission 2023) and to expand the definition of dealers and government securities dealers to participants engaging in "significant liquidity-providing roles" (U.S. Securities and Exchange Commission 2024). In our model, non-bank dealers are subject to the same inventory frictions as bank dealers.[4] Going forward, Duffie (2023) argues that non-bank dealers could help to stabilize U.S. Treasury markets and improve liquidity. In that sense, our model provides theoretical guidance

[4]In Appendix B we discuss non-banks with a lower cost of inventory than banks.

for a world in which non-banks that engage in dealer-like activities are treated similarly to bank dealers.

Our model focuses on competition in dealing markets and funding frictions. The use of competition and funding frictions has a long history in the literature (Ho and Stoll 1980; Brunnermeier and Pedersen 2009). These frictions act as a complementary friction to others in fixed income markets, such as the search frictions in Duffie et al. (2005). We choose inventory frictions because bank-owned dealers are known to face balance-sheet constraints in the fixed income dealing, even in highly traded markets such as U.S. Treasuries. One source of balance-sheet limit may be post–Great Financial Crisis (GFC) regulation (Cimon and Garriott 2019; Saar et al. 2023). Empirically, changes in liquidity from post-GFC regulation are well documented (Bao et al. 2018; Wu 2020). The ability of these dealers to fund trades, often using repo, is a key part of these constraints (Huh and Infante 2021) and is a key ingredient of our model.

We seek to explain why liquidity may differ sharply during different states of the world. During financial crises, dealers may lack the ability to perform their intermediation roles between buying and sellers. In the COVID-19 crisis, dealers in multiple markets were unable to fully intermediate buyers and sellers (Fontaine et al. 2021; Fleming et al. 2022). As a result, illiquidity may rise non-linearly as dealers approached their balance sheet limits (Allen and Wittwer 2023; Duffie et al. 2023). As a contributing factor, the inter-dealer market may itself become less liquid during times of stress (Benos and Žikeš 2018). Our model provides theoretical support, showing how liquidity may suffer from non-linearities between normal and stressed times and how the inter-dealer market may play a role.

In this paper, we model how central bank lending may differentially impact bank and non-bank dealers and how it impacts competitiveness and liquidity provision outside of crises. Existing work shows that central bank facilities for banks may not fully pass through to non-banks (d'Avernas et al. 2020) or may pass through to non-banks differently depending on if they are outright purchases as opposed to lending (Breckenfelder and Hoerova 2023). To that end, Liang and Parkinson (2020) propose that central banks could open a standing facility for non-bank dealers. However, central bank access for non-banks could have systemic risk implications which would require further regulatory engagement (Aramonte et al. 2023).

1 Model

There are four agents: an investor, a bank dealer, a non-bank dealer and an inter-dealer broker (IDB) participant. The bank dealer ("the bank"), non-bank dealer ("the non-bank") and IDB participant ("the IDB") are referred to collectively as "intermediaries."

There is one perfectly divisible asset with total supply I held initially by an investor. Similar to Baldauf et al. (2024), we consider agents who trade the asset on multiple marketplaces, at some spread from its true value, and who incur a quadratic cost to hold it. The investor faces a constant holding cost γ_I; intermediaries, however, face a state-dependent holding cost γ^i. We denote states as $i \in \{H, L\}$ where $\mathsf{Pr}(H) = \delta$, and let $\gamma^H > \gamma^L$.

There are two markets: a client market and an inter-dealer market. The client market may contain the investor, the bank and the non-bank. In the client market, the investor is a price taker who submits a price-quantity pair, while the bank and non-bank are Cournot competitors who each choose a quantity to buy. The inter-dealer market may contain the bank, the non-bank and the IDB. In the inter-dealer market, all participants are price takers who submit price-quantity schedules which clear in a continuously priced double auction. The markets clear at endogenous spreads s_C and s_D, respectively.

In $t = 0$, the investor submits their ask schedule in the client market. The bank then chooses whether to take on the investor as a client. In $t = 1$, the state-dependent cost γ^i is realized and becomes public knowledge. The non-bank views the bank's decision and the state and chooses whether to take on the investor as a client. In $t = 2$, all intermediaries submit trades simultaneously.

1.1 Investor

There is a bond investor who begins with the entire supply I of the bond. To interact with the investor, each dealer (bank and/or non-bank) must independently pay a cost wasteful c, which is not remitted to the investor. This cost proxies for all the costs of taking on a new client, such as regulatory and compliance issues.

In state i, the bond investor has a profit function:

$$\pi_I^i = -s_C^i * X - \frac{\gamma_I}{2} * (I - X)^2, \tag{1}$$

where s_C^i is the spread in the client market, taken as given by the investor, and X is the quantity they sell. For mathematical simplicity, we normalize $\gamma_I = 1$.

The investor is a price taker. They maximize their profit by selecting quantity $X(s_C^i)$, which they are willing to sell at any given price. This price-quantity ask schedule is then made available to the bank and non-bank prior to them selecting whether to take them on as a client.

1.2 Dealer: Bank

The bank serves as an incumbent. At the beginning of $t = 0$, the bank chooses whether to pay the cost c to take the investor as a client. This entry represents a designated market maker, who makes markets in all states of the world. Thus, the bank may select across investor traits, but not across states of the world. If they choose to interact with the investor, they may participate in the client and inter-dealer markets; otherwise, they exit the game.

The bank has an expected profit function

$$E_i[\pi_B^i] = \epsilon_B * \left(E_i \left[s_C^i(x_B^i) * x_B^i - s_D^i * q_B^i - \frac{\gamma^i}{2} * (x_B^i - q_B^i)^2 \right] - c \right), \tag{2}$$

where $s_C^i(x_B^i)$ and s_D^i are the spreads in client and inter-dealer markets, respectively, the latter of which they take as given. x_B^i is the amount the bank buys from the investor, q_B^i is the amount they sell to the IDB, and ϵ_B represents the bank's entry choice.

In the client market, the bank is a Cournot competitor. They view the investor's price-quantity ask schedule and select x_B^i to maximize their profit, taking the actions of the non-bank as given. In the inter-dealer market, the bank is a price taker. They maximize their profit by selecting a price-quantity ask schedule $q_B^i(s_D^i)$. To simplify derivation of the equilibrium, we allow the bank (and later the non-bank) to submit bids in each market which are contingent on their traded quantity in the other market. For example, the bank's quantity bought in the client market may be dependent on its quantity bought in the dealer market, and vice versa.[5]

The bank enters if their expected profit, given the probability of each γ_i and the non-bank's best response in each case, is greater than zero. If the bank enters $\epsilon_B = 1$, and $\epsilon_B = 0$, otherwise.

1.3 Dealer: Non-Bank

There is a non-bank dealer, referred to as the "non-bank," who serves as a challenger. After viewing the state-dependent funding cost γ^i in $t = 1$, and the bank's entry decision, the non-bank may choose to pay the cost c to take the investor as a client. This entry represents an opportunistic market maker: they may select across investor traits, states of the world and whether the investor is already served by the bank. If they choose to interact with the investor, they may participate in the client and inter-dealer markets; otherwise, they leave the game.

[5] In some respects, this creates a market structure which is similar in nature to multi-product auctions. See Klemperer (2010) for one example of such an auction format with differentiated goods.

For a given γ^i, the non-bank has a profit function:

$$\pi_N^i = \epsilon_N^i * \left(s_C^i(x_N^i) * x_N^i - s_D^i * q_N^i - \frac{\gamma^i}{2} * (x_N^i - q_N^i)^2 - c \right), \tag{3}$$

where x_N^i is the amount the non-bank buys from the investor, q_N^i is the amount they sell to the IDB and ϵ_N^i is the non-bank's entry choice. To isolate the impact of the non-bank's ability to participate in a state-dependent manner, we endow it with the same inventory cost γ^i as the bank. In practice, risk limits of various types play an important role at non-banks such as hedge funds (Kruttli et al. 2023). In Appendix B, we discuss some implications of a non-bank with a lower cost of inventory.

The non-bank competes in both the client and dealer markets identically to the bank. In the client market, they are a Cournot competitor. They view the client's price-quantity ask schedule and select x_N^i to maximize their profit, taking the actions of the bank as given. In the inter-dealer market, they are a price taker. They maximize their profit by selecting a price-quantity ask schedule $q_N^i(s_D^i)$.

The non-bank enters if its profit, given the observed γ_i and observed entry decision of the bank (ϵ_B), is greater than zero. In each state i, $\epsilon_N^i = 1$ if the non-bank enters and $\epsilon_N^i = 0$ otherwise.

1.4 IDB Participant

There is an IDB participant who may buy bonds in the inter-dealer market from the bank and non-bank. Their profit in state i is given:

$$\pi_D^i = s_D^i * Q^i - \frac{\gamma^i}{2} * (Q^i)^2, \tag{4}$$

where Q_i is the amount the IDB buys collectively from the bank and non-bank.

The IDB dealer is a price taker. They maximize their profit by selecting a price-quantity bid schedule $Q^i(s_D^i)$.

