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ON THE QUANTUM THEORY
OF RADIATION

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The formal similarity between the chromatic distribution curve for thermal radiation and the Maxwell velocity-distribution law is too striking to have remained hidden for long. In fact, it was this similarity which led W. Wien, some time ago, to an extension of the radiation formula in his important theoretical paper, in which he derived his displacement law

$$\varrho = \nu^3 f(\nu/T). \tag{1}$$

As is well known, he discovered the formula

$$\varrho = \alpha \nu^3 \exp(-h\nu/kT), \tag{2}$$

which is still accepted as correct in the limit of large values of ν/T (Wien's radiation formula). Today we know that no approach which is founded on classical mechanics and electrodynamics can yield a useful radiation formula. Rather, classical theory must of necessity lead to Rayleigh's formula

$$\varrho = \frac{k\alpha}{h} \nu^2 T. \tag{3}$$

Next, Planck in his fundamental investigation based his radiation formula

$$\varrho = \alpha \nu^3 \frac{1}{\exp(h\nu/kT) - 1} \tag{4}$$

on the assumption of discrete portions of energy, from which quantum theory developed rapidly. It was then only natural that Wien's argument, which had led to eq. (2), should have become forgotten.

Not long ago I discovered a derivation of Planck's formula which was closely related to Wien's original argument * and which was based on the fundamental assumption of quantum theory. This derivation displays the relationship between Maxwell's curve and the chromatic distribution curve and deserves attention not only because of its simplicity, but especially because it seems to throw some light on the mechanism of emission and absorption of radiation by matter, a process which is still obscure to us. By postulating some hypotheses on the emission and absorption of radiation by molecules, which suggested themselves from quantum theory, I was able to show that molecules with a quantum-theoretical distribution of states in thermal equilibrium, were in dynamical equilibrium with the Planck radiation; in this way, Planck's formula (4) could be derived in an astonishingly simple and general way. It was obtained from the condition that the internal energy distribution of the molecules demanded by quantum theory, should follow purely from an emission and absorption of radiation.

But if these hypotheses on the interaction between radiation and matter turn out to be justified, they must produce rather more than just the correct statistical distribution of the internal energy of the molecules: for there is also a momentum transfer associated with the emission and absorption of radiation; this produces, purely through the interaction between the radiation and the molecules, a certain velocity distribution for the latter. This must evidently be identical with the velocity distribution of the molecules which is entirely due to their collisions among themselves, i.e. it must agree with the Maxwell distribution. It has to be required that the mean kinetic energy of a molecule (per degree of freedom) should be equal to $\frac{1}{2}kT$ in a Planck radiation field of temperature T. This requirement should hold independently of the nature of the molecules under consideration and independently of the frequencies emitted or absorbed by them. We want to demonstrate in the present paper that this far-reaching requirement is in fact satisfied quite generally, thus lending new support to our simple hypotheses concerning the elementary processes of emission and absorption.

To obtain such a result however requires a certain extension of the hypotheses, which had been up to now solely concerned with an exchange

* Verh. d. Deutschen physikal. Gesellschaft **18** Nr. 13/14 (1916) 318. The arguments used in that paper are reproduced in the present discussion.

of *energy*. The question arises: does the molecule suffer an impulse when it emits or absorbs energy ε? As an example, let us consider the emission of radiation from the point of view of classical electrodynamics. When a body emits an energy ε, it has a recoil (momentum) ε/c, provided the whole of the radiation is emitted in the same direction. If, however, the emission process has spatial symmetry, such as in the case of spherical waves, no recoil is produced at all. This second possibility is also of importance in the quantum theory of radiation. If a molecule absorbs or emits energy ε in the form of radiation during its transition from one quantum-theoretically possible state to another, such an elementary process can be thought of as being partially or completely directional, or else symmetrical (non-directional). It will become apparent that we shall only then arrive at a theory which is free from contradictions, if we consider such elementary processes to be perfectly directional; this embodies the main result of the subsequent discussion.

