the light—only those who have experienced it can understand that.

orig. in German

PHYSICS AND REALITY

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I. General Consideration Concerning the Method of Science

It has often been said, and certainly not without justification, that the man of science is a poor philosopher. Why, then, should it not be the right thing for the physicist to let the philosopher do the philosophizing? Such might indeed be the right thing at a time when the physicist believes he has at his disposal a rigid system of fundamental concepts and fundamental laws which are so well established that waves of doubt cannot reach them; but, it cannot be right at a time when the very foundations of physics itself have become problematic as they are now. At a time like the present, when experience forces us to seek a newer and more solid foundation, the physicist cannot simply surrender to the philosopher the critical contemplation of the theoretical foundations; for, he himself knows best, and feels more surely where the shoe pinches. In looking for a new foundation, he must try to make clear in his own mind just how far the concepts which he uses are justified, and are necessities.

The whole of science is nothing more than a refinement of everyday thinking. It is for this reason that the critical thinking of the physicist cannot possibly be restricted to the examination of the concepts of his own specific field. He cannot proceed without considering critically a much more difficult problem, the problem of analyzing the nature of everyday thinking.

Our psychological experience contains, in colorful succession, sense experiences, memory pictures of them, images, and feelings. In contrast to psychology, physics treats directly only of sense experiences and of the "understanding" of their connection. But even the concept of the "real external world" of everyday thinking rests exclusively on sense impressions.

Now we must first remark that the differentiation between sense impressions and images is not possible; or, at least it is not possible with absolute certainty. With the discussion of this problem, which affects also the notion of reality, we will not concern ourselves but we shall take the existence of sense experiences as given, that is to say, as psychic experiences of a special kind.

I believe that the first step in the setting of a "real external world" is the formation of the concept of bodily objects and of bodily objects of various kinds. Out of the multitude of our sense experiences we take, mentally and arbitrarily, certain repeatedly occurring complexes of sense impressions (par conjunction with sense impressions which are interpret signs for sense experiences of others), and we correlate to a concept—the concept of the bodily object. Considered cally this concept is not identical with the totality of sen pressions referred to; but it is a free creation of the h (or animal) mind. On the other hand, this concept ov meaning and its justification exclusively to the totality sense impressions which we associate with it.

The second step is to be found in the fact that, in our ing (which determines our expectation), we attribute t concept of the bodily object a significance, which is to a high degree independent of the sense impressions which originally give rise to it. This is what we mean when we attribute to the bodily object "a real existence." The justification of such a setting rests exclusively on the fact that, by means of such concepts and mental relations between them, we are able to orient ourselves in the labyrinth of sense impressions. These notions and relations, although free mental creations, appear to us as stronger and more unalterable than the individual sense experience itself, the character of which as anything other than the result of an illusion or hallucination is never completely guaranteed. On the other hand, these concepts and relations, and indeed the postulation of real objects and, generally speaking, of the existence of "the real world," have justification only in so far as they are connected with sense impressions between which they form a mental connection.

The very fact that the totality of our sense experiences is such that by means of thinking (operations with concepts, and the creation and use of definite functional relations between them, and the coordination of sense experiences to these concepts) it can be put in order, this fact is one which leaves us in awe, but which we shall never understand. One may say "the eternal mystery of the world is its comprehensibility." It is one of the great realizations of Immanuel Kant that the postulation of a real external world would be senseless without this comprehensibility.

In speaking here of "comprehensibility," the expression is used in its most modest sense. It implies: the production of some sort of order among sense impressions, this order being produced by the creation of general concepts, relations between these concepts, and by definite relations of some kind between the concepts and sense experience. It is in this sense that the world of our sense experiences is comprehensible. The fact that it is comprehensible is a miracle.

In my opinion, nothing can be said *a priori* concerning the manner in which the concepts are to be formed and connected, and how we are to coordinate them to sense experiences. In guiding us in the creation of such an order of sense experiences, success alone is the determining factor. All that is necessary is to fix a set of rules, since without such rules the acquisition of knowledge in the desired sense would be impossible. One may compare these rules with the rules of a game in which, while the rules themselves are arbitrary, it is their rigidity alone which makes the game possible. However, the fixation will never be final. It will have validity only for a special field of application (i.e., there are no final categories in the sense of Kant).

The connection of the elementary concepts of everyday thinking with complexes of sense experiences can only be comprehended intuitively and it is unadaptable to scientifically logical fixation. The totality of these connections—none of which is expressible in conceptual terms—is the only thing which differentiates the great building which is science from a logical but empty scheme of concepts. By means of these connections, the purely conceptual propositions of science become general statements about complexes of sense experiences.

We shall call "primary concepts" such concepts as are directly and intuitively connected with typical complexes of sense experiences. All other notions are—from the physical point of view—possessed of meaning only in so far as they are connected, by propositions, with the primary notions. These propositions are partially definitions of the concepts (and of the statements derived logically from them) and partially propositions not derivable from the definitions, which express at least indirect relations between the "primary concepts," and in this way between sense experiences. Propositions of the latter kind are "statements about reality" or laws of nature, i.e., propositions which have to show their validity when applied to sense experiences covered by primary concepts. The question as to which of the propositions shall be considered as definitions and which as natural laws will depend largely upon the chosen representation. It really becomes absolutely necessary to make this differentiation only when one examines the degree to which the whole system of concepts considered is not empty from the physical point of view.

STRATIFICATION OF THE SCIENTIFIC SYSTEM

The aim of science is, on the one hand, a comprehension, as *complete* as possible, of the connection between the sense experiences in their totality, and, on the other hand, the accomplishment of this aim *by the use of a minimum of primary concepts and relations.* (Seeking, as far as possible, logical unity in the world picture, i.e., paucity in logical elements.)

Science uses the totality of the primary concepts, i.e., concepts directly connected with sense experiences, and propositions connecting them. In its first stage of development, science does not contain anything else. Our everyday thinking is satisfied on the whole with this level. Such a state of affairs cannot, however, satisfy a spirit which is really scientifically minded; because the totality of concepts and relations obtained in this manner is utterly lacking in logical unity. In order to supplement this deficiency, one invents a system poorer in concepts and relations, a system retaining the primary concepts and relations of the "first layer" as logically derived concepts and relations. This new "secondary system" pays for its higher logical unity by having elementary concepts (concepts of the second layer), which are no longer directly connected with complexes of sense experiences. Further striving for logical unity brings us to a tertiary system, still poorer in concepts and relations, for the deduction of the concepts and relations of the secondary (and so indirectly of the primary) layer. Thus the story goes on until we have arrived at a system of the greatest conceivable unity, and of the greatest poverty of concepts of the logical foundations, which is still compatible with the observations made by our senses. We do not know whether or not this ambition will ever result in a definitive system. If one is asked for his opinion, he is inclined to answer no. While wrestling with the problems, however, one will never give up the hope that this greatest of all aims can really be attained to a very high degree.