Remark 1 (Interpretation as Government Debt Issuance) *An alternative interpretation of the model is the primary market for government debt. The investor's role can be viewed as the government issuing new debt, while the IDB participant could be viewed as other market participants without direct access to the auction. Traditionally, government debt markets have been restricted to a core group of primary dealers. However, there have recently been calls to open government debt auctions more broadly, including to non-bank dealers. We expand on this interpretation later in Section 3.*

1.5 Market Clearing and Equilibrium

In the client market, in each state i, the market clears at a price $s_C^i(X^i)$, along the ask schedule submitted by the investor. The dealers compete in Cournot competition, submitting quantities x_B^i and x_N^i, given this ask schedule. The market clears at a quantity such that

$$X^i = x_B^i + x_N^i. \tag{5}$$

The inter-dealer market, in each state i, clears in a continuously priced double auction at a price s_D^i, given the price-quantity pairs submitted by all intermediaries, such that

$$Q^i(s_D^i) = q_B^i(s_D^i) + q_N^i(s_D^i). \tag{6}$$

The equilibrium is described in Definition 1, while the timing and decisions made by all agents are illustrated in Figure 1

Definition 1 (Definition of an Equilibrium) *An equilibrium, solved through backwards induction, consists of* *(i)* ***A triplet of entry decisions:*** $\boldsymbol{\epsilon} = (\epsilon_B, \epsilon_N^L, \epsilon_N^H)$*. An entry decision by the bank (*ϵ_B*), such that* $\epsilon_B = 1$ *if* $E_i[\pi_B^i] \geq 0$*, given the expected* ϵ_N^i*, traded quantities and market clearing prices, in each state* i*. An entry decision by the non-bank in each state* i *(*ϵ_N^i*), such that* $\epsilon_N^i = 1$ *if* $\pi_N^i \geq 0$ *given the realized* ϵ_B*, and expected traded quantities and market clearing prices.*

(ii) ***Investor ask schedule:*** *An investor bid-schedule* $X^i(s_C^i)$*, such that* $X^i(s_C^i)$ *maximizes Equation 1.*

(iii) ***Intermediary quantities:*** *In each state* i*, for any investor ask schedule* $X^i(s_C^i)$ *and each entry triplet* $\boldsymbol{\epsilon}$*: client-market quantities* x_B^i *and* x_N^i*, which maximize Equations 2 and 3, respectively, taking each other quantity as given; dealer-market quantities,* q_B^i*,* q_N^i *and* Q^i*, which maximize Equations 2, 3 and 4, respectively, given* s_D^i *and taking each other quantity as given.*

(iv) ***Market clearing prices:*** *In each state* i*, prices* s_C^i *and* s_D^i *such that Equations 5 and 6 hold.*

2 Baseline: Bank Dealer Only

First, we consider a baseline in which the non-bank is unable to enter ($\epsilon_N^i = 0 \; \forall \; i$). In this baseline, the investor can only possibly be served by the bank and, if so, receives service in both normal and stressed states. We use this baseline to illustrate the impacts when the non-bank is given the opportunity to enter in Section 3.

First, for a state i, the investor's first-order condition reveals a pricing curve:

$$s_C^i = I - X^i, \tag{7}$$

which is known to the bank. This pricing curve remains identical in all subsequent sections.

Figure 1: Timing and Decisions

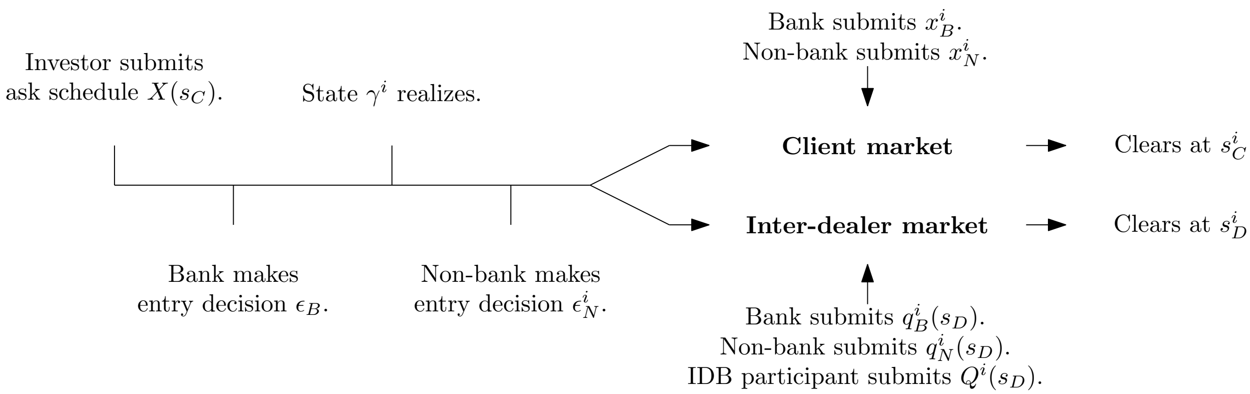

Figure 1 illustrates the timing of the model. In $t = 0$, the investor submits an ask schedule $X(s_C)$. The bank then makes its entry decision ϵ_B. In $t = 1$, the funding cost state γ^i is realized. The non-bank then views the bank's entry decision and the funding cost and makes its entry decision ϵ_N^i. In $t = 2$, the bank and non-bank submit quantities x_B^i and x_N^i to the client market, respectively, and the bank, non-bank and IDB participant submit quantities-price pairs $q_B^i(s_D)$, $q_N^i(s_D)$ and $Q^i(s_D)$ to the inter-dealer market, respectively. The client market clears at price s_C^i and the inter-dealer market clears at price s_D^i.

In each state i, the inter-dealer market clearing price (s_D^i), the amount supplied by the bank in the client market (x_B^i) and the inter-dealer market (q_B^i), and the amount supplied by the IDB in the inter-dealer market (Q^i), are defined by the joint solution to

$$0 = I - 2x_B^i - \gamma^i * (x_B^i - q_B^i), \tag{8}$$

$$s_D^i = \gamma^i * (x_B^i - q_B^i), \tag{9}$$

$$s_D^i = \gamma^i Q^i, \tag{10}$$

$$Q^i = q_B^i. \tag{11}$$

The first two are derived from the bank's first-order conditions from x_B^i and q_B^i, respectively; the third is from the IDB's first-order condition with respect to Q^i; and the fourth is an imposition of market clearing.

The resulting quantities in the client and inter-dealer market are

$$x_B^i = X^i = \frac{2I}{\gamma^i + 4}, \tag{12}$$

$$q_B^i = Q^i = \frac{I}{\gamma^i + 4}. \tag{13}$$

The spreads in the client and inter-dealer market are

$$s_C^i = \frac{I * (\gamma^i + 2)}{\gamma^i + 4}, \tag{14}$$

$$s_D^i = \frac{I\gamma^i}{\gamma^i + 4}. \tag{15}$$

Prices and quantities in both markets follow a common pattern. For a given γ^i, the dealer trades more in the client market than in the inter-dealer market, retaining some of the inventory on their own account. At the same time, the spread is larger in the client market than in the inter-dealer market, representing a spread earned by the dealer. Quantities are decreasing and spreads are increasing in the funding cost; thus, a lower quantity is traded, at a worse price for the investor, during the stress state.

The bank's profit in each state i, excluding the entry cost, is given:

$$\pi_B^i = \frac{I^2 * (\gamma^i + 8)}{2(\gamma^i + 4)^2}. \tag{16}$$

The bank chooses to enter if

$$c \leq \delta\pi_B^H + (1 - \delta)\pi_B^L. \tag{17}$$

3 Non-Bank Entry

Next, we consider a non-bank who may opportunistically enter after the state has been realized. The result is three within-state equilibrium types: competitive states where the bank and non-bank enter ($\epsilon_B = \epsilon_N^i = 1$), non-competitive states where only the bank or non-bank enter ($\epsilon_B = 1$ or $\epsilon_N^i = 1$), and finally unserved states, where there is no dealer present ($\epsilon_B = \epsilon_N^i = 0$). The bank entry decision and non-bank's state-dependent entry decisions can then be uniquely determined through backwards induction.

3.1 Competitive states ($\epsilon_B = \epsilon_N^i = 1$)

In competitive states, the bank and non-bank take each other's liquidity supply to the investor as given. In each state i, the inter-dealer market clearing price (s_D^i), the amount supplied by the bank in the client market (x_B^i) and the inter-dealer market (q_B^i), the amount supplied by the non-bank in the client market (x_N^i) and the inter-dealer market (q_N^i), and the amount supplied by the IDB in the inter-dealer market (Q^i) are defined by the joint solution to

$$0 = I - 2x_B^i - x_N^i - \gamma^i(x_B^i - q_B^i), \tag{18}$$

$$0 = I - 2x_N^i - x_B^i - \gamma^i(x_N^i - q_N^i), \tag{19}$$

$$s_D^i = \gamma^i * (x_B^i - q_B^i), \tag{20}$$

$$s_D^i = \gamma^i * (x_N^i - q_N^i), \tag{21}$$

$$s_D^i = \gamma^i Q^i, \tag{22}$$

$$Q^i = q_B^i + q_N^i. \tag{23}$$

The first four are derived from the bank and non-banks' first-order conditions from x_B^i, x_N^i, q_B^i and q_N^i respectively. The fifth is from the IDB's first-order condition with respect to Q^i, and the sixth is an imposition of market clearing.