1. Fundamental hypothesis of quantum theory. Canonical distribution of states

In quantum theory a molecule of a given kind can only exist in a discrete set of states $Z_1, Z_2, \ldots Z_n, \ldots$, with (internal) energies $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_n, \ldots$, apart from its orientation and translatory motion. If such molecules belong to a gas at temperature T, the relative frequency W_n of such states Z_n is given by the formula

$$W_n = p_n \exp\left(-\varepsilon_n/kT\right), \tag{5}$$

which corresponds to the canonical distribution of states in statistical mechanics. In this formula, $k=R/N$ is the well-known Boltzmann constant, and p_n is a number, independent of T and characteristic for the molecule and its nth quantum state, which can be called the statistical 'weight' of this state. Formula (5) can be derived from Boltzmann's principle, or from purely thermodynamical considerations. It expresses the most extreme generalisation of Maxwell's velocity-distribution law.

The latest fundamental developments in quantum theory are concerned with a theoretical derivation of the quantum-theoretically possible states Z_n and their weights p_n. For the present basic investigation, a detailed determination of the quantum states is not required.

2. Hypotheses on the radiative exchange of energy

Let Z_n and Z_m be two quantum-theoretically possible states of the gas molecule, whose energies are ε_n and ε_m, respectively, and satisfy the inequality $\varepsilon_m > \varepsilon_n$. Let us assume that the molecule is capable of a transition from state Z_n into state Z_m with an absorption of radiation energy $\varepsilon_m - \varepsilon_n$; that, similarly, the transition from state Z_m to state Z_n is possible, with emission of the same radiative energy. Let the radiation absorbed or emitted by the molecule have frequency v which is characteristic for the index combination (m, n) that we are considering.

For the laws governing this transition, we introduce a few hypotheses which are obtained by carrying over the known situation for a Planck resonator in classical theory to the as yet unknown one in quantum theory.

(a) *Emission of radiation.* According to Hertz, an oscillating Planck resonator radiates energy in the well-known way, regardless of whether or not it is excited by an external field. Correspondingly, let us assume that a molecule may go from state Z_m to a state Z_n and emit radiation energy $\varepsilon_m - \varepsilon_n$ with frequency μ, without excitation from external causes. Let the probability dW for this to happen during the time interval dt, be

$$dW = A_m^n \, dt, \tag{A}$$

where A_m^n is a constant characterising the index combination under consideration.

The statistical law which we assumed, corresponds to that of a radioactive reaction, and the above elementary process corresponds to a reaction in which only γ-rays are emitted. It need not be assumed here that the time taken for this process is zero, but only that this time should be negligible compared with the times which the molecule spends in states Z_1, etc.

(b) *Absorption of radiation.* If a Planck resonator is located in a radiation field, the energy of the resonator is changed through the work done on the resonator by the electromagnetic field of the radiation; this work can be positive or negative, depending on the phases of the resonator and the oscillating field. We correspondingly introduce the following quantum-theoretical hypothesis. Under the influence of a radiation density ϱ of frequency v, a molecule can make a transition from state Z_n to state Z_m by absorbing radiation energy

$\varepsilon_m - \varepsilon_n$, according to the probability law

$$dW = B_n^m \varrho \, dt. \tag{B}$$

We similarly assume that a transition $Z_m \to Z_n$, associated with a liberation of radiation energy $\varepsilon_m - \varepsilon_n$, is possible under the influence of the radiation field, and that it satisfies the probability law

$$dW = B_m^n \varrho \, dt. \tag{B'}$$

B_n^m and B_m^n are constants. We shall give both processes the name 'changes of state due to irradiation'.

