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How Does the Market-Based Intermediary Sector Affect the Business Cycle?—A Consideration Based on a DSGE Framework

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Abstract

This study describes the development of a new-Keynesian dynamic stochastic general equilibrium (DSGE) model in which market-based intermediaries or active investors have an interactive relationship with the ultimate borrowers and lenders. The study also seeks to assess the effects of the presence of active investors on the business cycle.

The theoretical analysis yields two important propositions related to market-based intermediaries: first, the greater the active investor's asset size, the higher will be the expected net profits; second, the steeper the yield curve, the greater is the asset size. These propositions together suggest that steeper yield curves will yield higher net profits to active investors.

Using the developed model and the U.S. quarterly data for 1990:Q1–2010:Q3, this study performs empirical analyses and thereby empirically proves the abovementioned propositions in the model. Furthermore, the analyses indicate that the active investor sector is not only a source of the business cycle but also a fluctuation amplifier, that active investors might impede the propagation of monetary policy effects, and that although rigorous financial regulation could forestall asset price bubbles, it might not necessarily lead to economic stability.

The results of the theoretical as well as empirical analyses indicate that the active investor sector has significant effects on the business cycle, thus supporting the view of Adrian, Moench and Shin [2010b].

Key words: dynamic stochastic general equilibrium (DSGE) model, market-based intermediaries, business cycle, asset prices, endogenous term structure

JEL Classification: E32, E43, E44, E52, G20

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I Introduction

It appears that since 2000, market-based intermediaries have played a crucial role in business cycles, especially boom-bust cycles, because nonbank financial intermediaries have become increasingly important sources of credit, particularly due to the growing popularity of securitization, as pointed out by many studies such as Adrian and Shin [2010a, 2011a] and Woodford [2010]. In that case, is it possible that the market-based intermediaries substantially impact business cycles? How would they do so? The research conducted on this issue so far has mainly focused on the behaviour of these intermediaries[1].

Recent studies tend to focus more on developing new-Keynesian macroeconomic models or dynamic stochastic general equilibrium (DSGE) models, wherein financial institutions play a crucial role and which allow for frictions that can impede an efficient supply of credit. Woodford [2010], Adrian and Shin [2011a], and Gertler and Kiyotaki [2011] provide surveys of the recent work in this area.

These models could be roughly classified into three groups. The first group focuses on financial frictions that arise from the behaviour of borrowers, but does not consider the behaviour of financial institutions themselves. There are many studies that can be categorized in this group, for example, Bernanke, Gertler and Gilchrist [1999], Christiano, Motto and Rostagno [2003, 2008, 2010], Aliaga-Diaz and Olivero [2007], Goodfriend and McCallum [2007], Kobayashi [2008], Teranishi [2008], Andrés and Arce [2009], and Gilchrist, Ortiz and Zakrajsek [2009].

The second group takes into account the influences of the behaviours of both financial intermediaries and borrowers on an economy; this group of studies includes Cúrdia and Woodford [2010], Gerali et al. [2010], Iacoviello and Neri [2010], and Verona, Martins and Drumond [2011]. However, the models in this group do not allow for the possibility that the intermediaries' behaviour will be affected by changes in the price of assets which they hold, although some of models regard the effects of changes in the value of assets pledged by borrowers as collateral—for example, houses and capital—on the behaviour of intermediaries.

The third group considers all the factors stated above, that is, the effects of not only the behaviour of borrowers and financial institutions but also of financial asset markets on the economy. The research carried out by Adrian, Moench and Shin [2010b] falls under this group. In their study, the researchers attempt to extend the standard new-Keynesian macroeconomic model by introducing the concepts of the macro risk

1 Works from the theoretical viewpoint include, for example, Danielsson, Shin and Zigrand [2009] and Adrian and Shin [2011b]; those from the empirical viewpoint include, for example, Brunnermeier [2009], Adrian, Moench and Shin [2010a], and Adrian and Shin [2010b].

premium and the risk appetite relevant to market-based intermediaries[2]. Their idea is derived from their vigorous studies related to market-based intermediaries and definitely contributes to the development of the macroeconomic model. However, it is regrettable that the relationships among the macro risk premium, the risk appetite and other macro variables such as real output and interest rates are specified ad hoc and therefore lack a microeconomic foundation.

This study has two objectives: first, to present a new-Keynesian DSGE model which will provide a microeconomic foundation to the interconnections among the ultimate borrowers, ultimate lenders and market-based financial intermediaries; second, to perform empirical analyses, using the U.S. data, and to assess the influences of the market-based intermediary sector on business cycles.

The paper is organized as follows. Section Ⅱ describes the model. The theoretically important features derived from the model are stated as propositions. Section Ⅲ describes the determination of the parameters of the model. In this study, we have adopted a combination of calibration and estimation for deciding these parameters. Hence, some parameters are set according to previous empirical studies, whereas others are estimated using the Bayesian inference method. Section Ⅳ presents the simulation results, empirically examines some of the propositions derived in Section Ⅱ, and assesses effects of the presence of the market-based intermediary sector on the business cycle. Section Ⅴ concludes the paper.

Ⅱ Model setup

In this section, we will consider a model economy composed of end-user borrowers, active investors, passive investors, firms and the central bank, and describe the behaviour of these agents. The framework of the economy follows that suggested by Adrian, Moench and Shin [2010b].

The starting point for our analysis is a hybrid new-Keynesian DSGE model with sticky prices and habits in consumption. Furthermore, we have further developed the standard model by making several modifications as follows. First, it is assumed that there are three different types of financial instruments—deposits, short-term bonds and long-term bonds. For trading in bonds, we have introduced a financial friction that makes these different types of bonds imperfect substitutes, as in Andrés, López-Salido and Nelson [2004a, b] (henceforth, ALSN), Marzo, Söderström and Zagaglia [2007], and Sudo [2010]. The friction reveals the endogenous term structure of interest rates in the

[2] Adrian and Shin [2011a] explain these concepts in detail.

sense that there exists bi-directional feedback between the yield curve and the economy.

Second, we will pay attention to the roles played by financial intermediary mechanisms and leveraged mechanisms in the economy, similar to Adrian, Moench and Shin [2010b]. Active investors play the role of intermediaries between end-user borrowers and passive investors. In addition, they issue short-term bonds to purchase more long-term bonds while complying with a minimum capital requirement. In this process, they make use of leveraged mechanisms such as shadow banks in order to maximize their net profits[3]. The leveraged mechanisms will be represented as a relationship between the active investor's assets and the endogenous term structure of interest rates.

Third, a collateral constraint has been incorporated into the model. While end-user borrowers as well as active investors raise funds by issuing bonds, the issues should be secured by using their assets as collateral, as suggested by Kiyotaki and Moore [1997], Pintus and Wen [2008], Gerali et al. [2010], and Iacoviello and Neri [2010].

Finally, the model considers the concept of probability of default. For this, we can presuppose two financial states of the economy: the first is the economy's state during stable times, while the second is its state during financial distress. Which state the economy is in depends on whether or not the end-user borrowers reach a situation of financial distress, which will have chain impacts on the behaviour of other agents, particularly the behaviour of active and passive investors.

In the following sections, we will present the objectives and constraints of different agents in the economy, paying special attention to specifying the behaviour of end-user borrowers as well as active and passive investors.

1. End-user borrowers

(1) Utility function and constraints

End-user borrowers basically behave like households; thus, they derive incomes from labour and consume goods. In this analysis, we will assume a continuum of identical and infinitely living borrowers, indexed by $i_1 \in [0, 1]$, and a continuum of consumption goods, indexed by $j \in [0, 1]$, which are produced by the firm j. These borrowers obtain utility from a bundle $C_{1,t}$, given by

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[3] In this study, it is supposed that the shadow banks are composed of asset-backed security (ABS) issuers, finance companies, funding companies, and agency- and GSE-backed mortgage pools, where 'GSE' is an abbreviation for 'government-sponsored enterprises'. For a comprehensive and up-to-date description of the shadow banking system, see Pozsar et al. [2010].

$$C_{1,t} = \left[\int_0^1 C_{1,t}(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $C_{1,t}(j)$ represents the quantity of goods j consumed by the borrower in period t, and ε is the elasticity of substitution across different varieties of goods. Moreover, during stable times, the borrowers invest in housing by issuing long-term bonds which are zero-coupon bonds, and thus obtain utility from their residential holdings as well.

As a result, during stable times, these borrowers have the period utility function of

$$U_{1,p}(C_{1,t}, C_{1,t-1}, N_{1,t}, S_{h,t}, e_{h,t}) \\ = \frac{1}{1-\sigma_{1,p}} \left(\frac{C_{1,t}}{C_{1,t-1}^{h_1}} \right)^{1-\sigma_{1,p}} + \frac{S_{h,t}^{1-\chi}}{1-\chi} e_{h,t} - \frac{\left(N_{1,t}^s\right)^{1+\varphi_{1,p}}}{1+\varphi_{1,p}}, \tag{2}$$

where the subscript 'p' stands for stable times. In the utility function, $S_{h,t}$ denotes housing stock holdings at the beginning of period t; $N_{1,t}^s$, hours worked by the borrower in period t; $\sigma_{1,p} > 0$, inverse of the elasticity of inter-temporal substitution; $\chi \geq 0$, inverse of the interest elasticity of the demand for housing; $\varphi_{1,p} \geq 0$, inverse of the Frisch labour supply elasticity; and $h_1 \geq 0$, the habit persistence parameter indicating the extent of habit formation. $e_{h,t}$ represents the shocks to the borrower's demand for housing in period t and follows the process

$$e'_{h,t} = \rho_h e'_{h,t-1} + \varepsilon_{h,t}, \tag{3}$$

where $e'_{h,t} \equiv \ln\left(e_{h,t}\right)$, $\rho_h \in (0, 1)$ and $\varepsilon_{h,t} \sim i.i.d.\left(0, \sigma_h^2\right)$.

