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Criticality in Biocomputation

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Abstract. Complexity in biological computation is one of the recognised means by which biological systems manage to function in a complex chaotic world. The ability to function and solve problems irrespective of scale and relative complexity, including higher-order interactions, is essential to the efficacy of biological systems. However, it has been unclear how the required complexity can be introduced to allow these functions to be realised. Nonlinear local interactions are required to combine into a global stable system. The property of criticality, that is exhibited by many nonlinear physical systems, can be exploited to allow local nonlinear oscillators to interact, resulting in a globally stable system. This concept introduces robustness, as well as, a means to control global stability.

1 Criticality

The property of physical systems to demonstrate criticality is based on local nonlinear interactions of many small components that contribute to the system [1]. These interactions are not significantly large at the scale of each component, but are contributing factors due to the nonlinear interactions of these components to the global stable state of the system. Criticality is seen in the global state of the system where the system changes state due to its apparent proximity to a critical point. This is often observed as rapid transitions from one state to the next. For example, in the case of a avalanche of snow or sand, where a previously apparent stable state rapidly changes to a different stable state. Similar dynamics have been observed in biological systems, in particular in neural dynamics [2]. The functional role of criticality is here under investigation. So far, it appears to be mostly related to network complexity and power law relations within those networks. Even though it is still not clear if power law relations are a true property of most systems of interest [3], there is certainly a potential for criticality to play an important role in many aspects of biological computation.

2 Nonlinear control

Although the concept of criticality has been known for some time now, its exact role and relation to dynamic systems has not yet been determined sufficiently. This can be attributed to the issue of reliably and robustly creating a dynamic system which exhibits criticality. Quite a lot is known about linear systems with relatively complex interactions, however complex interactions in nonlinear systems often quickly degenerate into unstable or chaotic systems. It is argued that biological systems are capable of exerting localised control on the many

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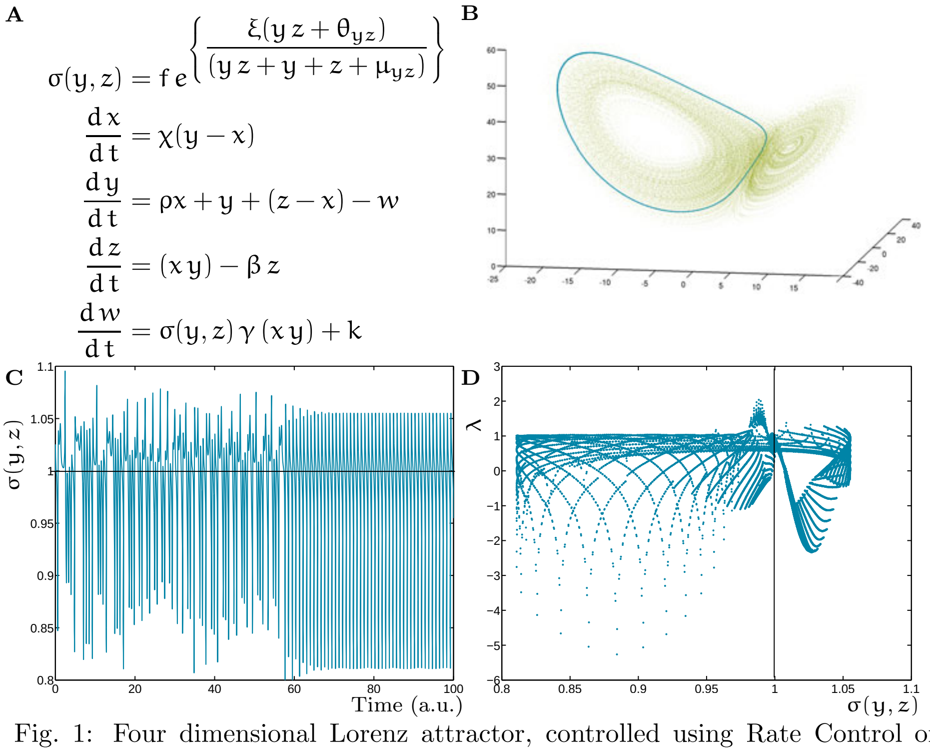
interacting parts resulting in a globally scale free stable system [4, 5]. This is due to the mechanism of rate control of chaos (RCC) which allows a complex nonlinear system to maintain dynamic control even when perturbed chaotically [5]. Furthermore, the mechanism of rate control of chaos is capable of stabilising a dynamic system if it crosses bifurcations. An illustration of this control principle is shown in figure 1, where the four dimensional Lorenz model [6] is shown controlled using the rate control of chaos method. In panel **A** the corresponding first order equations are shown including the RCC control function. In panel **B**, a three-dimensional projection of these equations are plotted, showing both chaos as well as the controlled unstable periodic orbit (UPO). In panel **C** is shown the control function $\sigma(y, z)$ in time when the control is enabled at $t = 50$. Lastly, in panel **D** is shown this control function plotted against the estimated maximal Lyapunov exponent, during the stable control [7]. This shows that the orbit is still dynamically unstable, and even weakly chaotic, demonstrating one of the interesting properties of this control mechanism.