The resulting quantities in the client and inter-dealer market are

$$x_B^i = x_N^i = \frac{3I}{2\gamma^i + 9}, \tag{24}$$

$$X^i = \frac{6I}{2\gamma^i + 9}, \tag{25}$$

$$q_B^i = q_N^i = \frac{I}{2\gamma^i + 9}, \tag{26}$$

$$Q^i = \frac{2I}{2\gamma^i + 9}. \tag{27}$$

The spreads in the client and inter-dealer market are

$$s_C^i = \frac{I(2\gamma^i + 3)}{2\gamma^i + 9}, \tag{28}$$

$$s_D^i = \frac{2I\gamma^i}{2\gamma^i + 9}. \tag{29}$$

Compared to Section 2, the quantity traded in the client market rises and the quantity traded in the inter-dealer market falls during competitive states. Similarly, spreads fall in the client market and rise in the inter-dealer market. The net result is that the two dealers, collectively, hold onto a larger inventory in competitive states and extract a smaller spread between the client and inter-dealer market.

The bank's and non-bank's profit in each state i, excluding the entry cost, is given:

$$\pi_j^i = \frac{I^2}{2\gamma^i + 9}. \tag{30}$$

3.2 Non-Competitive States ($\epsilon_B = 1$ or $\epsilon_N^i = 1$)

In non-competitive states, where only the bank or non-bank enter the market, results and entrant profitability are identical to those in Section 2. The sole difference is that it may be either the bank or non-bank providing intermediation between the client and inter-dealer markets. The traded quantities and spreads are then $x_j^i = X^i$, $q_j^i = Q^i$, s_C^i and s_D^i, where the entrant j is either the bank ($j = B$) or non-bank ($j = N$). These are otherwise identical in form to Equations 12 to 15.

3.3 Entry Decisions and Equilibrium

First, we address the non-bank's best responses to the bank's entry choice ϵ_B. For a given bank entry choice, the non-bank's profit is higher in the normal state γ^L. Thus, the non-bank will never choose to enter *only* in the stressed state. To simplify notation in the following section, we define $C = \frac{c}{I^2}$, which serves as a proxy for the size of an investor compared to the cost of acquiring them as a client.

If the bank enters ($\epsilon_B = 1$), the non-bank's best responses can be characterized as

$$(\epsilon_N^L, \epsilon_N^H) = \begin{cases} 0, 0 & C > \frac{1}{2\gamma^L+9} \\ 1, 0 & \frac{1}{2\gamma^L+9} \geq C > \frac{1}{2\gamma^H+9} \\ 1, 1 & \frac{1}{2\gamma^H+9} \geq C \end{cases} . \tag{31}$$

Alternatively, if the bank does not enter ($\epsilon_B = 0$), the non bank's best responses are

$$(\epsilon_N^L, \epsilon_N^H) = \begin{cases} 0, 0 & C > \frac{\gamma^L+8}{2(\gamma^L+4)^2} \\ 1, 0 & \frac{\gamma^L+8}{2(\gamma^L+4)^2} \geq C > \frac{\gamma^H+8}{2(\gamma^H+4)^2} \\ 1, 1 & \frac{\gamma^H+8}{2(\gamma^H+4)^2} \geq C \end{cases} . \tag{32}$$

Next, we can characterize the bank's entry decision, based on the non-bank's best response. In each case, given the non-bank's best response, the bank enters ($\epsilon_B = 1$) if

$$\delta\frac{\gamma^H+8}{2(\gamma^H+4)^2}+(1-\delta)\frac{\gamma^L+8}{2(\gamma^L+4)^2}\equiv C_{B,1}\geq C \qquad \text{and } (\epsilon_N^L,\epsilon_N^H)=(0,0) \tag{33}$$

$$\delta\frac{\gamma^H+8}{2(\gamma^H+4)^2}+(1-\delta)\frac{1}{2\gamma^L+9}\equiv C_{B,2}\geq C \qquad \text{and } (\epsilon_N^L,\epsilon_N^H)=(1,0) \tag{34}$$

$$\delta\frac{1}{2\gamma^H+9}+(1-\delta)\frac{1}{2\gamma^L+9}\equiv C_{B,3}\geq C \qquad \text{and } (\epsilon_N^L,\epsilon_N^H)=(1,1). \tag{35}$$

Proposition 1 (Liquidity Cross Section) *(i) If the investor has C sufficiently low, they are served by both the bank and the non-bank in at least one state and trade a weakly higher volume than in the baseline. If the investor has $C \in [C_{B,1}, \frac{\gamma^L+8}{2(\gamma^L+4)^2}]$, they are served by the non-bank in the normal state but would not have been served by the bank in the baseline, and trade a weakly higher volume.*

(ii) If $\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$, there exist C such that the investor trades less in the stressed state than in the baseline.

Proposition 1 characterizes the conditions under which the investor benefits from increased dealer market competition and those under which the investor suffers. We compare investor outcomes when the non-bank exists to the equilibrium from Section 2, where $\epsilon_B = 1$ if $C \leq C_{B,1}$ and $\epsilon_B = 0$, otherwise.

Investors with relatively low or high values of C may benefit. For low values of C, investors who would have been served by the bank in the baseline retain the services of the bank and are also served, at least occasionally, by the non-bank. These large investors represent clients who are too valuable for the bank to ignore, even if a non-bank provides additional competition. Alternatively, an investor with a higher value $C \in [C_{B,1}, \frac{\gamma^L+8}{2(\gamma^L+4)^2}]$ would have received no service in the baseline model and, in this section, would receive service from the non-bank in the normal state. These very small investors would previously have been ignored by bank dealers, but now receive opportunistic (but non-stress state) service from the non-bank.

While some investors trade at least as much in each state, the key result of this paper is that this is not universally the case: some investors may trade less. When $\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$, there exists a region where the investor's trading weakly falls. This special region exists when the best responses are such that the bank only enters if the non-bank never enters, and the non-bank enters in the normal state, regardless of the bank's entry choice. These region of investors with $C \in [C_{B,2}, min\{C_{B,1}, \frac{1}{2\gamma^L+9}\}]$ trade less than in the baseline. Previously, these investors would have been served by the bank. Now, were the bank to enter, the non-bank would follow in the normal state, pushing the bank's expected profit below zero. As a result, the bank does not enter and the non-bank serves them only in the normal state, leaving them unable to trade under the stress state.

This result is further illustrated in Figure 2. Panels (a) and (b) illustrate the case where $(\frac{1}{2\gamma^L+9} \leq \frac{\gamma^H+8}{2(\gamma^H+4)^2})$, and all investors are weakly better off. In this case, the quantity traded by the investor when the non-bank is able to enter are weakly greater in both states. The alternative case is illustrated in Panels (c) and (d). Specifically, in Panel (d), there exists a region where investor trading in the stress state is lower when non-bank entry is possible. Similar results for the spreads in the client market are illustrated in Figure 3.

In these figures, volume and spreads mostly increase linearly in the investor's initial endowment (I), with discontinuities in level and slope caused by dealer entry. When only the bank is present (represented by the blue dashed line), the single discontinuity represents the bank's entry point. When non-bank entry is permitted, there are two break points in each function. In the normal state (Panels (a) and (c)), the initial discontinuity is caused by the non-bank dealer entering in the normal state, while the second represents the point where both the bank and non-bank trade in the normal state. Alternatively, in the stress state (Panels (b) and (d)), the initial discontinuity is caused by the bank's entry in both states, while the second represents the non-bank's entry in the stress state.

Regardless of whether $\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$ or not, the change in investor trading volumes between Sections 3 and 2 follows a U-shaped piece-wise function. Those with either relatively high or relatively low values of C have improved service when non-banks enter, while those with more central values either receive no better or possibly worse service than before.

Remark 2 (Cross-Section of Debt Issuance) *In Remark 1, we discussed an alternative interpretation of the model, a government debt issuer. The results above can equally be applied to this context. Proposition 1 implies that very large (high I) issuances may benefit from the inclusion of non-bank dealers, as competition increases. At the same time, smaller issuances (inflation-protect securities, sub-national debt, etc.) may suffer during stressed states if marginal bank dealers leave the market. This interpretation implies that the entry of non-bank dealers into primary markets may worsen existing differences in demand for the largest most liquid government securities, when compared to other debt.*

3.4 Market Outcomes

Next, we look at the bifurcation of liquidity. First, in the case where the bank enters and the non-bank enters in both states, trading increases in both states. Moreover, the ratio of trading in stress to trading in normal states increases, representing a slight smoothing of liquidity in both states. Thus, for the lowest-cost (or largest-size) clients, liquidity may improve and become more predictable.

Next, in the case where the bank enters and the non-bank follows in normal times, trading increases in normal states but remains unchanged in stress. The result is a bifurcation, where the entry of the non-bank increases the differences between normal and stress times. However, this is a beneficial bifurcation, where liquidity only improves (though weakly so), rather than one in which the investor is harmed.

If the bank no longer enters the market and the non-bank only enters in normal times, a harmful bifurcation occurs. The investor receives no liquidity in stress times, while having an unimproved liquidity in normal times. Per Proposition 1, this harmful bifurcation exists for some investors if $\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$.

Proposition 2 (Bifurcation of Liquidity) *(i) If the non-bank enters in both states ($C \leq \frac{1}{2\gamma^H+9}$), investor trading volume is smoothed between states. (ii) If the non-bank enters only in the normal state, investor trading volume bifurcates.*

Next, we look at outcomes in the inter-dealer market. When both dealers enter, volume in the client market increases, though each dealer buys less than they would if they were the only dealer present. Each dealer is therefore less motivated to manage inventory through the inter-dealer market. The result is that inter-dealer market volume falls.