We now have to ask ourselves what is the momentum transfer to the molecule for such changes of state. Let us first discuss the case of absorption of radiation. If a radiation bundle in a given direction does work on a Planck resonator, the corresponding energy is removed from the radiation bundle. To this transfer of energy there also corresponds a momentum transfer from radiation bundle to resonator, by momentum conservation. The resonator is thus acted upon by a force in the beam direction of the radiation bundle. If the energy transfer is negative, then the force acts on the resonator in the opposite direction. If the quantum hypothesis holds, we can obviously interpret the process in the following way. If the incident radiation bundle produces the transition $Z_n \to Z_m$ by absorption of radiation, a momentum $(\varepsilon_m - \varepsilon_n)/c$ is transferred to the molecule in the direction of propagation of the beam. For the absorption process $Z_m \to Z_n$, the momentum transfer has the same magnitude, but is in the opposite direction. For the case where the molecule is acted upon simultaneously by several radiation bundles, we assume that total energy $\varepsilon_m - \varepsilon_n$ associated with an elementary process is removed from, or added to, a *single* such radiation bundle. Thus here, too, the momentum transferred to the molecule is $(\varepsilon_m - \varepsilon_n)/c$.

For an energy transfer by emission of radiation in the case of a Planck resonator, no momentum transfer to the resonator takes place, since emission occurs in the form of a spherical wave, according to classical theory. As was remarked previously, a quantum theory free from contradictions can only be obtained if the emission process, just as absorption, is assumed to be directional. In that case, for each elementary emission process $Z_m \to Z_n$ a momentum of magnitude $(\varepsilon_m - \varepsilon_n)/c$ is transferred to the molecule. If the latter is isotropic, we shall have to assume that all directions of emission are equally probable.

If the molecule is not isotropic, we arrive at the same statement if the orientation changes with time in accordance with the laws of chance. Moreover, such an assumption will also have to be made about the statistical laws for absorption, (B) and (B'). Otherwise the constants B_n^m and B_m^n would have to depend on the direction, and this can be avoided by making the assumption of isotropy or pseudo-isotropy (using time-averages).

3. Derivation of the Planck radiation law

We now look for that particular radiation density ϱ, for which the exchange of energy between radiation and molecules in keeping with the probability laws (A), (B), and (B') does not disturb the molecular distribution of states given by eq. (5). For this it is necessary and sufficient that the number of elementary processes of type (B) taking place per unit time should, on average, be equal to those of type (A) and (B') taken together. From this condition one obtains from (5), (A), (B), (B') the equation

$$p_n \exp\left(-\varepsilon_n/kT\right) B_n^m \varrho = p_m \exp\left(-\varepsilon_m/kT\right)(B_m^n \varrho + A_m^n)$$

for the elementary processes associated with the index combination (m, n).

If, in addition, ϱ tends to infinity with T, as will be assumed, the relation

$$p_n B_n^m = p_m B_m^n \tag{6}$$

has to hold between the constants B_n^m and B_m^n. We then obtain from our equation,

$$\varrho = \frac{A_m^n / B_m^n}{\exp\left[(\varepsilon_m - \varepsilon_n)/kT\right] - 1} \tag{7}$$

as the condition for dynamical equilibrium.

This expresses the temperature dependence of the radiation density according to Planck's law. From Wien's displacement law (1) it follows immediately that

$$\frac{A_m^n}{B_m^n} = \alpha\nu^3 \tag{8}$$

and

$$\varepsilon_m - \varepsilon_n = h\nu, \tag{9}$$

where α and h are universal constants. To compute the numerical value of the constant α, one would have to have an exact theory of

electrodynamic and mechanical processes; for the present, one has to confine oneself to a treatment of the limiting case of Rayleigh's law for high temperatures, for which the classical theory is valid in the limit.

Eq. (9) is of course the second principal rule in Bohr's theory of spectra. Since its extension by Sommerfeld and Epstein, this may well be claimed to have become a safely established part of our science. It also contains implicitly the photochemical principle of equivalence, as has been shown by me.

4. A method for calculating the motion of molecules in the radiation field

We now turn to a discussion of the motion of our molecules under the influence of radiation. For this we shall make use of a method which is well known from the theory of Brownian motion, and which I employed on several occasions for numerical computations of motions in a radiation field. To simplify the calculation we shall only consider the case where the motions take place in just one direction, the X-direction of the coordinate system. Furthermore, we shall confine ourselves to a calculation of the average value of the kinetic energy of the progressive motion, and we shall thus not attempt to prove that such velocities v obey the Maxwell distribution law. The mass M of the molecule is assumed sufficiently large, so that higher powers of v/c can be neglected in comparison with lower ones; we can then apply the laws of ordinary mechanics to the molecule. Finally, no real loss of generality is introduced if we perform the calculations as if the states with index m and n were the only possible states for the molecule.