An adherent to the theory of abstraction or induction might call our layers "degrees of abstraction"; but I do not consider it justifiable to veil the logical independence of the concept from the sense experiences. The relation is not analogous to that of soup to beef but rather of check number to overcoat.

The layers are furthermore not clearly separated. It is not even absolutely clear which concepts belong to the primary layer. As a matter of fact, we are dealing with freely formed concepts, which, with a certainty sufficient for practical use, are intuitively connected with complexes of sense experiences in such a manner that, in any given case of experience, there is no uncertainty as to the validity of an assertion. The essential thing is the aim to represent the multitude of concepts and propositions, close to experience, as propositions, logically deduced from a basis, as narrow as possible, of fundamental concepts and fundamental relations which themselves can be chosen freely (axioms). The liberty of choice, however, is of a special kind; it is not in any way similar to the liberty of a writer of

fiction. Rather, it is similar to that of a man engaged in solving a well-designed word puzzle. He may, it is true, propose any word as the solution; but, there is only *one* word which really solves the puzzle in all its parts. It is a matter of faith that nature —as she is perceptible to our five senses—takes the character of such a well-formulated puzzle. The successes reaped up to now by science do, it is true, give a certain encouragement for this faith.

The multitude of layers discussed above corresponds to the several stages of progress which have resulted from the struggle for unity in the course of development. As regards the final aim, intermediary layers are only of temporary nature. They must eventually disappear as irrelevant. We have to deal, however, with the science of today, in which these strata represent problematic partial successes which support one another but which also threaten one another, because today's system of concepts contains deep-seated incongruities which we shall meet later on.

It will be the aim of the following lines to demonstrate what paths the constructive human mind has entered, in order to arrive at a basis of physics which is logically as uniform as possible.

II. Mechanics and the Attempts to Base All Physics upon It

An important property of our sense experiences, and, more generally, of all of our experiences, is their temporal order. This kind of order leads to the mental conception of a subjective time, an ordering scheme for our experience. The subjective time leads then via the concept of the bodily object and of space to the concept of objective time, as we shall see later on.

Ahead of the notion of objective time there is, however, the concept of space; and ahead of the latter we find the concept of the bodily object. The latter is directly connected with complexes of sense experiences. It has been pointed out that one property which is characteristic of the notion "bodily object" is the property which provides that we coordinate to it an existence, independent of (subjective) time, and independent of the fact that it is perceived by our senses. We do this in spite of the fact that we perceive temporal alterations in it. Poincaré has justly emphasized the fact that we distinguish two kinds of alterations of the bodily object, "changes of state" and "changes of position." The latter, he remarked, are alterations which we can reverse by voluntary motions of our bodies.

That there are bodily objects to which we have to ascribe, within a certain sphere of perception, no alteration of state, but only alterations of position, is a fact of fundamental importance for the formation of the concept of space (in a certain degree even for the justification of the notion of the bodily object itself). Let us call such an object "practically rigid."

If, as the object of our perception, we consider simultaneously (i.e., as a single unit) two practically rigid bodies, then there exist for this ensemble such alterations as can *not* possibly be considered as changes of position of the whole, notwithstanding the fact that this is the case for each one of the two constituents. This leads to the notion of "change of relative position" of the two objects; and, in this way, also to the notion of "relative position" of the two objects. It is found moreover that among the relative positions, there is one of a specific kind which we designate as "contact." * Permanent contact of two bodies in three or more "points" means that they are united to a quasi-rigid compound body. It is permissible to say that the second body forms then a (quasi-rigid) continuation of the first body and may, in its turn, be continued quasi-rigidly. The possibility of the quasi-rigid continuation of a body is unlimited. The totality of all conceivable quasi-rigid continuations of a body B_0 is the infinite "space" determined by it.

In my opinion, the fact that every bodily object situated in any arbitrary manner can be put into contact with the quasi-rigid continuation of some given body B_0 (body of reference), this fact is the empirical basis of our conception of space. In pre-scientific thinking, the solid earth's crust plays the role of B_0 and its continuation. The very name geometry indicates

* It is in the nature of things that we are able to talk about these objects only by means of concepts of our own creation, concepts which themselves are not subject to definition. It is essential, however, that we make use only of such concepts concerning whose coordination to our experience we feel no doubt.

that the concept of space is psychologically connected with the earth as an ever present body of reference.

The bold notion of "space" which preceded all scientific geometry transformed our mental concept of the relations of positions of bodily objects into the notion of the position of these bodily objects in "space." This, of itself, represents a great formal simplification. Through this concept of space one reaches, moreover, an attitude in which any description of position is implicitly a description of contact; the statement that a point of a bodily object is located at a point P of space means that the object touches the point P of the standard body of reference B_0 (supposed appropriately continued) at the point considered.

In the geometry of the Greeks, space plays only a qualitative role, since the position of bodies in relation to space is considered as given, it is true, but is not described by means of numbers. Descartes was the first to introduce this method. In his language, the whole content of Euclidean geometry can axiomatically be founded upon the following statements: (1) Two specified points of a rigid body determine a segment. (2) We may associate triples of numbers X_1, X_2, X_3, to points of space in such a manner that for every segment $P' - P''$ under consideration, the coordinates of whose end points are X_1', X_2', X_3'; X_1'', X_2'', X_3'', the expression

$$s^2 = (X_1'' - X_1')^2 + (X_2'' - X_2')^2 + (X_3'' - X_3')^2$$

is independent of the position of the body, and of the positions of any and all other bodies.

The (positive) number s is called the length of the segment, or the distance between the two points P' and P'' of space (which are coincident with the points P' and P'' of the segment).

The formulation is chosen, intentionally, in such a way that it expresses clearly, not only the logical and axiomatic, but also the empirical content of Euclidean geometry. The purely logical (axiomatic) representation of Euclidean geometry has, it is true, the advantage of greater simplicity and clarity. It pays for this, however, by renouncing a representation of the connection between the conceptual construction and the sense experiences upon which connection, alone, the significance of geometry for physics rests. The fatal error that logical necessity, preceding all experience, was the basis of Euclidean geometry and the concept of space belonging to it, this fatal error arose from the fact that the empirical basis, on which the axiomatic construction of Euclidean geometry rests, had fallen into oblivion.

In so far as one can speak of the existence of rigid bodies in nature, Euclidean geometry is a physical science, which must be confirmed by sense experiences. It concerns the totality of laws which must hold for the relative positions of rigid bodies independently of time. As one may see, the physical notion of space also, as originally used in physics, is tied to the existence of rigid bodies.