Proposition 3 (Inter-dealer Markets) *(i) If the bank trades with the investor, liquidity is smoothed in the inter-dealer market when the non-bank enters, regardless of whether this is one or both states.*

(ii) If the bank does not trade with the investor, liquidity is bifurcated in the inter-dealer market.

Looking at these changes through the lens of liquidity bifurcation reveals different results than in the client market. Liquidity only bifurcates in the inter-dealer market when the bank does not enter and the non-bank participates only during normal times. If the bank enters, and the non-bank enters in either normal times or both states, liquidity is smoothed in the inter-dealer market. That is to say that, when liquidity bifurcates in the client market, liquidity may simultaneously smooth in the inter-dealer market.

Notably, non-bank entry may also be harmful to IDB participant profit. In the baseline, where only bank dealers are present, IDB profit is

$$\pi_D^i = \frac{I^2\gamma^i}{2(\gamma^i + 4)^2}. \tag{36}$$

When non-banks enter, the IDB profit is

$$\pi_D^i = \frac{2I^2\gamma^i}{(2\gamma^i + 9)^2}. \tag{37}$$

Comparison shows that, all else equal, states with only the bank dealer are more profitable for the IDB than ones with the bank and the non-bank. This follows directly from the fact that, when both dealers are present, less risk sharing is required as they both take on a lower inventory than if only one had entered.

Remark 3 (Trading following primary issuance) *If the model is interpreted through the lens of a primary issuance, the bifurcation volume and prices between the client and inter-dealer markets takes another interpretation. In this context, large issuers who attract both bank and non-bank dealers should expect their issuance demand to increase and the issuance price to improve. On the other hand, subsequent volumes between dealers and other end investors (ex: the When-Issued market) may decline, and spreads in this market may increase.*

4 Central Bank Access

Next, we consider a central bank which offers funding to banks during stress times. We model an exogenous central bank that replaces the bank and IDB participants' funding cost γ^H with funding from the central bank at cost γ^{CB} where $\gamma^L \leq \gamma^{CB} < \gamma^H$. We denote stress states where the central bank intervenes with the superscript CB to differentiate it from previous sections.

For a given entry decision by the bank and non-bank, trading outcomes when in the normal state remain identical to those in Section 3. Unlike Section 3, results for the bank and non-bank are non-symmetric during the stress state. If only the bank enters, the resulting quantities and prices are

$$x_B^{CB} = X^{CB} = \frac{2I}{\gamma^{CB} + 4}, \tag{38}$$

$$q_B^{CB} = Q^{CB} = \frac{I}{\gamma^{CB} + 4}, \tag{39}$$

$$s_C^{CB} = \frac{I * (\gamma^{CB} + 2)}{\gamma^{CB} + 4}, \tag{40}$$

$$s_D^{CB} = \frac{I\gamma^{CB}}{\gamma^{CB} + 4}. \tag{41}$$

If only the non-bank enters, the quantities and prices are

$$x_N^{CB} = X^{CB} = \frac{I * (\gamma^H + \gamma^{CB})}{\gamma^H\gamma^{CB} + 2 * (\gamma^H + \gamma^{CB})}, \tag{42}$$

$$q_N^{CB} = Q^{CB} = \frac{I\gamma^H}{\gamma^H\gamma^{CB} + 2 * (\gamma^H + \gamma^{CB})}, \tag{43}$$

$$s_C^{CB} = \frac{I * (\gamma^H\gamma^{CB} + \gamma^H + \gamma^{CB})}{\gamma^H\gamma^{CB} + 2 * (\gamma^H + \gamma^{CB})}, \tag{44}$$

$$s_D^{CB} = \frac{I\gamma^H\gamma^{CB}}{\gamma^H\gamma^{CB} + 2 * (\gamma^H + \gamma^{CB})}. \tag{45}$$

Finally, if both enter, the quantities and prices are

$$x_B^{CB} = x_N^{CB} = \frac{I * (2\gamma^H + \gamma^{CB})}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}, \tag{46}$$

$$X^{CB} = \frac{2I * (2\gamma^H + \gamma^{CB})}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}, \tag{47}$$

$$q_B^{CB} = \frac{I\gamma^{CB}}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}, \tag{48}$$

$$q_N^{CB} = \frac{I * (2\gamma^H - \gamma^{CB})}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}, \tag{49}$$

$$Q^{CB} = \frac{2I\gamma^H}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}, \tag{50}$$

$$s_C^{CB} = \frac{I * (2\gamma^H\gamma^{CB} + 2\gamma^H + \gamma^{CB})}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}, \tag{51}$$

$$s_D^{CB} = \frac{2I\gamma^H\gamma^{CB}}{2\gamma^H\gamma^{CB} + 6\gamma^H + 3\gamma^{CB}}. \tag{52}$$

Proposition 4 (Central Bank Pass-through to Non-Banks) *Given the bank's and non-bank's entry decisions: (i) when the central bank does not lend directly to non-banks, liquidity improves in the client market ($X^{CB} \geq X^H$), even if only the non-bank interacts with the investor; (ii) the ratio of client market volume between stressed states when the central bank acts, and stressed states where it does not act ($\frac{X^{CB}}{X^H}$), is greater when non-banks are not present.*

First, we look at improvements liquidity, taking participant entry as given. In all cases, as long as $\gamma^{CB} < \gamma^H$, liquidity in the client market increases. This would even be true in the off-equilibrium case where only the non-bank enters in the stress state; liquidity would improve through the inter-dealer market, and the non-bank would be better able to sell its own inventory to the IDB after buying it from the investor, even though it has no central bank access itself.

When non-banks are present, either with or without banks, the degree to which liquidity improves when the central bank intervenes ($\frac{X^{CB}}{X^H}$) is less than its improvement when only banks are present. This could be seen as dampening the impact of central bank interventions, since the degree of improvement is not as high compared to non-intervention stress states. However, this statement requires nuance; the central bank's actions increase trading volume in gross terms over the case when only the bank is present. It is simply the case that volume absent the central bank (X^H) is also higher, and the degree of improvement is lower.

Next, we look at the entry decisions of banks and non-banks. If the bank enters ($\epsilon_B = 1$), the non-bank's best responses can be characterized as follows:

$$(\epsilon_N^L, \epsilon_N^H) = \begin{cases} 0,0 & C > \frac{1}{2\gamma^L+9} \\ 1,0 & \frac{1}{2\gamma^L+9} \geq C > \hat{C}_{N,1} \\ 1,1 & \hat{C}_{N,1} \geq C \end{cases}, \tag{53}$$

where $\hat{C}_{N,1} = \frac{2\gamma^H(\gamma^{CB})^2 + (2\gamma^H+\gamma^{CB})^2}{(2\gamma^H\gamma^{CB}+6\gamma^H+3\gamma^{CB})^2}$.

If the bank does not enter ($\epsilon_B = 0$), the non bank's best responses are

$$(\epsilon_N^L, \epsilon_N^H) = \begin{cases} 0,0 & C > \frac{\gamma^L+8}{2(\gamma^L+4)^2} \\ 1,0 & \frac{\gamma^L+8}{2(\gamma^L+4)^2} \geq C > \hat{C}_{N,2} \\ 1,1 & \hat{C}_{N,2} \geq C \end{cases}, \tag{54}$$

where $\hat{C}_{N,2} = \frac{\gamma^H(\gamma^{CB})^2+2(\gamma^H+\gamma^{CB})^2}{2(\gamma^H\gamma^{CB}+2*(\gamma^H+\gamma^{CB}))^2}$.

Finally, as before, we can characterize the bank's entry decision, based on the non-bank's best response. In each case, given the non-bank's best response, the bank enters if

$$\delta\frac{\gamma^{CB}+8}{2(\gamma^{CB}+4)^2} + (1-\delta)\frac{\gamma^L+8}{2(\gamma^L+4)^2} \equiv \hat{C}_{B,1} \geq C \quad \text{and } (\epsilon_N^L, \epsilon_N^H) = (0,0) \tag{55}$$

$$\delta\frac{\gamma^{CB}+8}{2(\gamma^{CB}+4)^2} + (1-\delta)\frac{1}{2\gamma^L+9} \equiv \hat{C}_{B,2} \geq C \quad \text{and } (\epsilon_N^L, \epsilon_N^H) = (1,0) \tag{56}$$

$$\delta\frac{2(\gamma^H)^2\gamma^{CB}+(2\gamma^H+\gamma^{CB})^2}{(2\gamma^H\gamma^{CB}+6\gamma^H+3\gamma^{CB})^2} + (1-\delta)\frac{1}{2\gamma^L+9} \equiv \hat{C}_{B,3} \geq C \quad \text{and } (\epsilon_N^L, \epsilon_N^H) = (1,1). \tag{57}$$

As in Section 3, the equilibrium can take one of multiple forms, each of which is unique for a given parameter set. These cases are analogous to the previous section. Investors with a very high or very low value of C receive improved service as a result of the non-bank's entry, while those with a more central value of C may suffer a loss if they are only served by the non-bank in the normal state. Whether the investors with a central value of C lose, or are merely no better off, depends on the central bank's actions.