The momentum Mv of a molecule undergoes two different types of change during the short time interval τ. Although the radiation is equally constituted in all directions, the molecule will nevertheless be subjected to a force originating from the radiation, which opposes the motion. Let this be equal to Rv, where R is a constant to be determined later. This force would bring the molecule to rest, if it were not for the irregularity of the radiative interactions which transmit a momentum Δ of changing sign and magnitude to the molecule during time τ; such an unsystematic effect, as opposed to that previously mentioned, will sustain some movement of the molecule. At the end of the short

time interval τ, the momentum of the molecule will have the value

$$Mv - Rv\tau + \varDelta.$$

Since the velocity distribution is supposed to remain constant with time, the average of the absolute value of the above quantity must be equal to Mv; the mean values of the squares of these quantities, taken over a long time interval or over a large number of molecules, must therefore be equal:

$$\overline{(Mv - Rv\tau + \varDelta)^2} = \overline{(Mv)^2}.$$

Since we were specifically concerned with the systematic effect of v on the momentum of the molecule, we shall have to neglect the average value $\overline{v\varDelta}$. Expanding the left-hand side of the equation, one therefore obtains

$$\overline{\varDelta^2} = 2RM\overline{v^2}\tau. \tag{10}$$

The mean square value $\overline{v^2}$, which the radiation of temperature T produces in our molecules by interacting with them, must be of the same size as the mean square value $\overline{v^2}$ obtained from the gas laws for a gas molecule at temperature T in the kinetic theory of gases. For the presence of our molecules would otherwise disturb the thermal equilibrium between the thermal radiation and an arbitrary gas held at the same temperature. We must therefore have

$$\tfrac{1}{2}\overline{Mv^2} = \tfrac{1}{2}kT. \tag{11}$$

Eq. (10) thus becomes

$$\overline{\varDelta^2}/\tau = 2RkT. \tag{12}$$

The investigation is now continued as follows. For a given radiation $(\varrho(v))$, $\overline{\varDelta^2}$ and R can be calculated, using our hypotheses on the interaction between radiation and molecules. If the results are inserted in eq. (12), this equation must become an identity, if ϱ is expressed as a function of v and T, using Planck's equation (4).

5. Calculation of R

Consider a molecule of the kind discussed above, moving uniformly with velocity v along the X-axis of the coordinate system K. We wish to find the average momentum which is transferred from the radiation field to the molecule per unit time. In order to calculate it, we have to

describe the radiation in a coordinate system K' which is at rest relative to the molecule in question. For we had only formulated our hypotheses on emission and absorption for the case of stationary molecules. The transformation to the [coordinate] system K' has been carried out in a number of places in the literature, particularly accurately in Mosengeil's Berlin dissertation. For completeness, however, I shall reproduce these simple arguments at this point.

Referred to K, the radiation is isotropic, i.e. the radiation of frequency range $d\nu$ per unit volume, associated with a given infinitesimal solid angle $d\varkappa$ relative to its direction of propagation, is given by

$$\varrho \, d\nu \, \frac{d\varkappa}{4\pi}, \tag{13}$$

where ϱ depends only on the frequency ν, but not on the direction. To this particular radiation there corresponds a particular radiation in K' which is similarly characterised by a frequency range $d\nu'$ and a certain solid angle $d\varkappa'$. The volume density of this particular radiation is given by

$$\varrho'(\nu', \varphi') \, d\nu' \, \frac{d\varkappa'}{4\pi}. \tag{13'}$$

This defines ϱ'. It depends on the direction, which we shall define in the usual way by means of the angle φ' with the X'-axis and the angle ψ' which the projection in the $Y'Z'$-plane makes with the Y'-axis. To these angles correspond the angles φ and ψ, which determine the direction of $d\varkappa$ in K in an analogous manner.