The period budget constraint takes the form of

$$\frac{1}{P_t}\left\{W_t N_{1,t}^s + B_{l,t}^s + Q_{h,t}\left(1-\iota_h\right)S_{h,t-1}\right\} = C_{1,t} + \frac{1}{P_t}\left(H_{l,t-1}B_{l,t-1}^s + Q_{h,t}S_{h,t}\right), \tag{4}$$

where P_t stands for the price of the consumption goods in period t; W_t, the nominal wage in period t; $B_{l,t}^s$, long-term bond outstandings at the beginning of period t; $\iota_h \in (0, 1)$, the depreciation rate on housing stock; and $Q_{h,t}$, house prices in period t. $H_{l,t}$ represents the gross one-period nominal costs of issuing the long-term zero-coupon bond and is calculated by $H_{l,t} = \dfrac{Q_{l,t+1}}{Q_{l,t}}$, where $Q_{l,t}$ represents the price of the long-term bond in period t. The budget constraint (4) implies that, at the beginning of period t, borrowers sell their own house at price $Q_{h,t}$ and repurchase long-term bonds issued in previous periods at price $Q_{l,t}$; at the same time, they issue the bond and purchase a new house at the same prices, respectively[4].

At this point, we introduce a collateral constraint, related to the issuances of long bonds, as follows:

$$\frac{1}{P_t} R_t B_{l,t}^s \le \frac{1}{P_t} k_h Q_{h,t} S_{h,t}, \tag{5}$$

where R_t is the gross nominal one-period interest rate and $k_h \in (0, 1]$ measures the collateral value of houses owned by the borrower. Since, as shown below, R_t represents the interest rates which are applied to the deposits of passive investors and the reserve of active investors, $R_t B_{l,t}^s$ indicates the minimum expected returns that investors would require on long-term bond purchases. Consequently, the constraint (5) implies that the minimum required returns on long-term bonds investment should be secured by the collateral value of assets owned by the borrower. In other words, the constraint states that the amount of debt cannot exceed the collateral value of the borrower's assets discounted by the risk-free rate.

On the other hand, when borrowers are faced with financial distress with probability X_t in period t, they have to sell their houses at price $Q_{h,t}$ and redeem their previously issued long-term bonds $B_{l,t-1}^s$. This is expressed by the equation

$$Q_{h,t}(1 - \iota_h) S_{h,t-1} = \gamma H_{l,t-1} B_{l,t-1}^s, \tag{6}$$

where $\gamma \in (0, 1]$ denotes the rate at which borrowers can redeem long-term bonds by disposing of their own houses. Afterwards, during subsequent financial distress periods, the borrower will not purchase a house.

Consequently, when the economy is in a state of financial distress, the period utility function is

$$U_{1,d}(C_{1,t}, C_{1,t-1}, N_{1,t}) = \frac{1}{1-\sigma_{1,d}} \left(\frac{C_{1,t}}{C_{1,t-1}^{h_1}} \right)^{1-\sigma_{1,d}} - \frac{\left(N_{1,t}^s\right)^{1+\varphi_{1,d}}}{1+\varphi_{1,d}}, \tag{7}$$

where the subscript 'd' represents times of financial distress. The budget constraint is described as

[4] It is implicitly assumed that borrowers not only demand houses but also supply them. Furthermore, we assume the presence of some suppliers other than borrowers, for example, homeowners by inheritance.

$$\frac{1}{P_k}\left\{W_k N_{1,k}^s + Q_{h,k}(1-\iota_h)S_{h,k-1}\right\} = C_{1,k} + \frac{1}{P_k}\gamma H_{l,k-1} B_{l,k-1}^s, \text{ if } k = t,$$

and

$$\frac{1}{P_k} W_k N_{1,k}^s = C_{1,k}, \text{ if } k \geq t+1.$$

Using equation (6), the budget constraint in financial distress is described by

$$\frac{1}{P_t} W_t N_{1,t}^s = C_{1,t}, \tag{8}$$

which indicates that end-user borrowers face liquidity constraints during times of financial distress.

(2) Optimality conditions

According to the above setup, the end-user borrower solves the following problem:

$$\max_{\{C_{1,t}, N_{1,t}^s, S_{h,t}, B_{l,t}^s\}} E_t \sum_{i=0}^{\infty} \beta_1^{t+i} \left\{ (1 - X_{t+i}) U_{1,p}\left(C_{1,t+i}, C_{1,t+i-1}, N_{1,t+i}^s, S_{h,t+i}, e_{h,t+i}\right) + X_{t+i} U_{1,d}\left(C_{1,t+i}, C_{1,t+i-1}, N_{1,t+i}^s\right) \right\},$$

$$\text{s.t.} \quad \begin{aligned} &\left(\frac{1}{P_{t+i}}\right)\left[W_{t+i} N_{1,t+i}^s + (1 - X_{t+i})\left\{B_{l,t+i}^s + (1-\iota_h) Q_{h,t+i} S_{h,t+i-1}\right\}\right] \\ &- C_{1,t+i} - \left(\frac{1}{P_{t+i}}\right)(1 - X_{t+i})(H_{l,t+i-1} B_{l,t+i-1}^s + Q_{h,t+i} S_{h,t+i}) = 0 \end{aligned} \tag{9}$$

and

$$\left(\frac{1}{P_{t+i}}\right)(1 - X_{t+i})(k_h Q_{h,t+i} S_{h,t+i} - R_{t+i} B_{l,t+i}^s) \geq 0, \tag{10}$$

where the parameter $\beta_1 \in (0, 1)$ is a discount factor and E_t denotes the mathematical expectations operator conditional on information available in period t.

In what follows, for any variable Z_t, $\overline{Z}$ stands for a steady state of the sequence $\{Z_t\}$ and Z_t' represents a real variable in terms of consumer price, that is, $Z_t' \equiv Z_t / P_t$.

Assuming that $\sigma_{1,p} = \sigma_{1,d} = \sigma_1$ and $\varphi_{1,p} = \varphi_{1,d} = \varphi_1$, the first-order conditions for the optimizing problem given above can be written, in real terms, as follows:

$$\Lambda_{1,t} = \frac{C_{1,t}^{-\sigma_1}}{C_{1,t-1}^{h_1(1-\sigma_1)}} - \beta_1 h_1 E_t\left[\frac{C_{1,t+1}^{1-\sigma_1}}{C_{1,t}^{h_1(1-\sigma_1)+1}}\right], \tag{11}$$

$$\left(N_{1,t}^s\right)^{\varphi_1} = \Lambda_{1,t} W_t', \tag{12}$$

$$S_{h,t}^{-\chi} e_{h,t} = (\Lambda_{1,t} - k_h \Lambda_{2,t}) Q'_{h,t} - \beta_1 (1 - \iota_h) E_t \left[\Lambda_{1,t+1} Q'_{h,t+1} \left(\frac{1 - X_{t+1}}{1 - X_t} \right) \right], \tag{13}$$

$$\Lambda_{1,t} - \Lambda_{2,t} R_t = \beta_1 E_t \left[\Lambda_{1,t+1} H_{l,t} \left(\frac{1}{\Pi_{t+1}} \right) \left(\frac{1 - X_{t+1}}{1 - X_t} \right) \right], \tag{14}$$

$$C_{1,t} = W'_t N_{1,t}^s + (1 - X_t) \left\{ B'^s_{l,t} + (1 - \iota_h) Q'_{h,t} S_{h,t-1} - H_{l,t-1} \frac{B'^s_{l,t-1}}{\Pi_t} - Q'_{h,t} S_{h,t} \right\}, \tag{15}$$

$$B'^s_{l,t} = k_h \left(\frac{Q'_{h,t}}{R_t} \right) S_{h,t}, \tag{16}$$

where $\Pi_t \equiv P_t / P_{t-1}$, and $\Lambda_{1,t}$ and $\Lambda_{2,t}$ represent the Lagrange multipliers for the budget constraint and the collateral constraint, respectively. As shown by Pintus and Wen [2008], it can be proved that, as indicated by equation (16), the collateral constraint is binding around a steady state, under a condition imposed on the discount factors of the end-user borrower and the passive investor.

Considering a steady state of the economy, equations (12) and (14) imply the following, respectively,

$$\overline{\Lambda}_1 = \left(\overline{N}_1^s \right)^{\varphi_1} \left(\overline{W'} \right)^{-1} > 0, \tag{17}$$

$$\overline{\Lambda}_2 = \frac{1 - \beta_1 \overline{H}_l}{\overline{R}} \overline{\Lambda}_1. \tag{18}$$

As described in sub-section 3, a passive investor's discount factor, β_2, is assumed to be larger than β_1 and equal to $\overline{H}_l^{-1}$ in a steady state. Then,

$1 - \beta_1 \overline{H}_l = 1 - \beta_1 \beta_2^{-1} = \beta_2^{-1} (\beta_2 - \beta_1) > 0$;

hence, equations (17) and (18) indicate that

$\overline{\Lambda}_2 > 0$,

which implies that the collateral constraint is binding around the steady state[5].

We can derive the following proposition from the above first-order conditions:

Proposition 1: Weak or no collateral constraint would push up the housing stock (i.e. residential investments) in a steady state.

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[5] For example, the quarterly data on 10-year zero-coupon yields by the Federal Reserve System (FRB) indicate that $\overline{H}_l$ was around 1.014 during 1990:Q1 and 2010:Q3. Meanwhile, Iacoviello and Neri [2010] suggest that the impatient household's discount factor, β_1, is 0.97 for the U.S. Therefore, it could be said that the data pertaining to the U.S. bear out the condition of $1 - \beta_1 \overline{H}_l > 0$.

This proposition can be proved as follows. In a steady state, equation (13) implies that

$$\overline{S}_h^{-\chi} = \left[\left\{1-(1-\iota_h)\beta_1\right\}\overline{\Lambda}_1 - k_h \overline{\Lambda}_2\right]\overline{Q}_h'. \qquad (19)$$

Then,

$$\frac{\partial \overline{S}_h}{\partial k_h} = \frac{\overline{S}_h^{1+\chi}}{\chi}\overline{Q}_h'\overline{\Lambda}_2 > 0$$

because $\overline{\Lambda}_2 > 0$.

Furthermore, substituting equation (18) into equation (19), we obtain

$$\overline{S}_h^{-\chi} = \left\{1-(1-\iota_h)\beta_1 - k_h\left(\frac{1-\beta_1\overline{H}_l}{\overline{R}}\right)\right\}\overline{Q}_h'\overline{\Lambda}_1. \qquad (20)$$

On the other hand, when there is no collateral constraint, the following equation can be derived from the first-order conditions in the steady state[6]:

$$\left(\overline{S}_h^*\right)^{-\chi} = \left\{1-(1-\iota_h)\frac{1}{\overline{H}_l}\right\}\overline{Q}_h'\overline{\Lambda}_1, \qquad (21)$$

where $\overline{S}_h^*$ stands for the steady state housing stock without the collateral constraint. According to equations (20) and (21),

$$\overline{S}_h^{-\chi} - \left(\overline{S}_h^*\right)^{-\chi} = \frac{1-\beta_1\overline{H}_l}{\overline{R}}\left\{(1-\iota_h)\frac{\overline{R}}{\overline{H}_l} - k_h\right\},$$

which implies that if k_h is small enough to meet $k_h < (1-\iota_h)\dfrac{\overline{R}}{\overline{H}_l}$, then $\overline{S}_h^* > \overline{S}_h$[7].