Proposition 5 (Elimination of Harmful Liquidity Bifurcation) *The central bank's actions prevent harmful liquidity bifurcation if*

$$\frac{1}{2\gamma^L+9} \leq \frac{\gamma^{CB}+8}{2(\gamma^{CB}+4)^2}. \tag{58}$$

Proposition 5 illustrates the conditions under which the central bank's lending during the stress state eliminates harmful liquidity bifurcation. Recall that harmful liquidity bifurcation occurs if the non-bank's best response to enter in the normal state, given that the bank has entered, overlaps with the bank's best response to only enter if the non-bank will not enter. Loosely, this occurs when the bank's monopolist profits from being the only entrant in the stress state are less than its oligopoly profits from competing with the non-bank during the normal state. Since the central bank's actions reduce the cost of funding during the stress state, it can flip the aforementioned inequality and remove harmful bifurcation. It does so if its funding rate is sufficiently low.

4.1 Central Bank Access for Non-Bank Dealers

Increasingly, central banks have considered interventions that impact non-banks directly. Next, we examine the difference if the central bank's actions extend to non-bank dealers. The amounts traded in the normal state, given entry decisions, remain identical. The amounts traded by all agents in the stress state are identical in form to those in Section 3, with all instances of γ^H replaced by γ^{CB}.

Qualitatively, the results are similar to when non-banks do not have access to the central bank. There is a change in the pass-through of central bank actions to investors, as all dealers now have a lower funding cost. More interestingly, the central bank's ability to prevent harmful liquidity bifurcation remains identical. As before, harmful liquidity bifurcation exists if $\frac{1}{2\gamma^L+9} > \frac{\gamma^{CB}+8}{2(\gamma^{CB}+4)^2}$. This is because of the necessary conditions to cause a harmful bifurcation. Harmful bifurcation exists if the monopoly profits to the bank in the stress state are less than its oligopoly profits if it competes with the non-bank in the normal state. Neither of these conditions is impacted by the central bank's intervention (or not) with the non-bank in the stress state. Thus, harmful bifurcation exists when the central bank intervenes with the non-bank if it would have existed when the central bank intervened with only the bank.

Proposition 6 (Central Bank Access for Non-Banks) *(i) Central bank pass-through* $(\frac{X^{CB}}{X^H})$ *weakly improves when they intervene with non-banks as well as banks.*

(ii) There is no additional prevention of harmful bifurcation when non-banks have central bank access. It is identical to the case when only banks have access.

5 Conclusion

In this paper, we construct a two-market model of fixed income dealing. In the client market, two dealers—a bank incumbent and a non-bank entrant—compete for an investor's orders. These dealers then manage their inventory costs through an inter-dealer market, which is inaccessible to the investor. We show that the presence of a non-bank entrant may cause a bifurcation of liquidity between normal and stressed market states. This bifurcation of liquidity impacts some investors more than others; specifically, investors who were marginal clients of the bank before non-banks entered may be dropped by the bank and receive worse service.

Our model has implications for central banks that may wish to calm fixed income markets during periods of turmoil. The central bank's actions during times of stress may prevent the harmful bifurcation of liquidity caused by the entry of non-banks. At the same time, these actions may not create the same relative increase in liquidity, a loss of efficacy. If central banks expand their interventions to non-banks, they may mitigate the loss in relative efficacy at the cost of more lending.

Our model has implications for regulators that may be concerned about the role of non-banks. Other than their ability to selectively serve clients during different states of the world, we treat non-banks as identical to banks. Increasingly, regulators have proposed to treat non-banks more similarly to banks. For example, by increasing the requirements for non-banks to centrally clear trades or by increasing the scope of risk-based regulation. In that sense, our model represents the fruition of those efforts.

The expanded role of non-banks as liquidity markers in fixed income markets creates many open questions. Increasingly, liquidity has become concentrated in the most liquid bonds, such as on-the-run U.S. treasuries (G30 Working Group on Treasury Market Liquidity 2021). Whether the incentives for non-banks to make markets differ between these extremely liquid instruments and other less liquid bonds will be key to understanding the full distributional impact of non-bank entry.

Another open question is whether the rise of non-banks in fixed income markets is largely a function of the current levels of government debt issuance and how conditions will change if issuance levels decline. With high levels of government debt issuance, there may be more demands for dealing capacity in general. If banks are not able to scale up their own operations, non-bank entry may be optimal. The concern is whether these same non-banks will remain in the market if issuance levels (and therefore necessary dealer capacity) return to previous levels. If the initial entry of non-banks caused some bank dealers to scale down operations, it is unclear if they would be willing or able to scale back up following non-bank exit.

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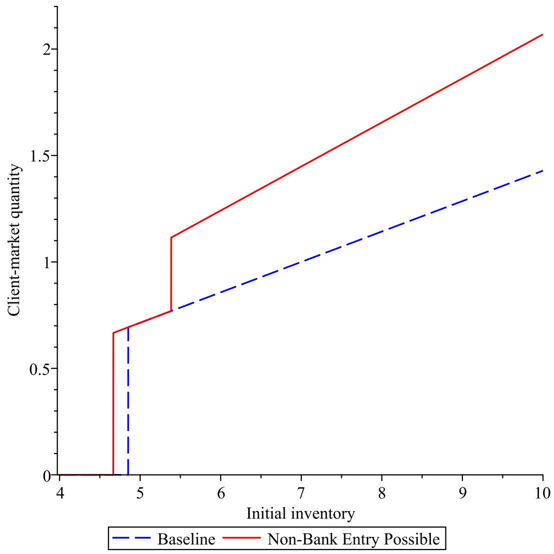
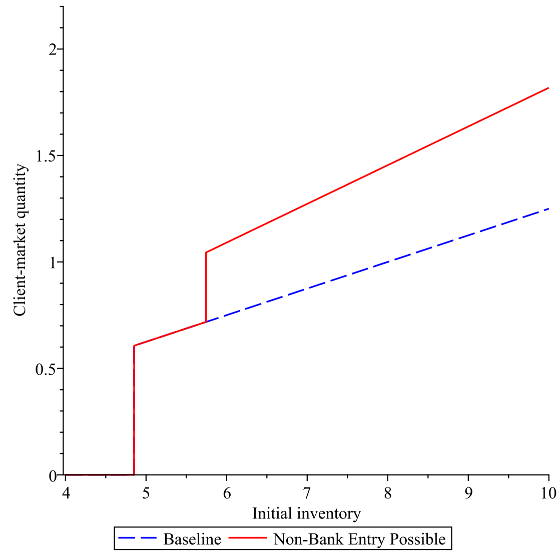
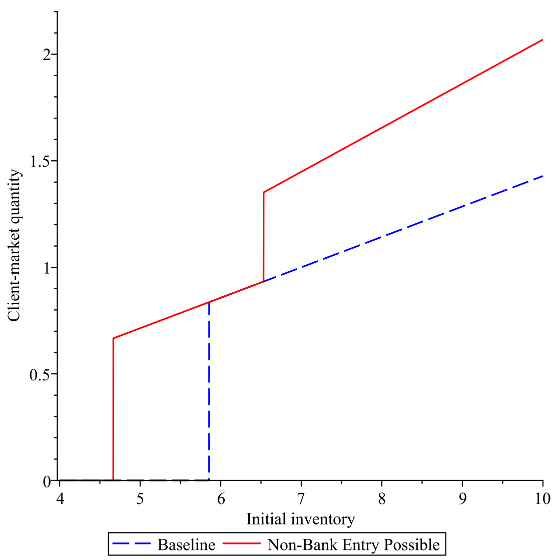
Figure 2: Quantities Traded in the Client Market



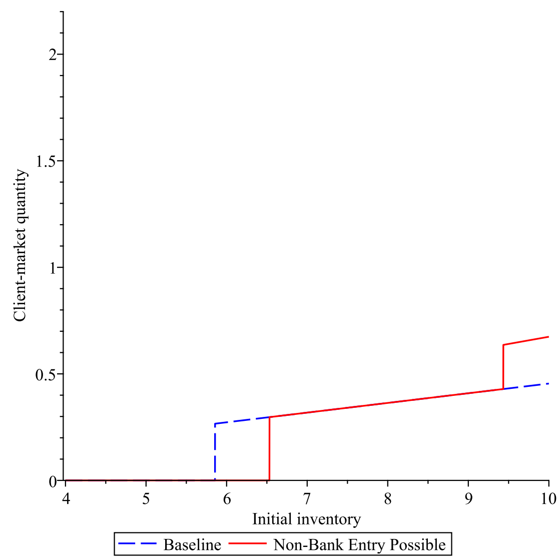
(a) Normal State



(b) Stress State



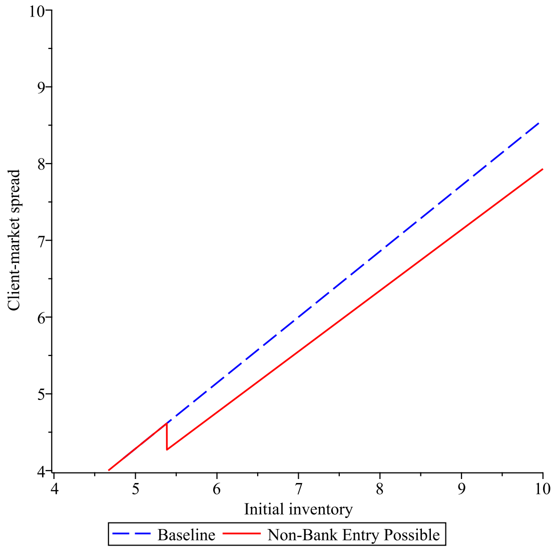
(c) Normal State with Harmful Bifurcation