First of all it is clear that the transformation law between (13) and (13') must be the same as that for the squares of the amplitudes, A^2 and A'^2, of a plane wave with corresponding direction. We therefore find, to the desired approximation, that

$$\frac{\varrho'(\nu', \varphi') \, d\nu' \, d\varkappa'}{\varrho(\nu) \, d\nu \, d\varkappa} = 1 - 2\, \frac{v}{c} \cos \varphi, \tag{14}$$

or

$$\varrho'(\nu', \varphi') = \varrho(\nu)\, \frac{d\nu}{d\nu'}\, \frac{d\varkappa}{d\varkappa'} \left(1 - 2\, \frac{v}{c} \cos \varphi\right). \tag{14'}$$

The theory of relativity further gives the following formulae, valid

to the desired approximation,

$$v' = v\left(1 - \frac{v}{c}\cos\varphi\right), \tag{15}$$

$$\cos\varphi' = \cos\varphi - \frac{v}{c} + \frac{v}{c}\cos^2\varphi, \tag{16}$$

$$\psi' = \psi. \tag{17}$$

With the same approximation, we have from (15)

$$v = v'\left(1 + \frac{v}{c}\cos\varphi'\right).$$

Therefore, again to the same approximation,

$$\varrho(v) = \varrho\left(v' + \frac{v}{c}v'\cos\varphi'\right)$$

or

$$\varrho(v) = \varrho(v') + \frac{\partial\varrho}{\partial v}(v')\cdot\frac{v}{c}v'\cos\varphi'. \tag{18}$$

Moreover, from (15), (16) and (17),

$$\frac{dv}{dv'} = 1 + \frac{v}{c}\cos\varphi',$$

$$\frac{d\varkappa}{d\varkappa'} = \frac{\sin\varphi\,d\varphi\,d\psi}{\sin\varphi'\,d\varphi'\,d\psi'} = \frac{d(\cos\varphi)}{d(\cos\varphi')} = 1 - 2\frac{v}{c}\cos\varphi'.$$

By means of these two relations and (18), we can write (14') in the form

$$\varrho'(v',\varphi') = \left[(\varrho)_v + \frac{v}{c}v'\cos\varphi'\left(\frac{\partial\varrho}{\partial v}\right)_v\right]\left(1 - 3\frac{v}{c}\cos\varphi'\right). \tag{19}$$

Using (19) and our hypothesis on the emission and absorption of radiation by the molecule, we can easily calculate the average momentum transferred to the molecule per unit time. Before doing so, however, we shall have to say a few words in justification of this approach. It could be objected that eqs. (14), (15), (16) are based on Maxwell's theory of the electromagnetic field which cannot be reconciled with quantum theory. But this objection relates more to the form than to the real essence of the matter. For whatever the shape of a future

theory of the electromagnetic processes, the Doppler principle and the aberration law will at all events remain preserved, and hence also eqs. (15) and (16). Furthermore, the validity of the energy relation (14) certainly extends beyond wave theory; according to the theory of relativity, this transformation law also holds, e.g., for the energy density of a mass moving with (almost) the velocity of light and having infinitesimally small rest density. Eq. (19) can therefore lay claim to being valid for any theory of radiation.

According to (B), the radiation associated with the solid angle $d\varkappa'$ would give rise to $B_n^m \varrho'(\nu', \varphi')\, d\varkappa'/4\pi$ elementary absorption processes of the type $Z_n \rightarrow Z_m$ per second, if the molecule were to be restored to the state Z_n immediately after each such elementary process. But in reality, the time for remaining in state Z_n per second is equal to $S^{-1} p_n \exp(-\varepsilon_n/kT)$ from (5), where the abbreviation

$$S = p_n \exp(-\varepsilon_n/kT) + p_m \exp(-\varepsilon_m/kT) \qquad (20)$$

has been used. The number of such processes per second is thus really

$$\frac{1}{S}\, p_n \exp(-\varepsilon_n/kT)\, B_n^m \varrho'(\nu', \varphi')\, \frac{d\varkappa'}{4\pi}.$$