The above argument indicates that in the case of weak or zero collateral constraints, borrowers may issue more long-term bonds and increase their housing investment, which would lead to financial as well as economic instability.

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[6] When there is no collateral constraint, the borrower's discount factor, $\beta*_1$, should be equal to $\overline{H}_l^{-1}$.

[7] For example, using the federal funds rate and the 10-year zero-coupon yield for the U.S., $\overline{R}/\overline{H}_l$ is calculated as around 1.0018 from 1990:Q1 to 2010:Q3. Meanwhile, Iacoviello and Neri [2010] suggest that the values of ι_h and k_h are 0.01 and 0.925, respectively. These data and parameter values support the condition that $k_h < (1-\iota_h)\overline{R}/\overline{H}_l$.

2. Active investors

(1) Net profits and constraints

Active investors consist of banks, security broker-dealers and shadow banks such as asset-backed security (ABS) issuers; hence, they play the dual roles of financial intermediaries and leveraged investors. In these two roles, they raise funds through deposits and issues of short-term bonds (ABS) to passive investors and allocate these funds in the form of reserves for the central bank and investments in long-term bonds issued by end-user borrowers. Here, we can assume that the short-term bond is a zero-coupon bond.

According to the active investor's behaviour, during stable times, the active investor's (expected) net profits in period t are defined as

$$F_p\left(B_{r,t}, B^d_{1,l,t}, B^s_{s,t}\right) = \left(R_t - 1\right)B_{r,t} + \left(H_{l,t} - 1\right)B^d_{1,l,t} - \left(R_t - 1\right)M_t - \left(H_{s,t} - 1\right)B^s_{s,t}, \quad (22)$$

where $B_{r,t}$ represents reserve outstandings at the beginning of period t; $B^d_{1,l,t}$ represents holdings of long-term bonds at the beginning of period t; and $B^s_{s,t}$ stands for short-term bond outstandings at the beginning of period t. M_t, the value of which is given, represents the deposits accepted from passive investors at the beginning of period t. $H_{s,t}$ stands for the gross one-period nominal cost of issuing short-term bonds and is calculated by $H_{s,t} = \dfrac{Q_{s,t+1}}{Q_{s,t}}$, where $Q_{s,t}$ is the price of the short bond in period t[8]. In equation (22), it should be noted that the central bank grants the same interest rates to its reserves as to its deposits.

Furthermore, it is supposed that active investors are risk-neutral but face two constraints: a budget constraint and a collateral constraint. The budget constraint is expressed as

$$R_t B_{r,t}\left(1 + AC_{l,t}\right) + H_{l,t} B^d_{1,l,t} - R_t M_t - H_{s,t} B^s_{s,t} = E_m, \quad (23)$$

where E_m is the minimum capital (capital adequacy) requirement[9, 10]. $AC_{l,t}$, which

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[8] This implies that at the beginning of period t, active investors repurchase short-term bonds issued in a previous period at a price $Q_{s,t}$, while they simultaneously issue the bond at the same price.

[9] Equation (23) and definition (24) imply that in a steady state, $\overline{R}\overline{B}_r + \overline{H}_l \overline{B}^d_{1,l} - \overline{R}\overline{M} - \overline{H}_s \overline{B}^s_s = E_m$. From the viewpoint of an optimal capital-to-assets ratio, ϕ, the capital adequacy requirement is expressed as $\dfrac{E_m}{\overline{R}\overline{B}_r + \overline{H}_l \overline{B}^d_{1,l}} = \phi$, where, for example, ϕ = 0.08 according to the Basel Accords.

[10] From the viewpoint of a value-at-risk (VaR) constraint, as suggested by Adrian and Shin [2011a],

denotes the cost function for investing in long-term bonds, is specified as follows:

$$AC_{l,t} \equiv \frac{\nu_{1,l}}{2}\left(\frac{B_{r,t}}{B^d_{1,l,t}}\kappa_{1,l} - 1\right)^2, \tag{24}$$

where $\nu_{1,l} > 0$ and $\kappa_{1,l} \equiv \frac{\bar{B}^d_{1,l}}{\bar{B}_r} > 0$ are parameters and $\kappa_{1,l}$ ensures that the cost $AC_{l,t}$ remains steady[11]. The implications of the cost function are that active investors perceive long-term bonds as riskier assets, entailing a loss of liquidity in relation to their reserves. When active investors invest in long-term bonds, they demand additional reserves (liquidity) to compensate themselves for the loss of liquidity. In other words, the agents have self-imposed 'liquidity requirements' of R_t ($B_{r,t} \times AC_{l,t}$) from their long-term bond investment, where, because deposits are redeemed with interests, it is assumed that the agents reserve the liquidity requirement multiplied by the same interest rate, R_t, as deposits[12].

The collateral constraint has the same implications as that on long-term bond issues—that is, short-term bond issues should be secured by the active investor's assets. Therefore, the collateral constraint is described as

$$R_t B^s_{s,t} \leq k_{bs} H_{l,t} B^d_{1,l,t}, \tag{25}$$

where $k_{bs} \in (0, 1]$ measures the collateral value of the long-term bond holdings.

On the other hand, when end-user borrowers face financial distress with probability X_t, this exerts a bad influence on the active investor's net profits and budget constraints. Based on equation (6), if borrowers face financial distress in period t, the active investors would receive $\gamma H_{l,t} B^d_{1,l,t}$ instead of $H_{l,t} B^d_{1,l,t}$. In that case, active investors would redeem all the previously issued short-term bonds by an amount equal to $k_{bs}\gamma H_{l,t} B^d_{1,l,t}$, according to the collateral agreement (25).

Consequently, in the state of financial distress, the active investor's (expected)

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we should incorporate $\gamma H_{l,t} B^d_{1,l,t}$ instead of $H_{l,t} B^d_{1,l,t}$ into the budget constraint (23), because if end-user borrowers face financial distress, they can repay $\gamma H_{l,t} B^d_{1,l,t}$ to active investors. However, since our model explicitly considers the probability of default, it adopts a normal budget constraint in stable times.

11 The parameter $\nu_{1,l}$ is conceptually similar to κ_{Kb} in Gerali et al. [2010], which indicates an adjustment coefficient on the quadratic cost that the bank pays whenever the capital-to-assets ratio moves away from the optimal value (in our context, ϕ).

12 In this study, since we do not consider a capital increase for the active investor, we can regard the liquidity requirement as playing the role of a kind of capital adequacy regulation.

net profits in period t are

$$F_d\left(B_{r,t},\, B^d_{1,l,t},\, B^s_{s,t}\right) = (R_t - 1)B_{r,t} + (\gamma H_{l,t} - 1)B^d_{1,l,t} - (R_t - 1)M_t - \left(k_{bs}\gamma H_{l,t}B^d_{1,l,t} - B^s_{s,t}\right). \tag{26}$$

In addition, the budget constraint in period t is given by

$$R_t B_{r,t}\left(1 + AC_{l,t}\right) + \gamma H_{l,t} B^d_{1,l,t} - R_t M_t - k_{bs}\gamma H_{l,t} B^d_{1,l,t} + L_t = E_m, \tag{27}$$

where L_t denotes capital injections through the central bank to maintain the minimum capital requirement.

Thereafter, during periods of financial distress, active investors only accept deposits and put them in reserve accounts in the central bank. From these, they would gain net profits $(R_{t+i} - 1)\left(B_{r,t+i} - M_{t+i}\right) = (R_{t+i} - 1)E_m$ $(i \geq 1)$ and repay the injected capital L_t by using these net profits. Therefore, the active investor's net profits continue to be zero during such periods.

(2) Optimality conditions

Based on the above setup, the active investor solves the following problem:

$$\max_{\left\{B_{r,t},\, B^d_{1,l,t},\, B^s_{s,t}\right\}} (1 - X_t)F_p\left(B_{r,t},\, B^d_{1,l,t},\, B^s_{s,t}\right) + X_t F_d\left(B_{r,t},\, B^d_{1,l,t},\, B^s_{s,t}\right)$$

$$\text{s.t.} \quad R_t B_{r,t}\left(1 + AC_{l,t}\right) - R_t M_t + (1 - X_t)\left(H_{l,t}B^d_{1,l,t} - H_{s,t}B^s_{s,t}\right) + X_t\left\{\gamma(1 - k_{bs})H_{l,t}B^d_{1,l,t} + L_t\right\} = E_m \tag{28}$$

and

$$(1 - X_t)\left(k_{bs}H_{l,t}B^d_{1,l,t} - R_t B^s_{s,t}\right) \geq 0\,. \tag{29}$$

The first-order conditions for the optimizing problem are described, in real terms, as follows:

$$-\Lambda_{3,t}R_t\left\{1 + \frac{v_{1,l}}{2}\left(\frac{B'_{r,t}}{B'^d_{1,l,t}}\kappa_{1,l} - 1\right)^2 + v_{1,l}\kappa_{1,l}\left(\frac{B'_{r,t}}{B'^d_{1,l,t}}\kappa_{1,l} - 1\right)\left(\frac{B'_{r,t}}{B'^d_{1,l,t}}\right)\right\} = R_t - 1, \tag{30}$$

$$(1 - X_t)\Lambda_{4,t}R_t = 1 - (1 - X_t)\left(1 + \Lambda_{3,t}\right)H_{s,t}, \tag{31}$$

$$-v_{1,l}\kappa_{1,l}\Lambda_{3,t}R_t\left(\frac{B'_{r,t}}{B'^d_{1,l,t}}\kappa_{1,l} - 1\right)\left(\frac{B'_{r,t}}{B'^d_{1,l,t}}\right)^2 = 1 - \left\{(1 - X_t)\left(1 + \Lambda_{3,t} + k_{bs}\Lambda_{4,t}\right) + \gamma(1 - k_{bs})X_t\left(1 + \Lambda_{3,t}\right)\right\}H_{l,t}, \tag{32}$$

$$R_t B'_{r,t}\left\{1+\frac{v_{1,l}}{2}\left(\frac{B'_{r,t}}{B'^{d}_{1,l,t}}\kappa_{1,l}-1\right)^2\right\}$$
$$\begin{aligned}&= E'_m + R_t M'_t + (1-X_t)\left(H_{s,t}B'^{s}_{s,t} - H_{l,t}B'^{d}_{1,l,t}\right)\\ &\quad -\gamma(1-k_{bs})X_t B'^{d}_{1,l,t} - X_t L'_t,\end{aligned} \tag{33}$$

$$B'^{s}_{s,t} = k_{bs}\left(\frac{H_{l,t}}{R_t}\right)B'^{d}_{1,l,t}, \tag{34}$$

where $\Lambda_{3,t}$ and $\Lambda_{4,t}$ represent the Lagrange multipliers for the budget constraint and the collateral constraint, respectively. It can be proved that, as shown in equation (34), the collateral constraint is binding around a steady state under the condition of $\overline{R} > (1-\overline{X})\overline{H}_s$, based on equations (30) and (31)[13].