From the physicist's point of view, the central importance of Euclidean geometry rests in the fact that its laws are independent of the specific nature of the bodies whose relative positions it discusses. Its formal simplicity is characterized by the properties of homogeneity and isotropy (and the existence of similar entities).

The concept of space is, it is true, useful, but not indispensable for geometry proper, i.e., for the formulation of rules about the relative positions of rigid bodies. By contrast, the concept of objective time, without which the formulation of the fundamentals of classical mechanics is impossible, is linked with the concept of the spatial continuum.

The introduction of objective time involves two postulates which are independent of each other.

1. The introduction of the objective local time by connecting the temporal sequence of experiences with the readings of a "clock," i.e., of a periodically recurring closed system.

2. The introduction of the notion of objective time for the events in the whole space, by which notion alone the idea of local time is extended to the idea of time in physics.

Note concerning 1. As I see it, it does not mean a "petitio principii" if one puts the concept of periodical recurrence ahead of the concept of time, while one is concerned with the clarification of the origin and of the empirical content of the concept of time. Such a conception corresponds exactly to the precedence of the concept of the rigid (or quasi-rigid) body in

the interpretation of the concept of space.

Further discussion of 2. The illusion which prevailed prior to the enunciation of the theory of relativity—that, from the point of view of experience the meaning of simultaneity in relation to spatially distant events and, consequently, that the meaning of physical time is *a priori* clear—this illusion had its origin in the fact that in our everyday experience we can neglect the time of propagation of light. We are accustomed on this account to fail to differentiate between "simultaneously seen" and "simultaneously happening"; and, as a result, the difference between time and local time is blurred.

The lack of definiteness which, from the point of view of its empirical significance, adheres to the notion of time in classical mechanics was veiled by the axiomatic representation of space and time as given independently of our sense experiences. Such a use of notions—independent of the empirical basis to which they owe their existence—does not necessarily damage science. One may, however, easily be led into the error of believing that these notions, whose origin is forgotten, are logically necessary and therefore unalterable, and this error may constitute a serious danger to the progress of science.

It was fortunate for the development of mechanics and hence also for the development of physics in general, that the lack of definiteness in the concept of objective time remained hidden from the earlier philosophers as regards its empirical interpretation. Full of confidence in the real meaning of the space-time construction, they developed the foundations of mechanics which we shall characterize, schematically, as follows:

(*a*) Concept of a material point: a bodily object which—as regards its position and motion—can be described with sufficient accuracy as a point with coordinates X_1, X_2, X_3. Description of its motion (in relation to the "space" B_0) by giving X_1, X_2, X_3, as functions of the time.

(*b*) Law of inertia: the disappearance of the components of acceleration for a material point which is sufficiently far away from all other points.

(*c*) Law of motion (for the material point): Force = mass $\times$ acceleration.

(*d*) Laws of force (interactions between material points).

In this, (*b*) is merely an important special case of (*c*). A real theory exists only when the laws of force are given. The forces must in the first place only obey the law of equality of action and reaction in order that a system of points—permanently connected to each other by forces—may behave like *one* material point.

These fundamental laws, together with Newton's law for the gravitational force, form the basis of the mechanics of celestial bodies. In this mechanics of Newton, and in contrast to the above conceptions of space derived from rigid bodies, the space B_0 enters in a form which contains a new idea; it is not for every B_0 that validity is asserted (for a given law of force) for (*b*) and (*c*), but only for a B_0 in an appropriate state of motion (inertial system). On account of this fact, the coordinate space acquired an independent physical property which is not contained in the purely geometrical notion of space, a circumstance which gave Newton considerable food for thought (pail-experiment).*

Classical mechanics is only a general scheme; it becomes a theory only by explicit indication of the force laws (*d*) as was done so very successfully by Newton for celestial mechanics. From the point of view of the aim of the greatest logical simplicity of the foundations, this theoretical method is deficient in so far as the laws of force cannot be obtained by logical and formal considerations, so that their choice is *a priori* to a large extent arbitrary. Also Newton's law of gravitation is distinguished from other conceivable laws of force exclusively by its *success*.

In spite of the fact that, today, we know positively that classical mechanics fails as a foundation dominating all physics, it still occupies the center of all of our thinking in physics. The reason for this lies in the fact that, regardless of important

* This defect of the theory could only be eliminated by such a formulation of mechanics as would claim validity for all B_0. This is one of the steps which led to the general theory of relativity. A second defect, also eliminated only by the introduction of the general theory of relativity, lies in the fact that there is no reason given by mechanics itself for the equality of the gravitational and inertial mass of the material point.

progress reached since the time of Newton, we have not yet arrived at a new foundation of physics concerning which we may be certain that the manifold of all investigated phenomena, and of successful partial theoretical systems, could be deduced logically from it. In the following lines I shall try to describe briefly how the matter stands.

First we try to get clearly in our minds how far the system of classical mechanics has shown itself adequate to serve as a basis for the whole of physics. Since we are dealing here only with the foundations of physics and with its development, we need not concern ourselves with the purely *formal* progresses of mechanics (equations of Lagrange, canonical equations, etc.). *One* remark, however, appears indispensable. The notion "material point" is fundamental for mechanics. If now we seek to develop the mechanics of a bodily object which itself can *not* be treated as a material point—and strictly speaking every object "perceptible to our senses" is of this category—then the question arises: How shall we imagine the object to be built up out of material points, and what forces must we assume as acting between them? The formulation of this question is indispensable, if mechanics is to pretend to describe the object *completely*.

It is in line with the natural tendency of mechanics to assume these material points, and the laws of forces acting between them, as invariable, since temporal changes would lie outside of the scope of mechanical explanation. From this we can see that classical mechanics must lead us to an atomistic construction of matter. We now realize, with special clarity, how much in error are those theorists who believe that theory comes inductively from experience. Even the great Newton could not free himself from this error (*"Hypotheses non fingo"**).

In order to save itself from becoming hopelessly lost in this line of thought (atomism), science proceeded first in the following manner. The mechanics of a system is determined if its potential energy is given as a function of its configuration. Now, if the acting forces are of such a kind as to guarantee the

* "I make no hypotheses."

maintenance of certain structural properties of the system's configuration, then the configuration may be described with sufficient accuracy by a relatively small number of configuration variables q_r; the potential energy is considered only in so far as it is dependent upon *these* variables (for instance, description of the configuration of a practically rigid body by six variables).

A second method of application of mechanics, which avoids the consideration of a subdivision of matter down to "real" material points, is the mechanics of so-called continuous media. This mechanics is characterized by the fiction that the density and the velocity of matter depend continuously upon coordinates and time, and that the part of the interactions not explicitly given can be considered as surface forces (pressure forces) which again are continuous functions of position. Herein we find the hydrodynamic theory, and the theory of elasticity of solid bodies. These theories avoid the explicit introduction of material points by fictions which, in the light of the foundation of classical mechanics, can only have an approximate significance.