For each such elementary process a momentum $[(\varepsilon_m - \varepsilon_n)/c] \cos \varphi'$ gets transmitted to the atom in the direction of the positive X'-axis. Analogously we find, starting from (B'), that the corresponding number, per second, of elementary processes for an absorption of the type $Z_m \rightarrow Z_n$ is

$$\frac{1}{S}\, p_m \exp(-\varepsilon_m/kT)\, B_m^n \varrho'(\nu', \varphi')\, \frac{d\varkappa'}{4\pi},$$

and in such a process a momentum $-[(\varepsilon_m - \varepsilon_n)/c] \cos \varphi'$ is transferred to the molecule. The total momentum transfer to the molecule produced by the absorption of radiation is therefore, per unit time,

$$\frac{h\nu'}{cS}\, p_n B_n^m [\exp(-\varepsilon_n/kT) - \exp(-\varepsilon_m/kT)] \int \varrho'(\nu', \varphi') \cos \varphi'\, \frac{d\varkappa'}{4\pi}.$$

This follows from (6) and (9), and the integration extends over all elementary solid angles. On integrating, one obtains from (19)

$$-\frac{h\nu}{c^2 S}\left(\varrho - \tfrac{1}{3}\nu\, \frac{\partial \varrho}{\partial \nu}\right) p_n B_n^m [\exp(-\varepsilon_n/kT) - \exp(-\varepsilon_m/kT)]\nu.$$

Here the effective frequency is again denoted by ν (instead of ν').

But this expression represents the whole of the average momentum transferred per unit time to a molecule moving with velocity v. For it it clear that the elementary radiative emission processes, which take place in K' without interaction with the radiation field, have no preferred direction, so that they cannot transmit any momentum to the molecule, on average. The final result of our discussion is therefore

$$R = \frac{h\nu}{c^2 S}\left(\varrho - \tfrac{1}{3}v\,\frac{\partial\varrho}{\partial v}\right) p_n B_n^m [\exp(-\varepsilon_n/kT)][1 - \exp(-h\nu/kT)]. \quad (21)$$

6. Calculation of $\overline{\Delta^2}$

It is much simpler to calculate the effect of the irregularity of the elementary processes on the mechanical behaviour of the molecule, because the calculation can be based on a molecule at rest, to the degree of approximation to which we had restricted ourselves from the beginning.

Consider an arbitrary event, causing a momentum transfer λ to a molecule in the X-direction. This momentum can be assumed of different sign and magnitude in different cases. Nevertheless λ is supposed to satisfy a certain statistical law, such that its average value vanishes. Now let $\lambda_1, \lambda_2, \ldots$ be the momenta transmitted to the molecule due to a number of mutually independent causes, so that the total momentum transfer Δ is given by

$$\Delta = \sum \lambda_v.$$

Then, if the averages $\overline{\lambda_v}$ of the individual λ_v vanish,

$$\overline{\Delta^2} = \sum \overline{\lambda_v^2}. \quad (22)$$

If the mean square values $\overline{\lambda_v^2}$ of the individual momenta are all equal $(\overline{\lambda_v^2} = \overline{\lambda^2})$, and if l is the total number of events producing these momenta, the relation

$$\overline{\Delta^2} = l\overline{\lambda^2} \quad (22a)$$

holds.

According to our hypotheses, a momentum $\lambda = (h\nu/c)\cos\varphi$ is transferred to the molecule for each absorption and emission process. Here, φ denotes the angle between the X-axis and a randomly chosen

direction. One therefore obtains

$$\overline{\lambda^2} = \tfrac{1}{3}(h\nu/c)^2. \tag{23}$$

Since we assume that all the elementary processes which occur can be regarded as mutually independent events, we are allowed to use (22a). Then l is the number of elementary events which occur in time τ. This is twice the number of absorption processes $Z_n \rightarrow Z_m$ taking place in time τ. We therefore have

$$l = \frac{2}{S} \, p_n B_n^m \exp(-\varepsilon_n/kT) \varrho \tau \tag{24}$$

and from (23), (24) and (22),

$$\frac{\overline{\Delta^2}}{\tau} = \frac{2}{3S} \left(\frac{h\nu}{c}\right)^2 p_n B_n^m \exp(-\varepsilon_n/kT)\varrho. \tag{25}$$