At this point, we should pay attention to equation (30). Denoting $\Gamma(B_{r,t},\ B^{d}_{1,l,t},\ B^{s}_{s,t},\ \Lambda_{3,t},\ \Lambda_{4,t})$ as the Lagrangean of the optimizing problem, which implicitly represents the active investor's expected net profits, we can show that $\frac{\partial \Gamma}{\partial E_m} = -\Lambda_{3,t}$. This indicates that $-\Lambda_{3,t}$ could be interpreted as the shadow value of the active investor's capital (i.e. equity). Meanwhile, equation (30) yields

$$-\Lambda_{3,t} = \left(\frac{R_t - 1}{R_t}\right)\left(\frac{1}{V_t}\right), \tag{35}$$

where

$$V_t \equiv 1 + \frac{v_{1,l}}{2}\left(\frac{B'_{r,t}}{B'^{d}_{1,l,t}}\kappa_{1,l}-1\right)^2 + v_{1,l}\kappa_{1,l}\left(\frac{B'_{r,t}}{B'^{d}_{1,l,t}}\kappa_{1,l}-1\right)\left(\frac{B'_{r,t}}{B'^{d}_{1,l,t}}\right). \tag{36}$$

Setting $Z_t = \left(\frac{B'_{r,t}}{B'^{d}_{1,l,t}}\right)\kappa_{1,l}$,

$$\begin{aligned}V_t &= \frac{v_{1,l}}{2}(Z_t-1)^2 + v_{1,l}Z_t(Z_t-1)+1\\ &= \frac{3v_{1,l}}{2}\left(Z_t-\frac{2}{3}\right)^2 + \frac{1}{6}(6-v_{1,l}).\end{aligned} \tag{37}$$

Consequently, for V_t to be positive for all t, the condition $0 < v_{1,l} < 6$ must hold, which is

[13] For example, using the 3-month TB rate and the federal funds rate for the U.S., $\overline{R}/\overline{H}_s$ = 1.0006 from 1990:Q1 to 2010:Q3. Consequently, the collateral constraint is binding around a steady state for any $\overline{X} \in [0, 1]$.

quite probable. Furthermore,

$$\frac{dV_t}{dZ_t} = 3v_{1,l}\left(Z_t - \frac{2}{3}\right) \tag{38}$$

and

$$\frac{d^2V_t}{dZ_t^2} = 3v_{1,l} \ > 0. \tag{39}$$

In equation (38), if $Z_t < \frac{2}{3}$, that is, if $B'^{d}_{1,l,t} > \frac{3}{2}\kappa_{1,l}B'_{r,t}$, then $\frac{dV_t}{dZ_t} < 0$. This result, together with equations (35), (37) and (39), suggests the following proposition.

Proposition 2: Under the condition of $0 < v_{1,l} < 6$, the marginal increases in capital (i.e. equity) would raise the expected net profits. Although rises in the active investor's risky assets, $B^{d}_{1,l,t}$, might lead to increases in capital, the larger risky assets would diminish the marginal increases in the net profits.

This proposition implies that the growth of the active investor's assets would push up the (expected) net profits under the abovementioned condition on $v_{1,l}$.

Furthermore, from the first-order conditions, we can derive a very important feature of the model. This feature is stated as proposition 3.

Proposition 3: When $X_t = \overline{X}$ for all t and $\overline{X} > 1 - \frac{\overline{R}}{\overline{H}_s}$, the active investor's holdings of risky assets are affected by the slope of the yield curve, and hence, the steeper the yield curve, the larger is its asset size[14].

This proposition can be proved as follows. In what follows, for any variables Z_t, we can define

$z_t \equiv \ln(Z_t)$.

Furthermore, with all variables z_t defined as above, we can define

$\hat{z}_t \equiv z_t - \overline{z}$,

where $\overline{z} = \ln(\overline{Z})$.

Log-linear approximations of equations (30) to (32) with $X_t = \overline{X}$ for all t yield the following:

[14] This study defines the slope of the yield curve in terms of holding returns instead of interest rates. The validity of this definition is given in Appendix A of Sudo [2010].

$$\hat{\lambda}_{3,t} = \frac{1}{\overline{R}-1}\hat{r}_t + v_{1,l}\left(\hat{b}^d_{1,l,t} - \hat{b}_{r,t}\right), \tag{40}$$

$$\hat{\lambda}_{4,t} = -\hat{r}_t + \frac{\left(1-\overline{X}\right)\overline{H}_s}{\overline{R}-\left(1-\overline{X}\right)\overline{H}_s}\left\{\left(\overline{R}-1\right)\hat{\lambda}_{3,t} - \hat{h}_{s,t}\right\}, \tag{41}$$

$$\begin{aligned}
&\left(\overline{R}-1\right)v_{1,l}\left(\frac{\overline{B}'_r}{\overline{B}'^d_{1,l}}\right)\left(\hat{b}^d_{1,l} - \hat{b}_{r,t}\right) \\
&\quad = -\hat{h}_{l,t} + \left\{\left(1-\overline{X}\right) + \gamma\left(1-k_{bs}\right)\overline{X}\right\}\left(\overline{R}-1\right)\left(\frac{\overline{H}_l}{\overline{R}}\right)\hat{\lambda}_{3,t} \\
&\qquad\qquad - k_{bs}\left\{\frac{\overline{R}-\left(1-\overline{X}\right)\overline{H}_s}{\overline{R}}\right\}\left(\frac{\overline{H}_l}{\overline{R}}\right)\hat{\lambda}_{4,t}.
\end{aligned} \tag{42}$$

Substituting equations (40) and (41) into equation (42) and rearranging the resultant equation, we obtain

$$\begin{aligned}
&\left(\overline{R}-1\right)v_{1,l}\left(1 - k_{bs}\frac{\overline{H}_l}{\overline{R}} - \frac{\overline{B}'_r}{\overline{B}'^d_{1,l}}\right)\left(\hat{b}^d_{1,l} - \hat{b}_{r,t}\right) \\
&\qquad = \left(\hat{h}_{l,t} - \hat{r}_t\right) - k_{bs}\frac{\left(1-\overline{X}\right)\overline{H}_s}{\overline{R}}\left(\frac{\overline{H}_l}{\overline{R}}\right)\left(\hat{h}_{s,t} - \hat{r}_t\right).
\end{aligned} \tag{43}$$

Equation (34) implies that

$$k_{bs} = \frac{\overline{R}\,\overline{B}'^s_s}{\overline{H}_l\overline{B}'^d_{1,l}}. \tag{44}$$

Hence, substituting equation (44) into equation (43), we obtain

$$\begin{aligned}
&\left(\overline{R}-1\right)v_{1,l}\left(1 - \frac{\overline{B}'^s_s}{\overline{B}'^d_{1,l}} - \frac{\overline{B}'_r}{\overline{B}'^d_{1,l}}\right)\left(\hat{b}^d_{1,l} - \hat{b}_{r,t}\right) \\
&\qquad = \left(\hat{h}_{l,t} - \hat{r}_t\right) - k_{bs}\frac{\left(1-\overline{X}\right)\overline{H}_s}{\overline{R}}\left(\frac{\overline{H}_l}{\overline{R}}\right)\left(\hat{h}_{s,t} - \hat{r}_t\right).
\end{aligned} \tag{45}$$

When $\overline{H}_l$, $\overline{H}_s$ and $\overline{R}$ are approximately equivalent, the budget constraint (33) implies that[15]

$$\begin{aligned}
\overline{B}'^d_{1,l} - \overline{B}'^s_s - \overline{B}'_r &\approx \overline{H}_l\overline{B}'^d_{1,l} - \overline{H}_s\overline{B}'^s_s - \overline{R}\,\overline{B}'_r \\
&= \frac{1}{1-\overline{X}}\left\{E'_m + \overline{R}\left(\overline{M}' - 2\overline{B}'_r\right)\right\} \\
&\qquad - \frac{\overline{X}}{1-\overline{X}}\left\{\gamma\left(1-k_{bs}\right)\overline{B}'^d_{1,l} + \overline{L}' - \overline{R}\,\overline{B}'_r\right\}.
\end{aligned} \tag{46}$$

[15] In next sub-section, it will be shown that $\overline{H}_l = \overline{H}_s = \overline{R}$ in a steady state.

Because $\overline{M}' > 2\overline{B}'_r$ would undoubtedly hold, equation (46) results in $\overline{B}'^{d}_{1,l} - \overline{B}'^{s}_{s} - \overline{B}'_r$ > 0 when $\overline{X}$ is substantially small. Therefore, equations (45) and (46) bear out proposition 3. The proposition theoretically establishes a relationship between 'the macro risk premium' and 'the risk appetite' advocated by Adrian, Moench and Shin [2010b], where the macro risk premium and the risk appetite correspond to the slope of the yield curve and the active investor's asset size, respectively, in the proposition[16].

3. Passive investors

(1) Utility function and budget constraint

Passive investors consist of households other than the end-user borrowers and institutional investors such as mutual funds, pension funds and insurance companies; hence, passive investors play the roles of consumers as well as funding sources in the economy.