In addition to their great *practical* significance, these categories of science have—by developing new mathematical concepts—created those formal tools (partial differential equations) which have been necessary for the subsequent attempts at a new foundation of all of physics.

These two modes of application of mechanics belong to the so-called "phenomenological" physics. It is characteristic of this kind of physics that it makes as much use as possible of concepts which are close to experience but, for this reason, has to give up, to a large extent, unity in the foundations. Heat, electricity, and light are described by separate variables of state and material constants other than the mechanical quantities; and to determine all of these variables in their mutual and temporal dependence was a task which, in the main, could only be solved empirically. Many contemporaries of Maxwell saw in such a manner of presentation the ultimate aim of physics, which they thought could be obtained purely inductively from experience on account of the relative closeness of the concepts

used to experience. From the point of view of theories of knowledge St. Mill and E. Mach took their stand approximately on this ground.

In my view, the greatest achievement of Newton's mechanics lies in the fact that its consistent application has led beyond this phenomenological point of view, particularly in the field of heat phenomena. This occurred in the kinetic theory of gases and in statistical mechanics in general. The former connected the equation of state of the ideal gases, viscosity, diffusion, and heat conductivity of gases and radiometric phenomena of gases, and gave the logical connection of phenomena which, from the point of view of direct experience, had nothing whatever to do with one another. The latter gave a mechanical interpretation of the thermodynamic ideas and laws and led to the discovery of the limit of applicability of the notions and laws of the classical theory of heat. This kinetic theory, which by far surpassed phenomenological physics as regards the logical unity of its foundations, produced, moreover, definite values for the true magnitudes of atoms and molecules which resulted from several independent methods and were thus placed beyond the realm of reasonable doubt. These decisive progresses were paid for by the coordination of atomistic entities to the material points, the constructively speculative character of these entities being obvious. Nobody could hope ever to "perceive directly" an atom. Laws concerning variables connected more directly with experimental facts (for example: temperature, pressure, speed) were deduced from the fundamental ideas by means of complicated calculations. In this manner physics (at least part of it), originally more phenomenologically constructed, was reduced, by being founded upon Newton's mechanics for atoms and molecules, to a basis further removed from direct experiment, but more uniform in character.

III. The Field Concept

In explaining optical and electrical phenomena, Newton's mechanics has been far less successful than it had been in the fields cited above. It is true that Newton tried to reduce light to the motion of material points in his corpuscular theory of light. Later on, however, as the phenomena of polarization, diffraction, and interference of light forced upon this theory more and more unnatural modifications, Huygens' undulatory theory of light prevailed. Probably this theory owes its origin essentially to the phenomena of crystal optics and to the theory of sound, which was then already elaborated to a certain degree. It must be admitted that Huygens' theory also was based in the first instance upon classical mechanics; the all-penetrating ether had to be assumed as the carrier of the waves, but no known phenomenon suggested the way in which the ether was built up from material points. One could never get a clear picture of the internal forces governing the ether, nor of the forces acting between the ether and "ponderable" matter. The foundations of this theory remained, therefore, eternally in the dark. The true basis was a partial differential equation, the reduction of which to mechanical elements remained always problematic.

electron

For the theoretical conception of electric and magnetic phenomena one introduced, again, masses of a special kind, and between these masses one assumed the existence of forces acting at a distance, similar to Newton's gravitational forces. This special kind of matter, however, appeared to be lacking in the fundamental property of inertia; and the forces acting between these masses and the ponderable matter remained obscure. To these difficulties there had to be added the polar character of these kinds of matter which did not fit into the scheme of classical mechanics. The basis of the theory became still more unsatisfactory when electrodynamic phenomena became known, notwithstanding the fact that these phenomena brought the physicist to the explanation of magnetic phenomena through electrodynamic phenomena and, in this way, made the assumption of magnetic masses superfluous. This progress had, indeed, to be paid for by increasing the complexity of the forces of interaction which had to be assumed as existing between electrical masses in motion.

The escape from this unsatisfactory situation by the electric field theory of Faraday and Maxwell represents probably the most profound transformation of the foundations of physics

since Newton's time. Again, it has been a step in the direction of constructive speculation which has increased the distance between the foundation of the theory and sense experiences. The existence of the field manifests itself, indeed, only when electrically charged bodies are introduced into it. The differential equations of Maxwell connect the spatial and temporal differential coefficients of the electric and magnetic fields. The electric masses are nothing more than places of non-vanishing divergence of the electric field. Light waves appear as undulatory electromagnetic field processes in space.

To be sure, Maxwell still tried to interpret his field theory mechanically by means of mechanical ether models. But these attempts receded gradually to the background following the representation of the theory—purged of any unnecessary trimmings—by Heinrich Hertz, so that in this theory the field finally took the fundamental position which had been occupied in Newton's mechanics by the material points. Primarily, however, this applied only for electromagnetic fields in empty space.

In its initial stage the theory was yet quite unsatisfactory for the interior of matter, because there, two electric vectors had to be introduced, which were connected by relations dependent on the nature of the medium, these relations being inaccessible to any theoretical analysis. An analogous situation arose in connection with the magnetic field, as well as in the relation between electric current density and the field.

Here H. A. Lorentz found a way out which showed, at the same time, the way to an electrodynamic theory of bodies in motion, a theory which was more or less free from arbitrary assumptions. His theory was built on the following fundamental hypotheses:

Everywhere (including the interior of ponderable bodies) the seat of the field is the empty space. The participation of matter in electromagnetic phenomena has its origin only in the fact that the elementary particles of matter carry unalterable electric charges, and, on this account, are subject on the one hand to the actions of ponderomotive forces and on the other hand possess the property of generating a field. The elementary particles obey Newton's law of motion for material points.

partial diff eqs

This is the basis on which H. A. Lorentz obtained his synthesis of Newton's mechanics and Maxwell's field theory. The weakness of this theory lies in the fact that it tried to determine the phenomena by a combination of partial differential equations (Maxwell's field equations for empty space) and total differential equations (equations of motion of points), which procedure was obviously unnatural. The inadequacy of this point of view manifested itself in the necessity of assuming finite dimensions for the particles in order to prevent the electromagnetic field existing at their surfaces from becoming infinitely large. The theory failed, moreover, to give any explanation concerning the tremendous forces which hold the electric charges on the individual particles. H. A. Lorentz accepted these weaknesses of his theory, which were well known to him, in order to explain the phenomena correctly at least in general outline.

total diff eqs.