7. Conclusion

We now have to show that the momenta transferred from the radiation field to the molecule according to our basic hypotheses, never disturb the thermodynamic equilibrium. For this, we need only insert the values for Δ^2/τ and R determined by (25) and (21), after replacing in (21) the expression

$$\left(\varrho - \tfrac{1}{3}\nu \, \frac{\partial \varrho}{\partial \nu}\right) [1 - \exp(-h\nu/kT)]$$

by $\varrho h\nu/3kT$, from (4). It is then seen immediately that our basic equation (12) is identically satisfied.

We have now completed the arguments which provide a strong support for the hypotheses stated in § 2, concerning the interaction between matter and radiation by means of absorption and emission processes, or in- or outgoing radiation. I was led to these hypotheses by my endeavour to postulate for the molecules, in the simplest possible manner, a quantum-theoretical behaviour that would be the analogue of the behaviour of a Planck resonator in the classical theory. From the general quantum assumption for matter, Bohr's second postulate (eq. 9) as well as Planck's radiation formula followed in a natural way.

Most important, however, seems to me to be the result concerning the momentum transfer to the molecule due to the absorption and

emission of radiation. If one of our assumptions about the momenta were to be changed, a violation of eq. (12) would be produced; it seems hardly possible to maintain agreement with this relation, imposed by the theory of heat, other than on the basis of our assumptions. The following statements can therefore be regarded as fairly certainly proved.

If a radiation bundle has the effect that a molecule struck by it absorbs or emits a quantity of energy hv in the form of radiation (ingoing radiation), then a momentum hv/c is always transferred to the molecule. For an absorption of energy, this takes place in the direction of propagation of the radiation bundle, for an emission in the opposite direction. If the molecule is acted upon by several directional radiation bundles, then it is always only a single one of these which participates in an elementary process of irradiation; this bundle alone then determines the direction of the momentum transferred to the molecule.

If the molecule undergoes a loss in energy of magnitude hv without external excitation, by emitting this energy in the form of radiation (outgoing radiation), then this process, too, is directional. Outgoing radiation in the form of spherical waves does not exist. During the elementary process of radiative loss, the molecule suffers a recoil of magnitude hv/c in a direction which is only determined by 'chance', according to the present state of the theory.

These properties of the elementary processes, imposed by eq. (12), make the formulation of a proper quantum theory of radiation appear almost unavoidable. The weakness of the theory lies on the one hand in the fact that it does not get us any closer to making the connection with wave theory; on the other, that it leaves the duration and direction of the elementary processes to 'chance'. Nevertheless I am fully confident that the approach chosen here is a reliable one.

There is room for one further general remark. Almost all theories of thermal radiation are based on the study of the interaction between radiation and molecules. But in general one restricts oneself to a discussion of the *energy* exchange, without taking the *momentum* exchange into account. One feels easily justified in this, because the smallness of the impulses transmitted by the radiation field implies that these can almost always be neglected in practice, when compared with other effects causing the motion. For a *theoretical* discussion, however, such small effects should be considered on a completely

equal footing with more conspicuous effects of a radiative *energy* transfer, since energy and momentum are linked in the closest possible way. For this reason a theory can only be regarded as justified when it is able to show that the impulses transmitted by the radiation field to matter lead to motions that are in accordance with the theory of heat.

Related papers

1a A. Einstein, *Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt.* Ann. d. Phys. **17** (1905) 132.

1b A. Einstein, *Zur Theorie der Lichterzeugung und Lichtabsorption.* Ann. d. Phys. **20** (1906) 199.

1c A. Einstein, *Beiträge zur Quantentheorie.* Verh. der D. Physikal. Ges. **16** (1914) 820.

1d A. Einstein, *Strahlungs-Emission und Absorption nach der Quantentheorie.* Verh. der D. Physikal. Ges. **18** (1916) 318.