In this study, we presume a continuum of identical and infinitely living passive investors indexed by $i_2 \in [0, 1]$. These investors obtain utility from a bundle $C_{2,t}$ given by

$$C_{2,t} = \left[\int_0^1 C_{2,t}(j)^{1-\frac{1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{47}$$

where $C_{2,t}(j)$ denotes the quantity of goods j consumed by the investor in period t.

These passive investors have the period utility function of

$$U_{2,k}\left(C_{2,t},\ C_{2,t-1}, N^s_{2,t}\right) = \frac{1}{1-\sigma_{2,k}}\left(\frac{C_{2,t}}{C^{h_2}_{2,t-1}}\right)^{1-\sigma_{2,k}} - \frac{\left(N^s_{2,t}\right)^{1+\varphi_{2,k}}}{1+\varphi_{2,k}}, \tag{48}$$

where k = p (stable times) or d (financial distress). $N^s_{2,t}$ represents the number of hours worked by the investor in period t; $\sigma_{2,k}$ > 0, the inverse of the elasticity of inter-temporal substitution; $\varphi_{2,k} \geq 0$, the inverse of the Frisch labour supply elasticity; and $h_2 \geq 0$, the habit persistence parameter indicating the extent of habit formation.

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[16] Adrian, Moench and Shin [2010b] state that when the financial intermediary's balance sheet constraints are loose, the risk premia are compressed. Their argument seems to contradict proposition 3. This is because they relate the intermediary's balance sheet size to the time difference in the weighted combination of yield and credit spreads (i.e. $r_t - r_{t-1}$ in their context), whereas we associate it with the term premium of holding returns (i.e. $r_{t+1} - r_t$ in their context). Proposition 3 is consistent with Woodford's [2010] view that a larger credit spread encourages financial intermediaries to increase the supply of credit.

In stable times, passive investors allocate their incomes for the purchase of short- and long-term bonds as well as consumption goods and deposits. Consequently, the period budget constraint in stable times takes the form of

$$\begin{aligned}&\frac{1}{P_t}\left(W_t N_{2,t}^s + H_{l,t-1}B_{2,l,t-1}^d + H_{s,t-1}B_{s,t-1}^d + R_{t-1}M_{t-1} + T_t\right)\\ &\qquad = C_{2,t} + \frac{1}{P_t}\left\{B_{2,l,t}^d + B_{s,t}^d + M_t\left(1 + AC_{m,t}\right)\right\},\end{aligned} \tag{49}$$

where $B_{2,l,t}^d$ denotes the holdings of long-term bonds at the beginning of period t; $B_{s,t}^d$, the holdings of short-term bonds at the beginning of period t; and M_t, the deposit amounts, which imply demand for money, at the beginning of period t. T_t represents lump sum transfers, which include dividends from firms and active investors of which the passive investors are the only owners, and is derived by

$$t_t = \rho_t t_{t-1} + \varepsilon_{t,t}, \tag{50}$$

where $t_t \equiv \ln(T_t)$, $\rho_t \in (0, 1)$ and $\varepsilon_{t,t} \sim i.i.d.\left(0, \sigma_t^2\right)$. $AC_{m,t}$ is the cost function for investing in short- and long-term bonds and is specified as follows:

$$AC_{m,t} \equiv \frac{v_{2,s}}{2}\left(\frac{M_t}{B_{s.t}^d}\kappa_{2,s} - 1\right)^2 + \frac{v_{2,l}}{2}\left(\frac{M_t}{B_{2,l,t}^d}\kappa_{2,l} - 1\right)^2, \tag{51}$$

where $v_{2,s} > 0$ and $v_{2,l} > 0$ are parameters. $\kappa_{2,s} > 0$ and $\kappa_{2,l} > 0$ are parameters ensuring that the cost $AC_{m,t}$ remains in the steady state; they are defined as $\kappa_{2,s} \equiv \overline{B}_s^d / \overline{M}$ and $\kappa_{2,l} \equiv \overline{B}_{2,l,t}^d / \overline{M}$.

The cost function has the same implication as that of the cost function $AC_{l,t}$ on the active investor's budget constraint (23). Hence, passive investors perceive both short- and long-term bonds as riskier assets, entailing a loss of liquidity in relation to their deposits. When passive investors invest in short- and long-term bonds, they demand additional money (i.e. deposits as risk-free assets) to compensate themselves for this loss of liquidity. In effect, the agents have self-imposed the 'reserve requirements' (as described in ALSN [2004a]) of $M_t \times AC_{m,t}$ on their short- and long-term bond investments[17].

On the other hand, when the end-user borrowers face financial distress in

[17] The cost function has another implication in that this functional form of $AC_{m,t}$ guarantees non-zero demand for these riskier assets, in terms of the passive investor budget constraint, under the condition that all $v_{2,s}$, $v_{2,l}$, $\kappa_{2,s}$ and $\kappa_{2,l}$ are positive.

period $t-1$, they redeem the long-term bonds of $\gamma H_{l,t-1} B^{s}_{l,t-1}$ instead of repurchasing those of $H_{l,t-1} B^{s}_{l,t-1}$, and this causes active investors to repay the short-term bonds of $k_{bs}\gamma H_{l,t-1} B^{d}_{1,l,t-1}$ instead of $H_{s,t-1} B^{d}_{s,t-1}$, as explained above. As a result, in the state of financial distress, the passive investor budget constraint in period t is

$$\frac{1}{P_t}\left(W_t N^{s}_{2,t} + \gamma H_{l,t-1} B^{d}_{2,l,t-1} + k_{bs}\gamma H_{l,t-1} B^{d}_{1,l,t-1} + R_{t-1} M_{t-1} + T_t\right) = C_{2,t} + \frac{1}{P_t} M_t \,. \quad (52)$$

Afterwards, since passive investors will not invest in short- and long-term bonds, the budget constraint in period $t+i\ (i \geq 1)$ becomes

$$\frac{1}{P_{t+i}}\left(W_{t+i} N^{s}_{2,t+i} + R_{t+i-1} M_{t+i-1} + T_{t+i}\right) = C_{2,t+i} + \frac{1}{P_{t+i}} M_{t+i} \,. \quad (53)$$

(2) Optimality conditions

According to the above setup, the problem solved by the passive investor can be expressed as follows:

$$\max_{\{C_{2,t}, N^{s}_{2,t}, B^{d}_{s,t}, B^{d}_{2,l,t}, M_t\}} E_t \sum_{i=0}^{\infty} \beta_2^{t+i} \Big\{ (1 - X_{t+i}) U_{2,p}\left(C_{2,t+i}, C_{2,t+i-1}, N^{s}_{2,t+i}\right) + X_{t+i} U_{2,d}\left(C_{2,t+i}, C_{2,t+i-1}, N^{s}_{2,t+i}\right) \Big\}$$

$$\text{s.t.} \quad \begin{aligned} &\left(\frac{1}{P_{t+i}}\right)\Big\{ W_{t+i} N^{s}_{2,t+i} + T_{t+i} + (1 - X_{t+i})\left(H_{s,t+i-1} B^{d}_{s,t+i-1} + H_{l,t+i-1} B^{d}_{2,l,t+i-1}\right) \\ &\quad + \xi_{t+i} X_{t+i} \gamma H_{l,t+i-1}\left(k_{bs} B^{d}_{1,l,t+i-1} + B^{d}_{2,l,t+i-1}\right) + R_{t+i-1} M_{t+i-1} \Big\} \\ &\quad - C_{2,t+i} - \left(\frac{1}{P_{t+i}}\right)\Big\{ M_{t+i} + (1 - X_{t+i})\left(B^{d}_{s,t+i} + B^{d}_{2,l,t+i} + M_{t+i} AC_{m,t+i}\right) \Big\} = 0, \end{aligned} \quad (54)$$

where the parameter $\beta_2 \in (0, 1)$ is a discount factor and we suppose $\beta_2 > \beta_1$ because the passive investor is a lender whereas the end-user borrower is a borrower, that is, the lender is likely to be more patient than the borrower. ξ_{t+i} denotes a dummy variable which takes the value of one if $i = 0$ and zero otherwise.

Assuming that $\sigma_{2,p} = \sigma_{2,d} = \sigma_2$ and $\varphi_{2,p} = \varphi_{2,d} = \varphi_2$, the first-order conditions for the optimizing problem given above can be written, in real terms, as follows:

$$\Lambda_{5,t} = \frac{C_{2,t}^{-\sigma_2}}{C_{2,t-1}^{h_2(1-\sigma_2)}} - \beta_2 h_2 E_t\left[\frac{C_{2,t+1}^{1-\sigma_2}}{C_{2,t}^{h_2(1-\sigma_2)+1}}\right], \quad (55)$$

$$\left(N_{2,t}^{s}\right)^{\varphi_2} = \Lambda_{5,t} W_t', \tag{56}$$

$$\begin{aligned}\Lambda_{5,t} - \beta_2 E_t\left[\Lambda_{5,t+1} H_{s,t}\left(\frac{1}{\Pi_{t+1}}\right)\left(\frac{1-X_{t+1}}{1-X_t}\right)\right] \\ = \Lambda_{5,t}\left\{v_{2,s}\kappa_{2,s}\left(\frac{M_t'}{B_{s,t}'^{d}}\kappa_{2,s}-1\right)\left(\frac{M_t'}{B_{s,t}'^{d}}\right)^2\right\},\end{aligned} \tag{57}$$

$$\begin{aligned}\Lambda_{5,t} - \beta_2 E_t\left[\Lambda_{5,t+1} H_{l,t}\left(\frac{1}{\Pi_{t+1}}\right)\left(\frac{1-X_{t+1}}{1-X_t}\right)\right] \\ = \Lambda_{5,t}\left\{v_{2,l}\kappa_{2,l}\left(\frac{M_t'}{B_{2,l,t}'^{d}}\kappa_{2,l}-1\right)\left(\frac{M_t'}{B_{2,l,t}'^{d}}\right)^2\right\},\end{aligned} \tag{58}$$