$$\sigma(y,z) = f\, e^{\left\{ \frac{\xi(yz+\theta_{yz})}{(yz+y+z+\mu_{yz})} \right\}}$$
$$\frac{dx}{dt} = \chi(y-x)$$
$$\frac{dy}{dt} = \rho x + y + (z-x) - w$$
$$\frac{dz}{dt} = (xy) - \beta z$$
$$\frac{dw}{dt} = \sigma(y,z)\,\gamma\,(xy) + k$$

Fig. 1: Four dimensional Lorenz attractor, controlled using Rate Control of Chaos. **A** shows the equations with the RCC controller; **B** the three dimensional controlled orbit within the chaotic attractor; **C** shows the control function $\sigma(y, z)$ in time when control is enabled at step 50; **D** shows the control function versus the estimated Lyapunov exponent λ.

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3 Criticality from Rate Controlled Chaos

Because the RCC method does not eliminate the occurrence of chaotic states, the controlled system is still capable of responding to weak non-linear perturbations. The RCC control considers any deviation from the stable orbit to be a chaotic exponential deviation, and will return the system to the unstable controlled orbit, at an exponential rate. However, the perturbation can have a nonlinear effect on the total dynamics of coupled RCC oscillators, resulting in different UPOs becoming stabilised. To investigate this property, four standard Rössler models [8] are controlled using RCC into unstable periodic orbits. In the following equations, the parameters ξ and μ determine the control function $\sigma(x, z)$ that applies the RCC to variable z. In the following modelling, the oscillators had slightly different control parameters, to avoid self-similarity between the oscillators ($\xi_1 = -2, \xi_2 = -2.1, \xi_3 = -2.25, \xi_4 = -2.5$).

$$\begin{aligned}
\sigma(x, z) &= f\, e^{\left(\frac{\xi_n (x\, z)}{x\, z + x + z + \mu}\right)} \\
\frac{dx}{dt} &= -(y + z) \\
\frac{dy}{dt} &= x + \frac{y}{\alpha} \\
\frac{dz}{dt} &= \frac{\beta}{\alpha} + (\sigma(x, z)\, z\, x) - (\gamma\, z)
\end{aligned}$$

The controlled oscillators are weakly connected using a simple finite difference scheme. By changing the connections from only two oscillators, to three, and finally four fully connected oscillators, the effect of the non-linear interactions become visible in the total unweighted summed dynamics of the four oscillators, such as an external observer might be able to determine. To ensure that criticality can emerge within the global dynamics of the summed oscillators, it is important that each individual oscillator remains stable despite the perturbations. The presence of the RCC control within each oscillator allows the total system to respond to changing network conditions, without destabilising. In figure 2 is shown the phase-plots of the four controlled oscillators, where each oscillator is individually shown for the three connection patterns, i.e. where oscillator 1 and 2 are connected (blue), oscillator 1, 2 and 3 are connected (green), and all four are connected (gold). The plots show only the stable oscillations, after the transients from the change in connectivity have disappeared. To compare the oscillators directly when connectivity changes, in figure 3 is shown the x variables of all four oscillators in time, where at time point $t = 1000$ the two connected oscillators 1 and 2 are joined by 3. At time point $t = 2000$, all four are connected. Notice the short transients in the dynamics before each of them stabilises again into a somewhat different, but stable, orbit.