Furthermore, there was one consideration which pointed beyond the frame of Lorentz's theory. In the environment of an electrically charged body there is a magnetic field which furnishes an (apparent) contribution to its inertia. Should it not be possible to explain the *total* inertia of the particles electromagnetically? It is clear that this problem could be worked out satisfactorily only if the particles could be interpreted as regular solutions of the electromagnetic partial differential equations. The Maxwell equations in their original form do not, however, allow such a description of particles, because their corresponding solutions contain a singularity. Theoretical physicists have tried for a long time, therefore, to reach the goal by a modification of Maxwell's equations. These attempts have, however, not been crowned with success. Thus it happened that the goal of erecting a pure electromagnetic field theory of matter remained unattained for the time being, although in principle no objection could be raised against the possibility of reaching such a goal. The lack of any systematic method leading to a solution discouraged further attempts in this direction. What appears certain to me, however, is that, in the foundations of any consistent field theory, the particle concept must not appear in addition to the field concept. The whole theory must be based

Same for Einstein field theory!

solely on partial differential equations and their singularity-free solutions.

IV. The Theory of Relativity

There is no inductive method which could lead to the fundamental concepts of physics. Failure to understand this fact constituted the basic philosophical error of so many investigators of the nineteenth century. It was probably the reason why the molecular theory and Maxwell's theory were able to establish themselves only at a relatively late date. Logical thinking is necessarily deductive; it is based upon hypothetical concepts and axioms. How can we expect to choose the latter so that we might hope for a confirmation of the consequences derived from them?

The most satisfactory situation is evidently to be found in cases where the new fundamental hypotheses are suggested by the world of experience itself. The hypothesis of the non-existence of perpetual motion as a basis for thermodynamics affords such an example of a fundamental hypothesis suggested by experience; the same holds for Galileo's principle of inertia. In the same category, moreover, we find the fundamental hypotheses of the theory of relativity, which theory has led to an unexpected expansion and broadening of the field theory, and to the superseding of the foundations of classical mechanics.

E+L impose

The success of the Maxwell-Lorentz theory has given great confidence in the validity of the electromagnetic equations for empty space, and hence, in particular, in the assertion that light travels "in space" with a certain constant speed c. Is this assertion of the constancy of light velocity valid for every inertial system? If it were not, then one specific inertial system or, more accurately, one specific state of motion (of a body of reference) would be distinguished from all others. This, however, appeared to contradict all mechanical and electromagnetic-optical experimental facts.

For these reasons it was necessary to raise to the rank of a principle the validity of the law of constancy of light velocity for all inertial systems. From this, it follows that the spatial coordinates X_1, X_2, X_3, and the time X_4, must be transformed according to the "Lorentz-transformation" which is characterized by the invariance of the expression

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2$$

(if the unit of time is chosen in such a manner that the speed of light $c = 1$).

By this procedure time lost its absolute character, and was adjoined to the "spatial" coordinates as of algebraically (nearly) similar character. The absolute character of time and particularly of simultaneity was destroyed, and the four-dimensional description was introduced as the only adequate one.

In order to account, also, for the equivalence of all inertial systems with regard to all the phenomena of nature, it is necessary to postulate invariance of all systems of physical equations which express general laws with respect to Lorentz transformations. The elaboration of this requirement forms the content of the special theory of relativity.

This theory is compatible with the equations of Maxwell; but it is incompatible with the basis of classical mechanics. It is true that the equations of motion of the material point can be modified (and with them the expressions for momentum and kinetic energy of the material point) in such a manner as to satisfy the theory; but, the concept of the force of interaction, and with it the concept of potential energy of a system, lose their basis, because these concepts rest upon the idea of absolute simultaneity. The field, as determined by differential equations, takes the place of the force.

Since the foregoing theory allows interaction only by fields, it requires a field theory of gravitation. Indeed, it is not difficult to formulate such a theory in which, as in Newton's theory, the gravitational fields can be reduced to a scalar which is the solution of a partial differential equation. However, the experimental facts expressed in Newton's theory of gravitation lead in another direction, that of the general theory of relativity.

It is an unsatisfactory feature of classical mechanics that in its fundamental laws the same mass constant appears in two different rôles, namely as "inertial mass" in the law of motion, and as "gravitational mass" in the law of gravitation. As a result, the acceleration of a body in a pure gravitational field is

independent of its material; or, in a uniformly accelerated coordinate system (accelerated in relation to an "inertial system") the motions take place as they would in a homogeneous gravitational field (in relation to a "motionless" system of coordinates). If one assumes that the equivalence of these two cases is complete, then one attains an adaptation of our theoretical thinking to the fact that the gravitational and inertial masses are equal.

From this it follows that there is no longer any reason for favoring, as a matter of principle, the "inertial systems"; and, we must admit on an equal footing also *non-linear* transformations of the coordinates (x_1, x_2, x_3, x_4). If we make such a transformation of a system of coordinates of the special theory of relativity, then the metric

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2$$

goes over into a general (Riemannian) metric of the form

$$ds^2 = g_{\mu\nu}\, dx_\mu\, dx_\nu \quad \text{(summed over } \mu \text{ and } \nu\text{)}$$

where the $g_{\mu\nu}$, symmetrical in μ and ν, are certain functions of $x_1 \ldots x_4$ which describe both the metric properties, and the gravitational field in relation to the new system of coordinates.

The foregoing improvement in the interpretation of the mechanical basis must, however, be paid for in that—as becomes evident on closer scrutiny—the new coordinates can no longer be interpreted as results of measurements on rigid bodies and clocks, as they could in the original system (an inertial system with vanishing gravitational field).

The passage to the general theory of relativity is realized by the assumption that such a representation of the field properties of space already mentioned, by functions $g_{\mu\nu}$ (that is to say, by a Riemann metric), is also justified in the *general* case in which there is no system of coordinates in relation to which the metric takes the simple quasi-Euclidean form of the special theory of relativity.

Now the coordinates, by themselves, no longer express metric relations, but only the "closeness" of objects whose coordinates differ but little from one another. All transformations of the coordinates have to be admitted so long as these transformations are free from singularities. Only such equations as are covariant in relation to arbitrary transformations in this sense have meaning as expressions of general laws of nature (postulate of general covariance).

The first aim of the general theory of relativity was a preliminary version which, while not meeting the requirements for constituting a closed system, could be connected in as simple a manner as possible with "directly observable facts." If the theory were restricted to pure gravitational mechanics, Newton's gravitational theory could serve as a model. This preliminary version may be characterized as follows:

1. The concept of the material point and of its mass is retained. A law of motion is given for it, this law of motion being the translation of the law of inertia into the language of the general theory of relativity. This law is a system of total differential equations, the system characteristic of the geodesic line.