$$\begin{aligned}\Lambda_{5,t} - \beta_2 R_t E_t\left[\Lambda_{5,t+1}\left(\frac{1}{\Pi_{t+1}}\right)\right] \\ = -\Lambda_{5,t}(1-X_t)\left\{\frac{v_{2,s}}{2}\left(\frac{M_t'}{B_{s,t}'^{d}}\kappa_{2,s}-1\right)^2 + \frac{v_{2,l}}{2}\left(\frac{M_t'}{B_{2,l,t}'^{d}}\kappa_{2,l}-1\right)^2\right. \\ \left. + v_{2,s}\kappa_{2,s}\left(\frac{M_t'}{B_{s,t}'^{d}}\kappa_{2,s}-1\right)\left(\frac{M_t'}{B_{s,t}'^{d}}\right) + v_{2,l}\kappa_{2,l}\left(\frac{M_t'}{B_{2,l,t}'^{d}}\kappa_{2,l}-1\right)\left(\frac{M_t'}{B_{2,l,t}'^{d}}\right)\right\},\end{aligned} \tag{59}$$

$$\begin{aligned}C_{2,t} = \Bigg[W_t' N_{2,t}^{s} + T_t' + (1-X_t)\left(H_{s,t-1}\frac{B_{s,t-1}'^{d}}{\Pi_t} + H_{l,t-1}\frac{B_{2,l,t-1}'^{d}}{\Pi_t}\right) \\ + \gamma X_t \xi_t H_{l,t}\left(k_{bs}\frac{B_{1,l,t-1}'^{d}}{\Pi_t} + \frac{B_{2,l,t-1}'^{d}}{\Pi_t}\right) + R_{t-1}\frac{M_{t-1}'}{\Pi_t} - M_t' \\ -(1-X_t)\left\{B_{s,t}'^{d} + B_{2,l,t}'^{d} + M_t\left\{\frac{v_{2,s}}{2}\left(\frac{M_t'}{B_{s,t}'^{d}}\kappa_{2,s}-1\right)^2 + \frac{v_{2,l}}{2}\left(\frac{M_t'}{B_{2,l,t}'^{d}}\kappa_{2,l}-1\right)^2\right\}\right\}\Bigg],\end{aligned} \tag{60}$$

where $\Lambda_{5,t}$ represents the Lagrange multiplier for the budget constraint.

For the first-order conditions, it is important to pay attention to two points. First, according to equation (59), the existence of the reserve requirement entails that money demand decisions are taken based on the relative supply of riskier bonds. In particular, an increase in the relative amount of riskier assets correspondingly raises the demand for money as liquid or risk-free assets.

Second, equations (57) to (59) imply the presence of an endogenous term structure relationship between the one-period nominal interest rate and the one-period nominal holding return on short- or long-term bonds. This feature is very important and is stated as the following proposition.

Proposition 4: The term structure of interest rates, in terms of the term premium between the one-period nominal interest rate and the one-period nominal holding return on short- or long-term bonds, is endogenously shifted by the modified ratio of money to each bond holding.

This proposition can be proved as follows. The log-linear approximation of equations (57) to (59) gives us

$$E_t\left[\hat{\lambda}_{5,t+1}\right]-\hat{\lambda}_{5,t}+\hat{h}_{s,t}-E_t\left[\hat{\pi}_{t+1}\right]=-v_{2,s}\left(\frac{\overline{M}'}{\overline{B}_s'^d}\right)(\hat{m}_t-\hat{b}_{s,t}^d)+\frac{\overline{X}}{1-\overline{X}}E_t\left[\hat{x}_{t+1}-\hat{x}_t\right], \quad (61)$$

$$E_t\left[\hat{\lambda}_{5,t+1}\right]-\hat{\lambda}_{5,t}+\hat{h}_{l,t}-E_t\left[\hat{\pi}_{t+1}\right]=-v_{2,l}\left(\frac{\overline{M}'}{\overline{B}_{2,l}'^d}\right)\left(\hat{m}_t-\hat{b}_{2,l,t}^d\right)+\frac{\overline{X}}{1-\overline{X}}E_t\left[\hat{x}_{t+1}-\hat{x}_t\right], \quad (62)$$

$$E_t\left[\hat{\lambda}_{5,t+1}\right]-\hat{\lambda}_{5,t}+\hat{r}_t-E_t\left[\hat{\pi}_{t+1}\right]=\left(1-\overline{X}\right)\left\{v_{2,s}\left(\hat{m}_t-\hat{b}_{s,t}^d\right)+v_{2,l}\left(\hat{m}_t-\hat{b}_{2,l,t}^d\right)\right\}. \quad (63)$$

By combining equations (63) with equation (61) or (62), we obtain equations (64) and (65), respectively.

$$\begin{aligned}\hat{h}_{s,t}=\hat{r}_t-v_{2,s}\left(\frac{\overline{M}'}{\overline{B}_s'^d}\right)\left(\hat{m}_t-\hat{b}_{s,t}^d\right)\\ -\left(1-\overline{X}\right)\left\{v_{2,s}\left(\hat{m}_t-\hat{b}_{s,t}^d\right)+v_{2,l}\left(\hat{m}_t-\hat{b}_{2,l,t}^d\right)\right\}+\frac{\overline{X}}{1-\overline{X}}E_t\left[\hat{x}_{t+1}-\hat{x}_t\right],\end{aligned} \quad (64)$$

$$\begin{aligned}\hat{h}_{l,t}=\hat{r}_t-v_{2,l}\left(\frac{\overline{M}'}{\overline{B}_{2,l}'^d}\right)\left(\hat{m}_t-\hat{b}_{2,l,t}^d\right)\\ -\left(1-\overline{X}\right)\left\{v_{2,s}\left(\hat{m}_t-\hat{b}_{s,t}^d\right)+v_{2,l}\left(\hat{m}_t-\hat{b}_{2,l,t}^d\right)\right\}+\frac{\overline{X}}{1-\overline{X}}E_t\left[\hat{x}_{t+1}-\hat{x}_t\right].\end{aligned} \quad (65)$$

It should be noted that one-period holding returns on the short- or long-term bond are affected by the demand not only for the correspond bond but also for the other bond. In these equations, $\hat{r}_t$ is endogenously determined by the central bank, as explained in sub-section 4. Therefore, these equations lead to proposition 4; hence, the entire shape of the yield curve in terms of the one-period holding returns is endogenously determined in the economy.

Proposition 4 has two implications. First, equations (64) and (65) capture an essential feature of Tobin's [1969] framework which maintains that spreads between interest rates should reflect the relative quantities of assets. Second, as shown in sub-section 2, the endogenous term structure would affect the active investor's holdings of risky assets, that is, the asset size. Hence, the endogenous term structure of interest rates can provide a bi-directional feedback between the active investor's behaviour and the real economy (economic activities).

4. Firms and the central bank

(1) Firms

In this sub-section, we will illustrate the derivation of the hybrid new-Keynesian Phillips curve (NKPC)[18].

As stated above, we assume a continuum of firms indexed by $j \in [0, 1]$. Each firm produces a differentiated consumption good $C_t^s(j)$ in period t; however, all firms use an identical technology, represented by the following production function:

$$C_t^s(j) = A_t\left(N_t^d(j)\right)^{1-\alpha}, \tag{66}$$

where we presuppose the absence of any capital accumulation in the firm[19]. A_t denotes the level of technology in period t, assumed to evolve exogenously over time as follows:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{67}$$

where $a_t \equiv \ln(A_t)$, $\rho_a \in (0, 1)$ and $\varepsilon_{a,t} \sim i.i.d.\left(0, \sigma_a^2\right)$, which indicates a shock to the technology. $N_t^d(j)$ stands for the number of work-hours hired from end-user borrowers and passive investors by firm j in period t, and $\alpha \in [0, 1]$ represents the share of capital in production.

It is assumed that all firms face an identical isoelastic demand schedule, in which they take the aggregate consumer price level P_t in period t and aggregate consumption index C_t^d in period t as given. The demand schedule is described as follows:

$$C_t^d(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t^d, \tag{68}$$

where $P_t(j)$ is the price of consumption goods j in period t, ε stands for constant price elasticity, $C_t^d = C_{1,t} + C_{2,t}$ and

$$P_t \equiv \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}. \tag{69}$$

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[18] For further details, see, for instance, Sudo [2010].

[19] In equation (66), $C_t^s(j)$ and $N_t^d(j)$ are interpreted as per capital in period t for firm j.

Here, let us consider the Calvo [1983] model of staggered price setting with the following modification. In the period between price reoptimizations, firms mechanically adjust their prices according to some indexation rule, described as the 'lagged inflation indexation' by Christiano, Eichenbanm and Evans [2005]. Formally, a firm that has the opportunity to reoptimize its price in period t with probability $1 - \eta$ sets an optimal price P_t* in that period. In subsequent periods (i.e. until the firm has the opportunity to reoptimize prices again), its price is adjusted according to the following rules of partial indexation to past inflation:

$$P_{t+k|t} = P_{t+k-1|t}\left(\Pi_{t+k-1}\right)^{\omega} \tag{70}$$

for $k = 1, 2, \dots$, and

$$P_{t,t} = P_t^*, \tag{71}$$

where $P_{t+k|t}$ denotes the price effective in period $t + k$ for the firm that last reoptimized its price in period t, and $\omega \in [0, 1]$ is a parameter measuring the degree of indexation.

Combining the definition of aggregate consumer price (69) with the firm's price-adjusting rules (70) and (71), the aggregate consumer price dynamics are described as

$$\Pi_t^{1-\varepsilon} = \eta\left(\Pi_{t-1}^{\omega}\right)^{1-\varepsilon} + (1-\eta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}. \tag{72}$$

Next, we will derive a firm's optimal price setting. A firm reoptimizing in period t will choose the price P_t* that maximizes the current market value of the profits generated, subject to a sequence of demand constraints and the rule of price adjustment. Based on the first-order conditions pertaining to the firm's optimizing problem, we can derive the following optimal price-setting equation:

$$\hat{p}_t^* - \hat{p}_t = (1-\beta\eta)\sum_{k=0}^{\infty}(\beta\eta)^k E_t\left[\hat{mc}_{t+k|t} + (\hat{p}_{t+k} - \hat{p}_t) - \omega(\hat{p}_{t+k-1} - \hat{p}_{t-1})\right], \tag{73}$$

where $\beta \in (0, 1)$ is a discount factor of the hypothetical aggregated consumers, as defined below, and

$$\hat{mc}_{t+k|t} = \ln\left(MC_{t+k|t}\right) - \ln\left(\overline{MC}\right),$$

where $MC_{t+k|t}$ represents the real marginal cost in period $t + k$ for a firm whose price was last set in period t.