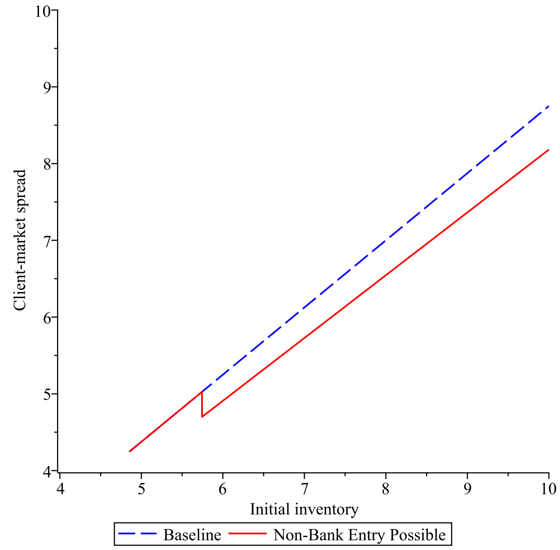
(d) Stress State with Harmful Bifurcation

Figure 2 illustrates the quantities traded in the client market, as initial inventory (I) rises. Panels (a) and (b) represent the normal and stress states, respectively, when there is no harmful bifurcation ($\frac{1}{2\gamma^L+9} \leq \frac{\gamma^H+8}{2(\gamma^H+4)^2}$). Panels (c) and (d) represent the normal and stress states, respectively, when harmful bifurcation is present ($\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$). In all cases, $c = 1$, $\gamma^L = 10$, $\delta = 0.5$. In panels (a) and (b) $\gamma^H = 12$, while in panels (c) and (d) $\gamma^H = 40$.

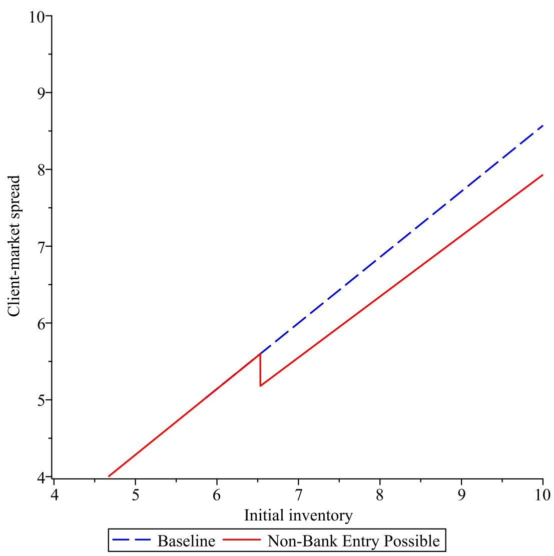
Figure 3: Spreads in the Client Market



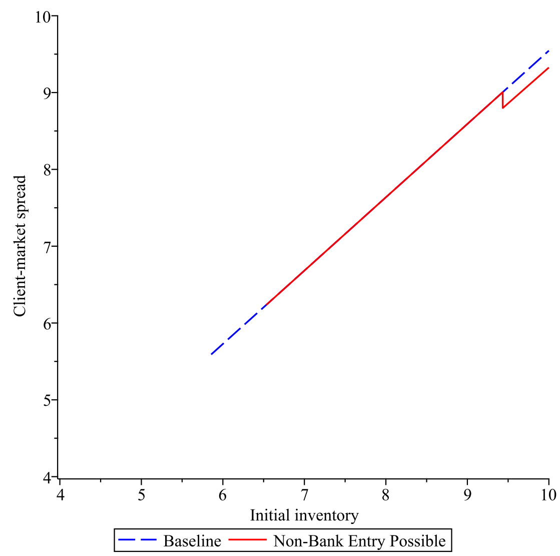
(a) Normal State



(b) Stress State



(c) Normal State with Harmful Bifurcation



(d) Stress State with Harmful Bifurcation

Figure 3 illustrates the spreads in the client market, as initial inventory (I) rises. Panels (a) and (b) represent the normal and stress states, respectively, when there is no harmful bifurcation ($\frac{1}{2\gamma^L+9} \leq \frac{\gamma^H+8}{2(\gamma^H+4)^2}$). Panels (c) and (d) represent the normal and stress states, respectively, when harmful bifurcation is present ($\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$). In all cases, $c = 1$, $\gamma^L = 10$, $\delta = 0.5$. In panels (a) and (b) $\gamma^H = 12$, while in panels (c) and (d) $\gamma^H = 40$.

A List of Notation

Variable	Description
Constants	
γ^i	bank, non-bank and IDB inventory cost in state i
γ_I	investor inventory cost
c	cost to connect to investor
δ	probability of the stress state
I	initial investor inventory
α	scaling factor for non-bank's inventory cost
Choice Variables and Equilibrium Values	
X^i	amount sold by investor in state i
x_B^i	amount bought by bank from investor in state i
x_N^i	amount bought by non-bank from investor in state i
Q^i	amount bought by the IDB in state i
q_B^i	amount sold by bank to the IDB in state i
q_N^i	amount sold by non-bank to the IDB in state i
s_C^i	spread in the client market in state i
s_D^i	spread in the inter-dealer market in state i
ϵ_B	bank's entry decision
ϵ_N^i	non-bank's entry decision in state i
Subscripts and Superscripts	
H	superscript for stress state
L	superscript for normal state
CB	superscript for stress state with a central bank
B	subscript for the bank
N	subscript for the non-bank

B Extension: Non-Banks with a Lower Inventory Cost

In this extension, we address a non-bank whose inventory cost is lower than that of the bank. The non-bank's profit function for a given γ^i is now

$$\pi_N^i = \epsilon_N^i * \left(s_C^i(x_N^i) * x_N^i - s_D^i * q_N^i - \frac{\alpha\gamma^i}{2}(x_N^i - q_N^i)^2 - c \right), \tag{B.1}$$

where $\alpha \in [0,1]$ represents the scaling of the non-bank's inventory cost relative to the bank. The equilibrium in this section follows identically to previous sections.

If the both the bank and non-bank enter, traded quantities and spreads are

$$x_B^i = x_N^i = \frac{I * (2\alpha + 1)}{2\alpha\gamma^i + 6\alpha + 3}, \tag{B.2}$$

$$X^i = \frac{2I * (2\alpha + 1)}{2\alpha\gamma^i + 6\alpha + 3}, \tag{B.3}$$

$$q_B^i = \frac{I}{2\alpha\gamma^i + 6\alpha + 3}, \tag{B.4}$$

$$q_N^i = \frac{I * (2\alpha - 1)}{2\alpha\gamma^i + 6\alpha + 3}, \tag{B.5}$$

$$Q^i = \frac{2\alpha I}{2\alpha\gamma^i + 6\alpha + 3}, \tag{B.6}$$

$$s_C^i = \frac{I * (2\alpha\gamma^i + 2\alpha + 1)}{2\alpha\gamma^i + 6\alpha + 3}, \tag{B.7}$$

$$s_D^i = \frac{2\alpha\gamma^i I}{2\alpha\gamma^i + 6\alpha + 3}. \tag{B.8}$$

If the non-bank is the sole entrant, traded quantities and spreads are

$$x_N^i = X^i = \frac{I * (\alpha + 1)}{\alpha\gamma^i + 2\alpha + 2}, \tag{B.9}$$

$$q_N^i = Q^i = \frac{\alpha I}{\alpha\gamma^i + 2\alpha + 2}, \tag{B.10}$$

$$s_C^i = \frac{I * (\alpha\gamma^i + \alpha + 1)}{\alpha\gamma^i + 2\alpha + 2}, \tag{B.11}$$

$$s_D^i = \frac{\alpha\gamma^i I}{\alpha\gamma^i + 2\alpha + 2}. \tag{B.12}$$

Alternatively, if only the bank enters, traded quantities and spreads are identical to those of Section 2.

As before, we can characterize the bank and non-bank's entry decisions. If the bank enters ($\epsilon_B = 1$), the non-bank's best responses can be characterized

$$(\epsilon_N^L, \epsilon_N^H) = \begin{cases} 0,0 & C > \frac{2\alpha\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \\ 1,0 & \frac{2\alpha\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \geq C > \frac{2\alpha\gamma^H+4\alpha^2+4\alpha+1}{(2\alpha\gamma^H+6\alpha+3)^2} \\ 1,1 & \frac{2\alpha\gamma^H+4\alpha^2+4\alpha+1}{(2\alpha\gamma^H+6\alpha+3)^2} \geq C \end{cases}. \tag{B.13}$$

If the bank does not enter ($\epsilon_B = 0$), the non bank's best responses are

$$(\epsilon_N^L, \epsilon_N^H) = \begin{cases} 0,0 & C > \frac{\alpha\gamma^L+2\alpha^2+4\alpha+2}{2(\alpha\gamma^L+2\alpha+2)^2} \\ 1,0 & \frac{\alpha\gamma^L+2\alpha^2+4\alpha+2}{2(\alpha\gamma^L+2\alpha+2)^2} \geq C > \frac{\alpha\gamma^H+2\alpha^2+4\alpha+2}{2(\alpha\gamma^H+2\alpha+2)^2} \\ 1,1 & \frac{\alpha\gamma^H+2\alpha^2+4\alpha+2}{2(\alpha\gamma^H+2\alpha+2)^2} \geq C \end{cases}. \tag{B.14}$$