2. Newton's law of interaction by gravitation is replaced by the system of the simplest generally covariant differential equations which can be set up for the $g_{\mu\nu}$-tensor. It is formed by equating to zero the once contracted Riemannian curvature tensor ($R_{\mu\nu} = 0$).

This formulation permits the treatment of the problem of the planets. More accurately speaking, it allows the treatment of the problem of motion of material points of practically negligible mass in the (centrally symmetric) gravitational field produced by a material point supposed to be "at rest." It does not take into account the reaction of the "moving" material points on the gravitational field, nor does it consider how the central mass produces this gravitational field.

Analogy with classical mechanics shows that the following is a way to complete the theory. One sets up as field equations

$$R_{ik} - \tfrac{1}{2}g_{ik}R = - T_{ik}$$

where R represents the scalar of Riemannian curvature, T_{ik} the energy tensor of the matter in a phenomenological representation. The left side of the equation is chosen in such a manner that its divergence disappears identically. The resulting disappearance of the divergence of the right side produces the "equations of motion" of matter, in the form of partial differential equations for the case where T_{ik} introduces, for the description of the matter, only *four* further independent functions (for instance, density, pressure, and velocity components, where there is between the latter an identity, and between pressure and density an equation of condition).

By this formulation one reduces the whole mechanics of gravitation to the solution of a single system of covariant partial differential equations. The theory avoids all the shortcomings which we have charged against the basis of classical mechanics. It is sufficient—as far as we know—for the representation of the observed facts of celestial mechanics. But it is similar to a building, one wing of which is made of fine marble (left part of the equation), but the other wing of which is built of low-grade wood (right side of equation). The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter.

There is no difficulty in connecting Maxwell's theory of the electromagnetic field with the theory of the gravitational field so long as one restricts himself to space free of ponderable matter and free of electric density. All that is necessary is to put on the right-hand side of the above equation for T_{ik} the energy tensor of the electromagnetic field in empty space and to adjoin to the so modified system of equations the Maxwell field equation for empty space, written in general covariant form. Under these conditions there will exist, between all these equations, a sufficient number of differential identities to guarantee their consistency. We may add that this necessary formal property of the total system of equations leaves arbitrary the choice of the sign of the member T_{ik}, a fact which later turned out to be important.

The desire to have, for the foundations of the theory, the greatest possible unity has resulted in several attempts to include the gravitational field and the electromagnetic field in one unified formal picture. Here we must mention particularly the five-dimensional theory of Kaluza and Klein. Having considered this possibility very carefully, I feel that it is more desirable to accept the lack of internal uniformity of the original theory, because I do not think that the totality of the hypotheses sion of energy values H_σ to a system specified in the sense of classical mechanics (the energy function is a given function of the coordinates q_r and the corresponding momenta p_r)? Planck's constant h relates the frequency H_σ/h to the energy values H_σ. It is therefore sufficient to assign to the system a succession of discrete *frequency* values. This reminds us of the fact that in acoustics a series of discrete frequency values is coordinated to a linear partial differential equation (for given boundary conditions) namely, the sinusoidal periodic solutions. In corresponding manner, Schrödinger set himself the task of coordinating a partial differential equation for a scalar function ψ to the given energy function $\mathcal{E}(q_r, p_r)$, where the q_r and the time t are independent variables. In this he succeeded (for a complex function ψ) in such a manner that the theoretical values of the energy H_σ, as required by the statistical theory, actually resulted in a satisfactory manner from the periodic solutions of the equation.

To be sure, it did not happen to be possible to associate a definite movement, in the sense of mechanics of material points, with a definite solution $\psi(q_r, t)$ of the Schrödinger equation. This means that the ψ function does not determine, at any rate *exactly*, the story of the q_r as functions of the time t. According to Born, however, an interpretation of the physical meaning of the ψ functions was shown to be possible in the following manner: $\psi\bar{\psi}$ (the square of the absolute value of the complex function ψ) is the probability density at the point under consideration in the configuration-space of the q_r, at the time t. It is therefore possible to characterize the content of the Schrödinger equation in a manner, easy to be understood, but not quite accurate, as follows: it determines how the probability density of a statistical ensemble of systems varies in the configuration-space with the time. Briefly: the Schrödinger equation determines the change of the function ψ of the q_r with time.

EINSTEIN'S VIEW

It must be mentioned that the results of this theory contain —as limiting values—the results of particle mechanics if the wave-lengths encountered in the solution of the Schrödinger problem are everywhere so small that the potential energy varies by a practically infinitely small amount for a distance of one

config space = 6n coords.
—relate to Hilbert space?

wave-length in the configuration-space. Under these conditions the following can in fact be shown: We choose a region G_0 in the configuration-space which, although large (in every direction) in relation to the wave-length, is small in relation to the relevant dimensions of the configuration-space. Under these conditions it is possible to choose a function ψ for an initial time t_0 in such a manner that it vanishes outside the region G_0, and behaves, according to the Schrödinger equation, in such a manner that it retains this property—approximately at least—also for a later time, but with the region G_0 having passed at that time t into another region G. In this manner one can, with a certain degree of approximation, speak of the motion of the region G as a whole, and one can approximate this motion by the motion of a point in the configuration-space. This motion then coincides with the motion which is required by the equations of classical mechanics.

Xray

Experiments on interference made with particle rays have given a brilliant proof that the wave character of the phenomena of motion as assumed by the theory does, really, correspond to the facts. In addition to this, the theory succeeded, easily, in demonstrating the statistical laws of the transition of a system from one quantum state to another under the action of external forces, which, from the standpoint of classical mechanics, appears as a miracle. The external forces were here represented by small time dependent additions to the potential energy. Now, while in classical mechanics, such additions can produce only correspondingly small changes of the system, in the quantum mechanics they produce changes of any magnitude however large, but with correspondingly small probability, a consequence in perfect harmony with experience. Even an understanding of the laws of radioactive decay, at least in broad outline, was provided by the theory.

Probably never before has a theory been evolved which has given a key to the interpretation and calculation of such a heterogeneous group of phenomena of experience as has quantum theory. In spite of this, however, I believe that the theory is apt to beguile us into error in our search for a uniform basis for physics, because, in my belief, it is an *incomplete* representation of real things, although it is the only one which can be built out of the fundamental concepts of force and material points (quantum corrections to classical mechanics). The incompleteness of the representation leads necessarily to the statistical nature (incompleteness) of the laws. I will now give my reasons for this opinion.