Finally, we will derive the NKPC. By combining the firm's production function (66), demand function (68) and market-clearing condition on consumption goods, we can derive the following approximate aggregated production function:

$$c_t = a_t + (1-\alpha) n_t^d , \tag{74}$$

where $n_t^d \equiv \ln\left(N_t^d\right)$ and $N_t^d \equiv \int_0^1 N_t^d(j)dj$. Based on equation (74), an individual firm's marginal cost in terms of the economy's average real marginal cost can be defined as

$$mc_t = w_t - p_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \ln(1-\alpha) . \tag{75}$$

Using equations (73) and (75), we can derive the following NKPC:

$$\hat{\pi}_t = \frac{\omega}{1+\omega\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\omega\beta}E_t\hat{\pi}_{t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\omega\beta)}\Theta\,\hat{mc}_t , \tag{76}$$

where $\Theta \equiv \dfrac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$ and

$$\hat{mc}_t = -\hat{\lambda}_{6,t} + \frac{\varphi+\alpha}{1-\alpha}\hat{c}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t . \tag{77}$$

Here, assuming that $h_1 \approx h_2$ and $\sigma_1 \approx \sigma_2$, we can approximately express β, φ and $\hat{\lambda}_{6,t}$ as follows:

$$\beta \approx \frac{\overline{C}_1}{\overline{C}}\beta_1 + \frac{\overline{C}_2}{\overline{C}}\beta_2 , \tag{78}$$

$$\varphi = \left(\frac{\overline{N}^s}{\overline{N}_1^s}\right)\left(\frac{\overline{C}_1}{\overline{C}}\right)\varphi_1 \left(= \left(\frac{\overline{N}^s}{\overline{N}_2^s}\right)\left(\frac{\overline{C}_2}{\overline{C}}\right)\varphi_2 \right), \tag{79}$$

$$\hat{\lambda}_{6,t} \approx \frac{\overline{C}_1}{\overline{C}}\hat{\lambda}_{1,t} + \frac{\overline{C}_2}{\overline{C}}\hat{\lambda}_{5,t} . \tag{80}$$

For the derivation of equations (78) to (80), see Appendix 1.

(2) The central bank

We assume that the central bank sets the one-period nominal (risk-free) interest rate as a policy rate based on an augmented Taylor-type interest rate rule. The interest rate in period t, R_t, responds not only to the rate in the previous period, and to deviations of the output and inflation rate from their steady-state values, but also to the ratio of the demand for money to the sum of base money and capital injection to the active investor with financial distress. Formally,

$$\ln\left(\frac{R_t}{\overline{R}}\right) = \rho_r \ln\left(\frac{R_{t-1}}{\overline{R}}\right) + (1-\rho_r)\left\{\rho_\pi \ln\left(\frac{\Pi_t}{\overline{\Pi}}\right) + \rho_y \ln\left(\frac{Y_t}{\overline{Y}}\right) + \rho_\mu \ln\left(\frac{\Phi_t}{\overline{\Phi}}\right)\right\} + e_{R,t}, \quad (81)$$

where $\rho_r \in (0, 1)$, $\rho_\pi > 1$ (for fulfilling the Taylor principle), $\rho_y > 0$ and $\rho_\mu > 0$. Y_t denotes the real GDP in period t and is defined as follows[20]:

$$Y_t = C_{1,t} + C_{2,t} + (1 - X_t)\{S_{h,t} - (1-\iota_h)S_{h,t-1}\}Q'_{h,t}. \quad (82)$$

$e_{R,t}$ is a shock to monetary policy and is expressed as

$$e_{R,t} = \rho_R e_{R,t-1} + \varepsilon_{R,t}, \quad (83)$$

where $\rho_R \in (0, 1)$ and $\varepsilon_{R,t} \sim i.i.d.(0, \sigma_R^2)$. In addition, Φ_t is defined as

$$\Phi_t \equiv \frac{M'_t}{B'_{r,t} + X_t L'_t}, \quad (84)$$

where L'_t is derived from equation (27).

5. Complete model

To close the model, we need to specify the market-clearing conditions on consumption goods, labour, and short- and long-term bonds. These conditions are as follows:

$$C_t = C_{1,t} + C_{2,t}, \quad (85)$$

$$N_t^d = N_{1,t}^s + N_{2,t}^s, \quad (86)$$

$$B'^s_{s,t} = B'^d_{s,t}, \quad (87)$$

$$B'^s_{l,t} = B'^d_{1,l,t} + B'^d_{2,l,t}. \quad (88)$$

In addition, we must decide the processes of house prices $Q_{h,t}$ and the probability of default X_t. First, let us assume that the real house prices in period t depend on previous ones and housing stock, which is expressed by

$$Q'_{h,t} = G Q'^{\rho_{qh1}}_{h,t-1} S^{\rho_{qh2}}_{h,t-1} e_{qh,t}, \quad (89)$$

where $G > 0$, $\rho_{qh1} \in (0, 1)$ and $\rho_{qh2} \in (0, 1)$ are parameters, and ρ_{qh1} and ρ_{qh2} should be $\rho_{qh1} + \rho_{qh2} < 1$. $e_{qh,t}$ denotes a shock to the real house prices and is expressed as

$$e'_{qh,t} = \varepsilon_{qh,t}, \quad (90)$$

[20] According to the definition of Y_t, the monetary policy implicitly takes house prices (i.e. asset prices) into consideration.

where $e'_{qh,t} \equiv \ln(e_{qh,t})$ and $\varepsilon_{qh,t} \sim i.i.d.(0, \sigma^2_{qh})$.

Second, in order to link the process of $\{X_t\}$ to the business cycle, we presuppose that

$$X_t = \overline{X}\left(\overline{Y} / Y_{t-1}\right)^{\rho_x}, \tag{91}$$

where $\overline{X} \in [0, 1]$ represents a steady state of the sequence $\{X_t\}$, and $\rho_x > 0$ is a parameter.

6. Model simplification

In order to explore our model, let us log-linearize the equations describing the model around the steady state. However, the entire model is so complicated that it is not easy to fulfil the Blanchard and Kahn [1980] condition, that is, to carry out the empirical analysis of the model. Hence, let us incorporate some additional assumptions to simplify the model.

First, we assume that $\dfrac{C_{1,t}}{C_t} = \dfrac{N^s_{1,t}}{N^s_t} = \tau_1$, where $\tau_1 \in (0, 1)$ is constant. Second, the lump sum transfer, T_t, is ignored in the case of passive investors. Third, since ξ_t is the dummy variable in equation (60), let us ignore the term multiplied by ξ_t for the sake of simplicity.

The final assumption is that active investors always choose a constant ratio of accepted deposits as reserves. This implies that in the active investor's first-order conditions, $B_{r,t}$ is replaced by $\tau_2 M_t$, where $\tau_2 \in (0, 1)$ denotes the constant ratio. Although this assumption imposes a strong restriction on the active investor's behaviour, it can be supported by two points: first, the fact that banks actually behave thus, and second, the fact that it helps maintain propositions 2 and 3.

Ⅲ Estimation

1. Preliminary setting

In this study, we have used the U.S. quarterly data for the period extending from 1990:Q1 to 2010:Q3. The data pertain to the following: real consumption, hours in business sector, compensation per hour in business sector, house prices, housing stock, real residential investment, consumer prices, money supply, interest rates (one-period and long-term), bond outstandings (short- and long-term), reserves and central bank loans to financial institutions. The data series and their sources are listed in Appendix 2.

The trends have been removed from the variables by using the Hodrick-Prescott filter. In the study, we have defined active investors as comprising commercial banking, securities dealer-brokers, ABS issuers, finance companies, funding corporations, and agency- and GSE-backed mortgage pools in the *Flow of Funds Accounts of the United States*[21].

We have not estimated all the parameters considered in the model; the values of some of these parameters have been set on the basis of previous studies. These parameters and their values are given in Table 1.

Since the present study's end-user borrowers and passive investors correspond to the impatient and patient households, respectively, in Gerali et al. [2010] and Iacoviello and Neri [2010], some parameters associated with these agents are set based on these studies, that is, β_1 = 0.97, k_h = 0.925, χ = 1 and $\varphi_1 = \varphi_2 = \varphi$ = 1[22]. Moreover, τ_1 is set as 0.35. Iacoviello and Neri [2010] estimate the labour income share of collateral constrained agents to be 21%. While both collateral constrained (borrowers) and unconstrained (lenders) households invest in residential assets in their model, the former invest in residential assets and the latter possess financial assets in our model. Therefore, we regard the income share of the former as not much lower than that of the latter and retain its prior mean of 35%, sourced from the estimation by Iacoviello [2005].

This study has formulated the utility on real consumption in a manner different from Gerali et al. [2010] and Iacoviello and Neri [2010] but identical to ALSN [2004a, b], wherein households correspond to passive investors (i.e. patient households) in our model. Consequently, based on ALSN [2004a], we have set $\sigma_1 = \sigma_2 = 2$ and $h_1 = h_2 = 0.9$. Moreover, β_2 is set as 0.99 based on $\frac{1}{\overline{H}_l}$, and ι_h is set as 0.016 based on the definition of depreciation $\iota_h \equiv \frac{\overline{I}_h}{\overline{Q}_h' \overline{S}_h}$, where $\overline{I}_h$ represents a steady state value on real residential investment.

With regard to the active investor part, τ_2 and k_{bs} are set based on $\frac{\overline{B}_r}{\overline{M}}$ and $\frac{\overline{R}\overline{B}_s^s}{\overline{H}_l \overline{B}_{1,l}}$, respectively, as τ_2 = 0.037 and k_{bs} = 0.3. The setting of k_{bs} should be noted. In the repo market, a haircut of 2% is traditionally applied to AAA through AA

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21 $B_{2,l,t}^d$ is calculated as $B_{2,l,t}^d = B_{l,t}^s - B_{1,l,t}^d$, because we have not clearly defined passive investors.

22 As mentioned above, we assume that $C_{1,t} / C_t = N_{1,t} / N_t = \tau_1$; hence, φ_1, φ_2 and φ are equivalent based on equation (79).

grade bonds. In practice, however, not all short-term bonds are secured by collateral, so we can calibrate k_{bs} as above, based on equation (34).

The setup of the firm is the same as described in Sudo [2010]. Therefore, the related parameters are set on the basis of his study: α = 0.36, ω = 0.6, η = 0.85 and Θ = 0.2. Furthermore, based on equation (78), β is calculated as 0.983.