It should be emphasised that each of the oscillators is unchanged in every respect, except for the additional perturbing input to each variable from the other oscillators. Each oscillator goes through a transient before stabilising in a similar, but significantly different orbit. The local response from each oscillator

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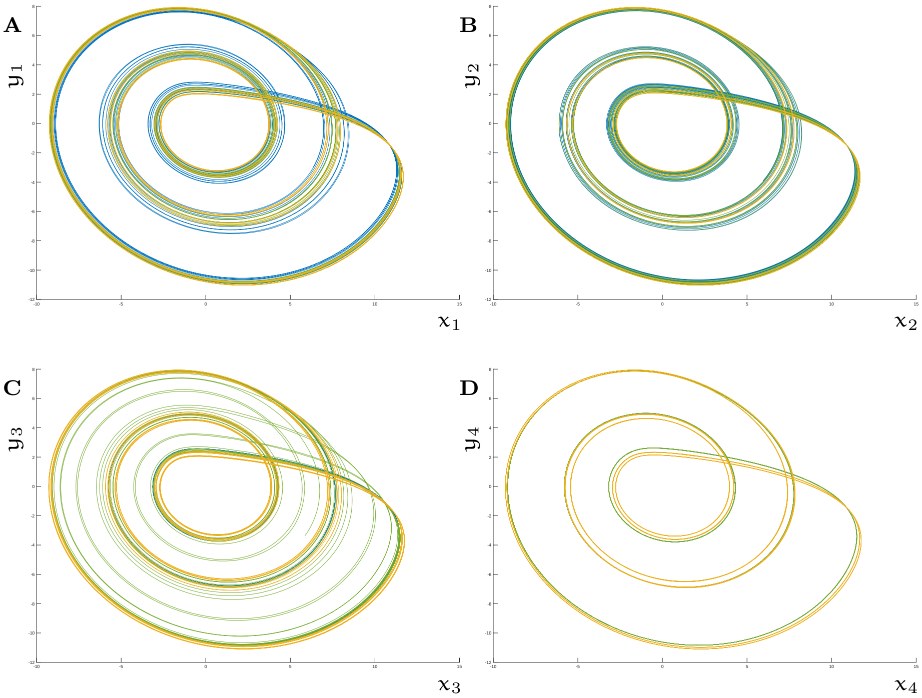

Fig. 2: Four coupled RCC controlled Rössler oscillators, under different connection regimes. **A** First oscillator x_1 versus y_1 where blue indicates the two connected oscillators, green three connected, and gold all four connected oscillators. In **B**, **C**, and **D** are shown the same for oscillators $2, 3$ and 4, respectively.

varies due to the difference in control parameter, but this difference is not sufficiently large to cancel out the effects from the perturbations. To appreciate the effect that the change in connectivity has to the network, the total unweighted sum of each of the variables is determined, and in figure 4 is shown the phase plots of those sums (minus transients). As is immediately obvious, the change in connectivity appears to affect the global dynamics profoundly, stabilising into different orbits for each of the three connectivities possible. In particular, the three connected oscillators show different multi-orbits, at different scale, than the other two connectivities. Also, the four connectivity is naturally larger than the two connectivity, but this is not the case for the three connectivity scenario. Due to the nonlinear interactions, and differences in the stable orbit of each oscillator, the total is not the linear sum of the three connectivity, and neither is the four connectivity. Furthermore, the difference between the global state for the three connectivities is not attributable to relative phase shifts; see figure 3 where no significant phase differences are apparent. In figure 5 is shown the total summed oscillator variables x_{total}, y_{total} and z_{total} in time, with changes to different connectivities at $t = 1000$, and $t = 2000$. The transients are visible, but disappear, resulting in a different stable global state for the entire network.