I ask first: How far does the ψ function describe a real state of a mechanical system? Let us assume the ψ_r to be the periodic solutions (put in the order of increasing energy values) of the Schrödinger equation. I shall leave open, for the time being, the question as to how far the individual ψ_r are *complete* descriptions of physical states. A system is first in the state ψ_1 of lowest energy $\mathcal{E}_1$. Then during a finite time a small disturbing force acts upon the system. At a later instant one obtains then from the Schrödinger equation a ψ function of the form

$$\psi = \Sigma\, c_r \psi_r$$

where the c_r are (complex) constants. If the ψ_r are "normalized," then $|c_1|$ is nearly equal to 1, $|c_2|$ etc. is small compared with 1. One may now ask: Does ψ describe a real state of the system? If the answer is yes, then we can hardly do otherwise than ascribe * to this state a definite energy $\mathcal{E}$, and, in particular, an energy which exceeds $\mathcal{E}_1$ by a small amount (in any case $\mathcal{E}_1 < \mathcal{E} < \mathcal{E}_2$). Such an assumption is, however, at variance with the experiments on electron impact such as have been made by J. Franck and G. Hertz, if one takes into account Millikan's demonstration of the discrete nature of electricity. As a matter of fact, these experiments lead to the conclusion that energy values lying between the quantum values do not exist. From this it follows that our function ψ does not in any way describe a homogeneous state of the system, but represents rather a statistical description in which the c_r represent probabilities of the individual energy values. It seems to be clear, therefore, that Born's statistical interpretation of quantum theory is the only possible one. The ψ function does not in any way describe a state which could be that of a single system; it relates rather to many systems, to "an en-

* Because, according to a well-established consequence of the relativity theory, the energy of a complete system (at rest) is equal to its inertia (as a whole). This, however, must have a well-defined value.

semble of systems" in the sense of statistical mechanics. If, except for certain special cases, the ψ function furnishes only *statistical* data concerning measurable magnitudes, the reason lies not only in the fact that the *operation of measuring* introduces unknown elements, which can be grasped only statistically, but because of the very fact that the ψ function does not, in any sense, describe the state of *one* single system. The Schrödinger equation determines the time variations which are experienced by the ensemble of systems which may exist with or without external action on the single system.

Such an interpretation eliminates also the paradox recently demonstrated by myself and two collaborators, and which relates to the following problem.

Consider a mechanical system consisting of two partial systems A and B which interact with each other only during a limited time. Let the ψ function before their interaction be given. Then the Schrödinger equation will furnish the ψ function after the interaction has taken place. Let us now determine the physical state of the partial system A as completely as possible by measurements. Then quantum mechanics allows us to determine the ψ function of the partial system B from the measurements made, and from the ψ function of the total system. This determination, however, gives a result which depends upon *which* of the physical quantities (observables) of A have been measured (for instance, coordinates *or* momenta). Since there can be only *one* physical state of B after the interaction which cannot reasonably be considered to depend on the particular measurement we perform on the system A separated from B it may be concluded that the ψ function is *not* unambiguously coordinated to the physical state. This coordination of several ψ functions to the same physical state of system B shows again that the ψ function cannot be interpreted as a (complete) description of a physical state of a single system. Here also the coordination of the ψ function to an ensemble of systems eliminates every difficulty.*

* A measurement on A, for example, thus involves a transition to a narrower ensemble of systems. The latter (hence also its ψ function) depends upon the point of view according to which this reduction of the ensemble of systems is carried out.

The fact that quantum mechanics affords, in such a simple manner, statements concerning (apparently) discontinuous transitions from one state to another without actually giving a description of the specific process—this fact is connected with another, namely, the fact that the theory, in reality, does not operate with the single system, but with a totality of systems. The coefficients c_r of our first example are really altered very little under the action of the external force. With this interpretation of quantum mechanics one can understand why this theory can easily account for the fact that weak disturbing forces are able to produce changes of any magnitude in the physical state of a system. Such disturbing forces produce, indeed, only correspondingly small changes of the *statistical density* in the ensemble of systems, and hence only infinitely weak changes of the ψ functions, the mathematical description of which offers far less difficulty than would be involved in the mathematical description of finite changes experienced by part of the single systems. What happens to the single system remains, it is true, entirely unclarified by this mode of consideration; this enigmatic event is entirely eliminated from the description by the statistical approach.

But now I ask: Is there really any physicist who believes that we shall never get any insight into these important changes in the single systems, in their structure and their causal connections, regardless of the fact that these single events have been brought so close to us, thanks to the marvelous inventions of the Wilson chamber and the Geiger counter? To believe this is logically possible without contradiction; but, it is so very contrary to my scientific instinct that I cannot forego the search for a more complete conception.

To these considerations we should add those of another kind which also appear to indicate that the methods introduced by quantum mechanics are not likely to give a useful basis for the whole of physics. In the Schrödinger equation, absolute time, and also the potential energy, play a decisive rôle, while these two concepts have been recognized by the theory of relativity as inadmissible in principle. If one wishes to escape from this difficulty, he must found the theory upon field and field laws instead of upon forces of interaction. This leads us to apply the statistical methods of quantum mechanics to fields, that is, to systems of infinitely many degrees of freedom. Although the attempts so far made are restricted to linear equations, which, as we know from the results of the general theory of relativity, are insufficient, the complications met up to now by the very ingenious attempts are already terrifying. They certainly will multiply if one wishes to obey the requirements of the general theory of relativity, the justification of which in principle nobody doubts.

To be sure, it has been pointed out that the introduction of a space-time continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. It is maintained that perhaps the success of the Heisenberg method points to a purely algebraical method of description of nature, that is, to the elimination of continuous functions from physics. Then, however, we must also give up, on principle, the space-time continuum. It is conceivable that human ingenuity will some day find methods which will make it possible to proceed along such a path. At the present time, however, such a program looks like an attempt to breathe in empty space.

There is no doubt that quantum mechanics has seized hold of a good deal of truth, and that it will be a touchstone for any future theoretical basis, in that it must be deducible as a limiting case from that basis, just as electrostatics is deducible from the Maxwell equations of the electromagnetic field or as thermodynamics is deducible from classical mechanics. However, I do not believe that quantum mechanics can serve as a *starting point* in the search for this basis, just as, vice versa, one could not find from thermodynamics (resp. statistical mechanics) the foundations of mechanics.

In view of this situation, it seems to be entirely justifiable seriously to consider the question as to whether the basis of field physics cannot by *any* means be put into harmony with quantum phenomena. Is this not the only basis which, with the presently available mathematical tools, can be adapted to the requirements of the general theory of relativity? The belief, prevailing among the physicists of today, that such an attempt would be hopeless, may have its root in the unwarranted assumption that such a theory must lead, in first approximation, to the equations of classical mechanics for the motion of corpuscles, or at least to total differential equations. As a matter of fact, up to now we have never succeeded in a field-theoretical description of corpuscles free of singularities, and we can, *a priori,* say nothing about the behavior of such entities. *One thing,* however, is certain: if a field theory results in a representation of corpuscles free of singularities, then the behavior of these corpuscles in time is determined solely by the differential equations of the field.