Finally, the bank enters if

$$\delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{\gamma^L+8}{2(\gamma^L+4)^2} \geq C \qquad \text{and } (\epsilon_N^L, \epsilon_N^H) = (0,0) \tag{B.15}$$

$$\delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \geq C \qquad \text{and } (\epsilon_N^L, \epsilon_N^H) = (1,0) \tag{B.16}$$

$$\delta\frac{2\alpha^2\gamma^H+4\alpha^2+4\alpha+1}{(2\alpha\gamma^H+6\alpha+3)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \geq C \qquad \text{and } (\epsilon_N^L, \epsilon_N^H) = (1,1). \tag{B.17}$$

Proposition 7 (Harmful Bifurcation with Lower-Cost Non-Banks) *When non-banks have a lower cost of inventory than banks, there exists an investor who trades less in the stressed state than in the baseline if both*

$$\frac{\alpha\gamma^H+2\alpha^2+4\alpha+2}{2(\alpha\gamma^H+2\alpha+2)^2} < \frac{2\alpha\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \tag{B.18}$$

$$\frac{\alpha\gamma^H+2\alpha^2+4\alpha+2}{2(\alpha\gamma^H+2\alpha+2)^2} < \delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{\gamma^L+8}{2(\gamma^L+4)^2}. \tag{B.19}$$

When non-banks have a lower inventory cost, harmful bifurcation may continue to occur. Unlike the baseline, this is now dependent on two conditions. The first condition is similar, but not identical, to that of Proposition 1. Harmful bifurcation is only possible if the non-bank's monopoly profits in the stress state are less than its oligopoly profits if it competes with the bank in the normal state. The second condition indicates that harmful bifurcation is only possible if the non-bank's monopoly profits in the stress state are less than the bank's total monopoly profits.

In Section 3, the second condition holds trivially, as $\alpha = 1$, but for $\alpha < 1$, it may be violated, which rules out harmful bifurcation. This possibility brings us the primary result in this appendix: a sufficiently low inventory cost for the non-bank eliminates the possibility of harmful bifurcation. For α sufficiently low (ex: $\alpha = 0$), the second statement in Proposition 7 is violated. When this is the case, the non-bank simply improves over the bank. If the bank chooses not to enter, the non-bank enters in both states and provides a higher volume to the client.

This result presents a potential dilemma for regulators. If non-banks have sufficiently low inventory costs relative to banks, no harmful bifurcation occurs and non-banks may trade in higher volumes than the bank had previously. A regulator who imposes additional burden on non-banks for reasons outside the model (ex: reducing systemic risk) may increase inventory costs by increasing the non-banks' effective value of α. If the increase in α is such that the conditions in Proposition 7 now hold, it may impose harmful bifurcation on the investor, creating a discontinuous loss of liquidity during times of stress.

C Proofs

C.1 Section 2

C.1.1 Existence of an Equilibrium

Given the bank has entered, the first-order conditions of the bank and IDB's profit functions, along with the market clearing conditions, are given by Equations 8 – 11. Solving this system of equations results in unique quantities and prices given by Equations 12 – 15, for each γ^i. The bank then enters if $E[\pi_B^i] \geq c$ where π_B^i is given by Equation 16.

C.2 Section 3

C.2.1 Existence of an Equilibrium

We search for an equilibrium in pure strategies, by backwards induction. In each state i, there are four possible outcomes in the client and dealer markets, depending on whether the bank and/or non-bank have entered. First, if there is no entrant, the traded quantities are zero. The second and third cases are states where only one of the banks or the non-bank enters as the lone dealer. In these cases, the equilibrium within the client and dealer markets is functionally identical to that in Section 2, with Equations 12 and 13 replacing x_B^i and q_B^i with x_N^i and q_N^i, respectively, if the lone entrant is the non-bank. Finally, given that both the bank and non-bank have entered, the first-order conditions of the bank, non-bank and IDB's profit functions, along with the market clearing conditions, are given by Equations 18 – 23. Solving this system results in unique quantities and prices given by Equations 24 – 29.

Given the outcomes of the client and dealer markets, we turn to entry. If the bank has entered, the non-bank's entry decision in each state is characterized by Equation 31, while if it has not entered, it is characterized by Equation 32. In both cases, these intervals are mutually exclusive and collectively exhaustive, representing a unique state-dependent best response. Given each possible combination of non-bank entry decision, the bank's entry decision is characterized by one of Equations 33 – 35. Again, the bank's optimal pure strategy decision is unique, given the non-bank's unique best response.

C.2.2 Proof of Proposition 1

(i) In the baseline $\boldsymbol{\epsilon} = (1,0,0)$ and the bank enters if $C \leq C_{B,1}$, otherwise $\boldsymbol{\epsilon} = (0,0,0)$. In this section, the investor trades weakly more in each state if $\boldsymbol{\epsilon} = (1,1,1)$ or $\boldsymbol{\epsilon} = (1,1,0)$, or alternatively if both $\boldsymbol{\epsilon} = (0,1,0)$ and $C \geq C_{B,1}$. Since $\frac{1}{2\gamma^H+9} \leq C_{B,3} \leq C_{B,1}$, there exists a region $C \in [0, \frac{1}{2\gamma^H+9}]$ such that (a) the non-bank's best response, if the bank enters, is to enter in both states, and (b) the bank's best response, if the non-bank enters in both states, is to enter. Therefore, in this region $\boldsymbol{\epsilon} = (1,1,1)$ when in the baseline $\boldsymbol{\epsilon} = (1,0,0)$.[6]

Since $C_{B,1} \leq \frac{\gamma^L+8}{2(\gamma^L+4)^2}$, there exists a region $C \in [C_{B,1}, \frac{\gamma^L+8}{2(\gamma^L+4)^2}]$ such that in the baseline $\boldsymbol{\epsilon} = (0,0,0)$, but where the non-bank's best response to the bank not entering, is to enter in the normal state. Therefore, in this region $\boldsymbol{\epsilon} = (0,1,0)$ when in the baseline $\boldsymbol{\epsilon} = (0,0,0)$.

(ii) We search for a case where, in the baseline $\boldsymbol{\epsilon} = (1,0,0)$, but when the non-bank is present, either $\boldsymbol{\epsilon} = (0,1,0)$ or $\boldsymbol{\epsilon} = (0,0,0)$. First consider the interval $C \in [0, C_{B,3}]$. In this interval, the bank enters regardless of the non-bank's entry, and thus, $\epsilon_B = 0$ cannot occur. Second, consider the interval $C \in [C_{B,3}, C_{B,2}]$, where the bank enters if the non-bank either does not enter or enters only in the normal state. The non-bank's entry threshold in the stress state, given the bank has entered, is $\frac{1}{2\gamma^H+9} \leq C_{B,3}$. Thus, over this interval $\epsilon_B = 0$ cannot occur. Finally, consider the interval $C \in [C_{B,2}, C_{B,1}]$, where the bank enters only if the non-bank does not enter. The non-bank would enter, given the bank has entered if $C \leq \frac{1}{2\gamma^L+9}$. This falls within the interval if $\frac{\gamma^H+8}{2(\gamma^H+4)^2} \leq \frac{1}{2\gamma^L+9}$ and affects a portion of the interval $C \in [C_{B,2}, min\{\frac{1}{2\gamma^L+9}, C_{B,1}\}]$ Thus, this interval has no trading in the stress state, whereas in the baseline the bank was present.

C.2.3 Proof of Proposition 2

The ratio of client market trading in stress and normal states in the baseline is $\frac{X^H}{X^L} = \frac{\gamma^L+4}{\gamma^H+4}$. When non-banks are present, this value depends on their entry and the bank's entry:

$$\frac{X^H}{X^L} = \begin{cases} \frac{2\gamma^L+9}{2\gamma^H+9} & \boldsymbol{\epsilon} = (1,1,1) \\ \frac{2\gamma^L+9}{3\gamma^H+12} & \boldsymbol{\epsilon} = (1,1,0) \\ 0 & \boldsymbol{\epsilon} = (0,1,0) \end{cases} \tag{C.1}$$

[6] There also exists a region where $\boldsymbol{\epsilon} = (1,1,0)$, where trading volume increases in the normal state. For simplicity, we omit this element of the proof.

Algebraic manipulation shows that when $\boldsymbol{\epsilon} = (1,1,1)$ $\frac{X^H}{X^L}$ increases compared to the baseline, whereas when $\boldsymbol{\epsilon} = (1,1,0)$ or $\boldsymbol{\epsilon} = (0,1,0)$ it declines.

C.2.4 Proof of Proposition 3

In the baseline, $\frac{Q^H}{Q^L} = \frac{\gamma^L+4}{\gamma^H+4}$. When non-banks are present, this value depends on their entry and the bank's entry:

$$\frac{Q^H}{Q^L} = \begin{cases} \frac{2\gamma^L+9}{2\gamma^H+9} & \boldsymbol{\epsilon} = (1,1,1) \\ \frac{2\gamma^L+9}{2\gamma^H+8} & \boldsymbol{\epsilon} = (1,1,0) \\ 0 & \boldsymbol{\epsilon} = (0,1,0) \end{cases}. \tag{C.2}$$

Algebraic manipulation shows that when $\boldsymbol{\epsilon} = (1,1,1)$ and $\boldsymbol{\epsilon} = (1,1,0)$, $\frac{Q^H}{Q^L}$ rises compared to the baseline, whereas when $\boldsymbol{\epsilon} = (0,1,0)$, it falls.