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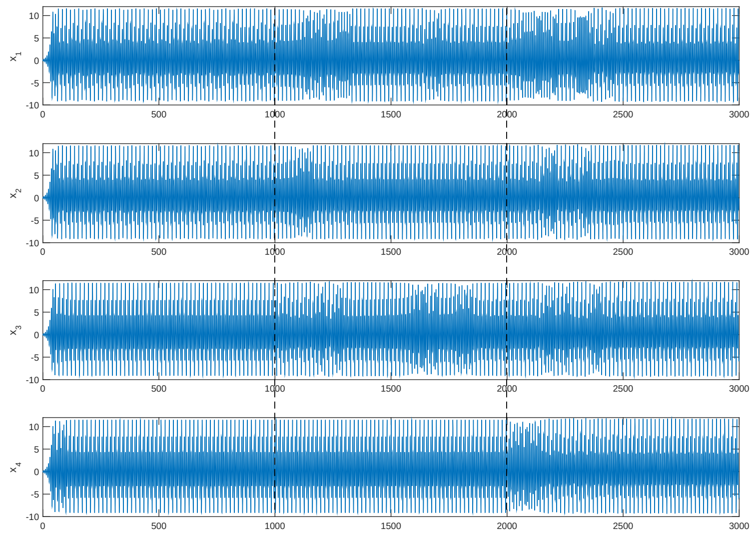

Fig. 3: The four connected oscillators in time, with connections from two oscillators to three enabled at $t = 1000$, and from three to four at $t = 2000$.

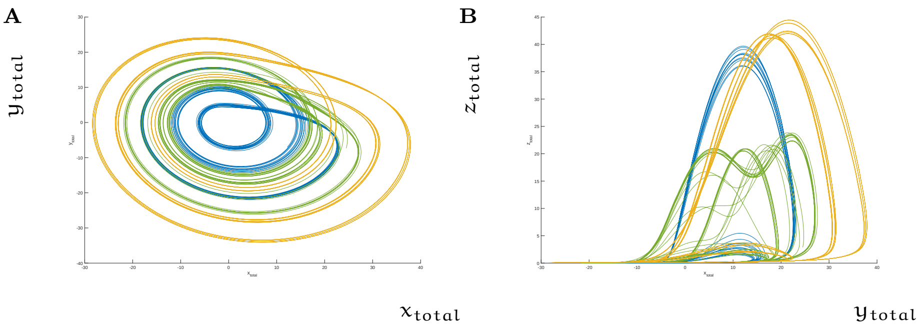

Fig. 4: Phase plots of summed oscillators, with **A** showing the x_{total} versus y_{total}, and z_{total}. Notice the nonlinear global stable states of each of the connectivities.

4 Conclusion

The minimal model described previously, illustrates the ability of Rate Controlled Chaotic systems to form a seemingly critical system. The nonlinear interactions, communicated via diffusion, allow the emergence of a critical state, which does not happen if the connectivity is linear or low-pass filtered. Further investigations will focus on demonstrating the ability of the emergent critical system to have global multi-state conditions in a scale-free domain. This will support the concept of many small nonlinear oscillators resulting in a large scale

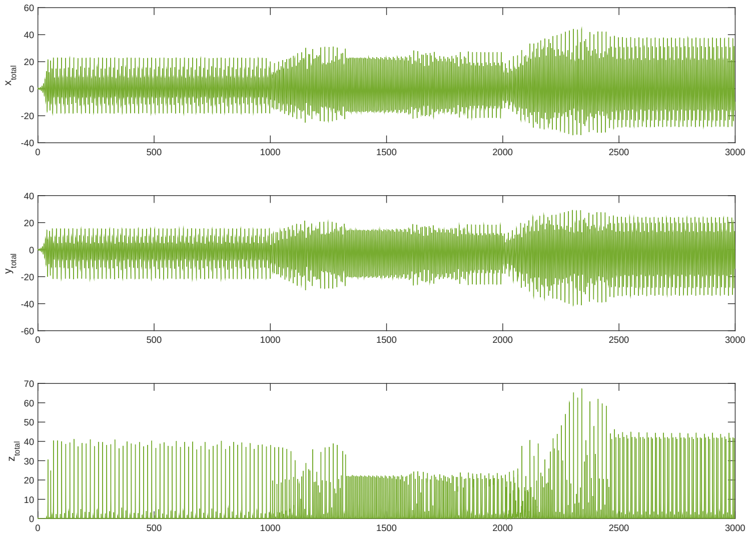

Fig. 5: Total summed oscillators variables in time, with switches from two to three connectivity at $t = 1000$, and from three to four connectivity at $t = 2000$.

controlled stable system. By exploiting this concept, a better understanding of low-level biological computation and control may be developed that can complete the black boxes in established concept, such as homeostasis. In further work, the generation of localised representation which represents higher order information in a criticality network can be exploited to address non-linear scaling aspects of neural information processing.

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