VI. Relativity Theory and Corpuscles

I shall now show that, according to the general theory of relativity, there exist singularity-free solutions of field equations which can be interpreted as representing corpuscles. I restrict myself here to neutral particles because, in another recent publication in collaboration with Dr. Rosen, I have treated this question in detail, and because the essentials of the problem can be completely exhibited in this case.

The gravitational field is entirely described by the tensor $g_{\mu\nu}$. In the three-index symbols $\Gamma^{\sigma}_{\mu\nu}$, there appear also the contravariant $g^{\mu\nu}$ which are defined as the minors of the $g_{\mu\nu}$ divided by the determinant $g(=|g_{\alpha\beta}|)$. In order that the R_{ik} shall be defined and finite, it is not sufficient that there shall be, in the neighborhood of every point of the continuum, a system of coordinates in which the $g_{\mu\nu}$ and their first differential quotients are continuous and differentiable, but it is also necessary that the determinant g shall nowhere vanish. This last restriction disappears, however, if one replaces the differential equations $R_{ik}=0$ by $g^2R_{ik}=0$, the left-hand sides of which are *whole* rational functions of the g_{ik} and of their derivatives.

These equations have the centrally symmetrical solution given by Schwarzschild

$$ds^2 = -\frac{1}{1-2m/r}\,dr^2 - r^2(d\theta^2+\sin^2\theta d\varphi^2) + \left(1-\frac{2m}{r}\right)dt^2$$

This solution has a singularity at $r=2m$, since the coefficient of dr^2 (i.e., g_{11}), becomes infinite on this hypersurface. If, however, we replace the variable r by ρ defined by the equation

$$\rho^2 = r - 2m$$

we obtain

$$ds^2 = -4(2m+\rho^2)d\rho^2 - (2m+\rho^2)^2(d\theta^2+\sin^2\theta d\varphi^2) + \frac{\rho^2}{2m+\rho^2}\,dt^2$$

This solution behaves regularly for all values of ρ. The vanishing of the coefficient of dt^2 (i.e., g_{44}) for $\rho=0$ results, it is true, in the consequence that the determinant g vanishes for this value; but, with the methods of writing the field equations actually adopted, this does not constitute a singularity.

If ρ varies from $-\infty$ to $+\infty$, then r varies from $+\infty$ to $r=2m$ and then back to $+\infty$, while for such values of r as correspond to $r<2m$ there are no corresponding real values of ρ. Hence the Schwarzschild solution becomes a regular solution by representing the physical space as consisting of two identical "sheets" in contact along the hypersurface $\rho=0$ (i.e., $r=2m$), on which the determinant g vanishes. Let us call such a connection between the two (identical) sheets a "bridge." Hence the existence of such a bridge between the two sheets in the finite realm corresponds to the existence of a material neutral particle which is described in a manner free from singularities.

The solution of the problem of the motion of neutral particles evidently amounts to the discovery of such solutions of the gravitational equations (written free of denominators), as contain several bridges.

The conception sketched above corresponds, *a priori,* to the atomistic structure of matter in so far as the "bridge" is by its nature a discrete element. Moreover, we see that the mass constant m of the neutral particles must necessarily be positive, since no solution free of singularities can correspond to the Schwarzschild solution for a negative value of m. Only the examination of the several-bridge-problem can show whether or not this theoretical method furnishes an explanation of the empirically demonstrated equality of the masses of the particles found in nature, and whether it takes into account the facts which the quantum mechanics has so wonderfully comprehended.

In an analogous manner, it is possible to demonstrate that the combined equations of gravitation and electricity (with appropriate choice of the sign of the electrical member in the gravitational equations) produce a singularity-free bridge-representation of the electric corpuscle. The simplest solution of this kind is that for an electrical particle without gravitational mass.

So long as the considerable mathematical difficulties concerned with the solution of the several-bridge-problem are not overcome, nothing can be said concerning the usefulness of the theory from the physicist's point of view. However, it constitutes, as a matter of fact, the first attempt toward the consistent elaboration of a field theory which presents a possibility of explaining the properties of matter. In favor of this attempt one should also add that it is based on the simplest possible relativistic field equations known today.

Summary

Physics constitutes a logical system of thought which is in a state of evolution, whose basis cannot be distilled, as it were, from experience by an inductive method, but can only be arrived at by free invention. The justification (truth content) of the system rests in the verification of the derived propositions by sense experiences, whereby the relations of the latter to the former can only be comprehended intuitively. Evolution is proceeding in the direction of increasing simplicity of the logical basis. In order further to approach this goal, we must resign to the fact that the logical basis departs more and more from the facts of experience, and that the path of our thought from the fundamental basis to those derived propositions, which correlate with sense experiences, becomes continually harder and longer.

Our aim has been to sketch, as briefly as possible, the development of the fundamental concepts in their dependence upon the facts of experience and upon the endeavor to achieve internal perfection of the system. These considerations were intended to illuminate the present state of affairs, as it appears to me. (It is unavoidable that a schematic historic exposition is subjectively colored.)

I try to demonstrate how the concepts of bodily objects, space, subjective and objective time, are connected with one another and with the nature of our experience. In classical mechanics the concepts of space and time become independent. The concept of the bodily object is replaced in the foundations by the concept of the material point, by which means mechanics becomes fundamentally atomistic. Light and electricity produce insurmountable difficulties when one attempts to make mechanics the basis of all physics. We are thus led to the field theory of electricity, and, later on to the attempt to base physics entirely upon the concept of the field (after an attempted compromise with classical mechanics). This attempt leads to the theory of relativity (evolution of the notion of space and time into that of the continuum with metric structure).

I try to demonstrate, furthermore, why in my opinion quantum theory does not seem capable to furnish an adequate foundation for physics: one becomes involved in contradictions if one tries to consider the theoretical quantum description as a *complete* description of the individual physical system or event.

On the other hand, the field theory is as yet unable to explain the molecular structure of matter and of quantum phenomena. It is shown, however, that the conviction of the inability of field theory to solve these problems by its methods rests upon prejudice.

FIELD

THE FUNDAMENTS OF THEORETICAL PHYSICS

From Science, *Washington, D. C. May 24, 1940.*

Science is the attempt to make the chaotic diversity of our sense-experience correspond to a logically uniform system of thought. In this system single experiences must be correlated with the theoretic structure in such a way that the resulting coordination is unique and convincing.

The sense-experiences are the given subject-matter. But the theory that shall interpret them is man-made. It is the result of an extremely laborious process of adaptation: hypothetical,