C.3 Section 4

C.3.1 Existence of an Equilibrium

The equilibrium in this section follows from Section 3. To avoid repetition, the first-order conditions are functionally identical to those in Section 3, with all instances of γ^H replaced by γ^{CB} in those of the bank and IDB participant. Unlike previous sections, the traded quantities and prices now differ between the bank and non-bank. If only the bank enters, prices and quantities are characterized by Equations 38 – 41. If only the non-bank enters, prices and quantities are characterized by Equations 42 – 45. Finally, if both enter, prices and quantities are given by Equations 46 – 52.

Entry decisions are also characterized in a similar manner to Section 3. The non-bank's entry if the bank has entered is characterized by Equation 53, while its entry if the bank does not enter is characterized by Equation 54. The bank's entry is then characterized by Equations 55 – 57.

C.3.2 Proof of Proposition 4

(i) In Section 2, $X^H = \frac{2I}{\gamma^H+4}$ if only the bank enters and $X^H = \frac{6I}{2\gamma^H+9}$ if both dealers enter. In Section 4, if only the bank enters, $X^{CB} = \frac{2I}{\gamma^{CB}+4} \geq \frac{2I}{\gamma^H+4}$, while if both dealers enter, $X^{CB} = \frac{I*(2\gamma^H+\gamma^{CB})}{2\gamma^H\gamma^{CB}+6\gamma^H+3\gamma^{CB}} \geq \frac{6I}{2\gamma^H+9}$.

(ii) When only banks are present, the ratio of trading in client market, with and without the central bank, is $\frac{X^{CB}}{X^H} = \frac{\gamma^H+4}{\gamma^{CB}+4}$. In states where the non-bank is present, trading with the client is

$$\frac{X^{CB}}{X^H} = \begin{cases} \frac{4(\gamma^H)^2+2\gamma^H\gamma^{CB}+18\gamma^H+9\gamma^{CB}}{6\gamma^H\gamma^{CB}+18\gamma^H+9\gamma^{CB}} & \boldsymbol{\epsilon} = (1,1,1) \text{ or } \boldsymbol{\epsilon} = (1,0,1) \\ \frac{(\gamma^H)^2+\gamma^H\gamma^{CB}+4\gamma^H+4\gamma^{CB}}{2\gamma^H\gamma^{CB}+4\gamma^H+4\gamma^{CB}} & \boldsymbol{\epsilon} = (0,0,1) \end{cases}. \tag{C.3}$$

Algebraic manipulation shows that, in both cases when the non-bank is present, $\frac{X^{CB}}{X^H}$ is less than when only the bank is present. It should be noted that it is never optimal for only the non-bank to enter in the stressed state and thus neither $\boldsymbol{\epsilon} = (1, 0, 1)$ nor $\boldsymbol{\epsilon} = (0, 0, 1)$ are equilibrium entry cases.

C.3.3 Proof of Proposition 5

We search for parameter sets where harmful bifurcation is possible. That is, where in the baseline $\boldsymbol{\epsilon} = (1, 0, 0)$, but in the presence of the non-bank, either $\boldsymbol{\epsilon} = (0, 1, 0)$ or $\boldsymbol{\epsilon} = (0, 0, 0)$. First, for $C < \hat{C}_{B,3}$, the bank will enter regardless of the non-bank's response. Second, we search in the interval $C \in [\hat{C}_{B,3}, \hat{C}_{B,2}]$, where the bank will enter as long as the non-bank enters only in the normal state, or not at all. The non-bank's entry threshold in the stress state, given the bank enters, is $C \leq \frac{2\gamma^H(\gamma^{CB})^2+(2\gamma^H+\gamma^{CB})^2}{(2\gamma^H\gamma^{CB}+6\gamma^H+3\gamma^{CB})^2}$. Algebraic manipulation shows that $\frac{2\gamma^H(\gamma^{CB})^2+(2\gamma^H+\gamma^{CB})^2}{(2\gamma^H\gamma^{CB}+6\gamma^H+3\gamma^{CB})^2} \leq \hat{C}_{B,3}$ and thus, $\epsilon_B = 0$ cannot occur over this interval. Finally, we search in the interval $C \in [\hat{C}_{B,2}, \hat{C}_{B,1}]$ where the bank will enter only if the non-bank does not enter. The non-bank's entry threshold in the normal state, given the bank has entered is $C \leq \frac{1}{2\gamma^L+9}$. Algebraic manipulation shows that it can fall within this interval (i.e. $\frac{1}{2\gamma^L+9} \geq \hat{C}_{B,2}$) when Equation 58 does not hold.

C.3.4 Proof of Proposition 6

(i) When the bank and non-bank enter ($\boldsymbol{\epsilon} = (1, 1, 1)$), $\frac{X^{CB}}{X^H} = \frac{2\gamma^L+9}{2\gamma^{CB}+9}$. Algebraic manipulation shows that $\frac{2\gamma^L+9}{2\gamma^{CB}+9} \geq \frac{4(\gamma^H)^2+2\gamma^H\gamma^{CB}+18\gamma^H+9\gamma^{CB}}{6\gamma^H\gamma^{CB}+18\gamma^H+9\gamma^{CB}}$. When only the bank enters ($\boldsymbol{\epsilon} = (1, 0, 0)$), $\frac{X^{CB}}{X^H}$ is identical to Section 4.

(ii) By construction, the entry choices in Section 4.1 are identical to those in Section 3, with all instances of γ^H replaced by γ^{CB}. From Proposition 1, harmful bifurcation exists if $\frac{1}{2\gamma^L+9} > \frac{\gamma^H+8}{2(\gamma^H+4)^2}$, which creates the analogous condition of $\frac{1}{2\gamma^L+9} > \frac{\gamma^{CB}+8}{2(\gamma^{CB}+4)^2}$ in this section. This condition is identical to the one presented in Proposition 5.

C.4 Section B

C.4.1 Proof of Proposition 7

Comparing Equations B.3 and B.9 to Equation 12 when $\gamma^i = \gamma^H$, client trading volume is at least as high in the stress state when $\alpha < 1$ and $\boldsymbol{\epsilon} = (1, 1, 1)$ or $\boldsymbol{\epsilon} = (1, 1, 0)$, compared to when $\boldsymbol{\epsilon} = (1, 0, 0)$. Thus, as before, we search for a parametrization where, in the baseline $\epsilon_B = 1$, but in Section B, $\boldsymbol{\epsilon} = (0, 1, 0)$.

First, if $\delta\frac{2\alpha^2\gamma^H+4\alpha^2+4\alpha+1}{(2\alpha\gamma^H+6\alpha+3)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \geq C$, the bank enters regardless of the non-bank's actions, and it cannot be the case that $\boldsymbol{\epsilon} = (0, 1, 0)$ or $\boldsymbol{\epsilon} = (0, 0, 0)$. Second, if $\delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} \geq C > \delta\frac{2\alpha^2\gamma^H+4\alpha^2+4\alpha+1}{(2\alpha\gamma^H+6\alpha+3)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2}$, the bank does not enter if the non-bank enters during stress. Since the non-bank's monopoly profits in stress (were the bank not to enter) are greater than its oligopoly profits in stress, were the bank to enter, it cannot be the case that $\boldsymbol{\epsilon} = (0,1,0)$ or $\boldsymbol{\epsilon} = (0,0,0)$. Finally, if $\delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{\gamma^L+8}{2(\gamma^L+4)^2} \geq C > \delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2}$, the bank does not enter if the non-bank enters at all. We search this final region for both $(\epsilon_N^L, \epsilon_N^H) = (1,0)$ as a best response to $\epsilon_B = 1$, and $(\epsilon_N^L, \epsilon_N^H) = (1,0)$ as a best response to $\epsilon_B = 0$, resulting in $\boldsymbol{\epsilon} = (0,1,0)$.

Within this region, two conditions must hold simultaneously. First, it must be that $\epsilon_B = 0$ is the best response to $\epsilon_N^L = 1$ while $\epsilon_N^L = 1$ is the best response to $\epsilon_B = 1$ over the same region, which causes the bank not to enter the market in this region. The first occurs for $C > \delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2}$, while the second occurs for $C \leq \frac{2\alpha\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2}$. This region exists if $\frac{2\alpha\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2} > \delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{2\alpha^2\gamma^L+4\alpha^2+4\alpha+1}{(2\alpha\gamma^L+6\alpha+3)^2}$. Thus, if this condition holds, there is a region where the bank chooses not to enter the market because of the non-bank's response.

Second, and different from Section 3, it must be the case that the non-bank's best response of $(\epsilon_N^L, \epsilon_N^H) = (1,1)$ to $\epsilon_B = 0$ do not occur at higher values of C to the bank's best response of $\epsilon_B = 1$ to $(\epsilon_N^L, \epsilon_N^H) = (0,0)$. If the reverse was true, any state where the bank does not enter solely because of the response of the non-bank, the non-bank would enter in both states. This condition is such that $\frac{\alpha\gamma^H+2\alpha^2+4\alpha+2}{2(\alpha\gamma^H+2\alpha+2)^2} < \delta\frac{\gamma^H+8}{2(\gamma^H+4)^2} + (1-\delta)\frac{\gamma^L+8}{2(\gamma^L+4)^2}$. This forms the second condition of Proposition 7. Of note, this condition does not occur in Section 3 because the non-bank's monopoly profit in the stress state would be strictly less than the bank's monopoly profits across both states. When $\alpha < 1$, this is no longer necessarily true.