The interest rate rule is similar to that in Iacoviello and Neri [2010], except for the inclusion of the term $\rho_\mu \ln\left(\frac{\Phi_t}{\Phi}\right)$ in equation (81). Consequently, the related parameters are set based on the above study; ρ_r = 0.8, ρ_π = 1.5 and ρ_y = 0.3. ρ_μ will be estimated in this study.

Finally, we set $\overline{X}$ = 0.2, because in the period between 1990 and 2010, we will assume that the end-user borrowers have started facing financial distress after 2007.

2. Parameters estimation

The remaining parameters— γ, $v_{1,l}$, $v_{2,s}$, $v_{2,l}$, ρ_μ, ρ_{qh1}, ρ_{qh2}, ρ_x, ρ_h, ρ_a, ρ_R and the standard deviation on disturbances—still need to be estimated. To estimate unknown parameters, we will perform Bayesian inference using the Markov Chain Monte Carlo (MCMC) method, which is now a standard technique for estimating the DSGE model[23]. The observable data used to estimate the unknown parameters include real balances, federal funds rates, real house prices and inflation rates; the estimation results are presented in Table 2. For all parameters except for the standard deviation on disturbances, the estimated values fulfil relevant conditions[24].

All shocks are highly persistent. Real house prices are also highly persistent, while the elasticity of real house prices with respect to housing stock is very low.

$v_{1,l}$ indicates the degree of self-imposed liquidity requirement when active investors invest in risky assets. The larger the value of $v_{1,l}$, the higher is the liquidity requirement imposed by active investors. Although the $v_{1,l}$ estimate fulfils the condition of 0 < $v_{1,l}$ < 6 in proposition 2, its value may not be high. Moreover, defining the term premia between one-period holding returns on long- or short-term bonds and one-period interest rates as $\hat{TP}_{l,t} \equiv \hat{h}_{l,t} - \hat{r}_t$ and $\hat{TP}_{s,t} \equiv \hat{h}_{s,t} - \hat{r}_t$, respectively, we can calculate the semi-elasticity of the real risky asset holding by active investors, $\hat{b}^d_{1,l,t}$, with respect

[23] In conducting Bayesian MCMC estimation, we have employed the DYNARE software (version 4.04) for MATLAB.

[24] In addition, results from posterior maximization indicate that the estimated standard deviation of parameters is low enough to be statistically significant.

to the term premia according to equation (45) as follows: $\frac{\partial \hat{b}^{d}_{1,l,t}}{\partial \hat{TP}_{l,t}}$ = 215.06 and $\frac{\partial \hat{b}^{d}_{1,l,t}}{\partial \hat{TP}_{s,t}}$ = −51.615. The former value indicates that if the term premia on long-term returns increase by 1%, the real risky asset holdings (i.e. active investors' assets) would correspondingly increase by 2.15%, on average. The latter shows that a 1% increase in the term premia on short-term returns would depress the real risky asset holdings by 0.52%.

Since $v_{2,s}$ and $v_{2,l}$ are estimated as positive, they have the following implications. First, changes in the real demand for short- and long-term bonds influence the real balance. Second, the relative weight of the real balance, in terms of real holdings of short- or long-term bonds by passive investors, affects not only the one-period holding returns on the corresponding bond but also the other bonds.

ρ_{μ} indicates the effect of changes in the money multiplier on the policy rate setting. The estimate of ρ_{μ} shows that changes in the money multiplier have a very small impact on the policy rate decision, as compared to changes in the output and the inflation rate.

Ⅳ Analysis

In this section, we will discuss the dynamics of the linearized model using impulse responses, while focusing on the active investor sector. Our aim is to empirically examine propositions 2–4 and to assess the effects of the presence of the active investor sector on the business cycle.

Before proceeding to the main results, let us have an overview of the role of each shock in generating fluctuations in the main variables by calculating their forecast error decomposition. Table 3 presents the results. Housing preference shocks, ε_{h}, have large influences on housing stock (i.e. residential investment), house prices, active investor profits, and the supply and demand of both short- and long-term bonds. The shock accounts for over 70% of the variance in these variables; moreover, it comprises around one-half of the variance in the term premia on short- and long-term bonds. On the other hand, shocks to house prices, ε_{qh}, and policy rates (i.e. monetary policy), ε_{R}, have some small impacts on the main variables. The former contributes less than 10% to the variance in all the main variables except for real output, while the latter contributes less than 1% to their variance, except in the case of term premia on short- and long-term bonds.

Although shocks to house prices and policy rates have little influence on generating fluctuations in the main variables, we emphasize these shocks as well as housing preference shocks because this study focuses on the effects of variations in the housing market and changes in monetary policy on the economy with the active investor sector. Figure 1 presents the impulse responses of the main variables to the shocks to housing preference, house prices and policy rates.

1. Properties of the model and empirical examination of the propositions

(1) Positive shocks to housing preference

Positive housing preference shocks raise house prices and housing stock (i.e. residential investment), which increase both real output and inflation rates, thus inducing rises in the policy rates. Meanwhile, increases in the housing stock lead to greater issuances of long-term bonds, which push up long yields, that is, one-period holding returns on the long-term bonds. Although this induces the long-term premia to decrease, initially, because one-period holding returns rise to a lesser extent than policy rates, they increase soon enough. Increases in the long-term premia (i.e. a steeping yield curve) apply downward pressure on the main variables mentioned above, which finally causes the premia to decrease.

With respect to the active investor sector, its profits and demand for long-term bonds change in accordance with the long-term premia. This empirically proves propositions 2 and 3. In addition, the increasing demand for long-term bonds stimulates the issuing of short-term bonds, which in turn increases the short-term premia through increases in the one-period holding returns on short-term bonds. Therefore, if we combine this fact with the explanation given in the previous paragraph, we can say that there exists a bi-directional feedback system between the yield curve and the economy; this empirically proves proposition 4.

(2) Positive shocks to house prices

The shape of the impulse responses of the main variables is similar to those in the positive housing preference shocks. Accordingly, in this case, we can say that propositions 2–4 have been empirically proved as well.

However, the period of impulse responses is much shorter than that of positive housing preference shocks. Although the reason for this is not clear, we can infer it as follows. Although the house prices initially rise due to the shocks, they gradually fall to reach equilibrium soon afterward. Consequently, while the initial impact has the continuous effect of expanding the economy, the subsequent price reductions counteract this effect. This interaction is likely to shorten the period of impulse responses.

(3) Positive shocks to policy rates (tightening monetary policy)

The impulse responses of the main variables in this study are different from the standard results in previous studies. In particular, the real output, real consumption, housing stock and house prices are initially found to move upwards in spite of increasing policy rates. We infer the reason as follows. First, let us consider the reason for the initial increases in housing stock and house prices. On the one hand, a rise in policy rates induces corresponding increases in the one-period holding returns on long-term bonds or long yields, thereby decreasing the issuance of long-term bonds. On the other hand, passive investors increase their long-term bond holdings or investment in long-term bonds because of improving holding returns, while active investors reduce such investment because their assets are almost entirely composed of long-term bonds and rising long yields leads to deteriorations in their balance sheet. Due to the changing supply and demand of long-term bonds, their holding returns initially increase, but take a rapid downturn soon afterward[25]. Consequently, the real returns decrease, which accordingly pushes up the housing stock as well as house prices.

Second, let us consider the reason for the initial increases in real consumption. As stated above, the rise in policy rates causes active investors to cut down on their long-term bond investment, thus reducing the issuing of short-term bonds accordingly. This induces the passive investors to reduce their short-term bond purchases and consequently push up real consumption.

After its initial expansion, the economy goes through a downturn through the following process. Increases in housing stock lead to increases in the supply of long-term bonds, which cause the real holding returns on the bond (i.e. real long yields) and term premia on the bond to go up, thereby pushing down residential investment or housing stock. On the other hand, increases in term premia stimulate long-term bond investment and short-term bond issues by active investors, and the rise in short-term bond issuances leads to greater short-term bond investment by passive investors and accordingly pushes down real consumption. Furthermore, falls in housing stock affect both long- and short-term bond markets and consequently the term premia through the holding returns on these bonds.

The process described above conveys the very important implication that it

[25] This would imply what has been described as a 'conundrum' by Alan Greenspan.

takes some time for a monetary policy to penetrate through an economy. In addition, it indicates the existence of a bi-directional relationship between the yield curve and the economy; that is, it confirms the validity of proposition 4. Finally, Figure 1 suggests that propositions 2 and 3 are also empirically relevant.

2. Effects of the presence of an active investor sector

In order to investigate the effects of the presence of an active investor sector on the economy, we will introduce another model. In this model, there are no active investors, and instead, we implicitly assume a financial institution that raises funds from passive investors in the form of deposits, M_t, and short-term bond issues, $B_{s,t}$, in period t[26]. Following this, the institution transfers these funds to the central bank as reserves to yield interest and repays the funds with their interest, $R_t M_t$ and $R_t B_{s,t}$, respectively, to passive investors in period $t+1$[27].

Computing each shock's forecast error variance decomposition, we find that the impacts of policy rate shocks become much larger in this model than in the model with active investors, while the influences of housing preference shocks are much lower. Therefore, we can examine the effect of the existence of an active investor sector by comparing the impulse responses of the main variables to shocks to housing preference and policy rates between economies with and without active investors. With respect to the model without the active investor sector, the impulse responses of the main variables are shown in Figure 2.

(1) Positive shocks to housing preference

Positive shocks to housing preference push up housing stock, house prices and real output, thereby increasing the supply and demand of long-term bonds, policy rates, one-period holding returns on long-term bonds and term premia on the bond. The increasing demand for residences and long-term bonds decreases real consumption, which results in slight initial reductions in inflation rates.

The initial responses of the main variables are similar to those in the base

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[26] In this economy, short-term bonds are also treated as risk-free assets.

[27] In the model without active investors, it is assumed that $X_t = \left(\overline{Y}/Y_t\right)^{\rho_x}$ instead of $X_t = \left(\overline{Y}/Y_{t-1}\right)^{\rho_x}$, in order to meet the Blanchard and Kahn [1980] condition. Since the structure of the model without active investors is considerably different from that of our basic model, we need to re-estimate the following relevant parameters: γ, $v_{2,l}$, ρ_{qh1}, ρ_{qh2}, ρ_x, ρ_h, ρ_a, ρ_R and the standard deviation on disturbances.

model with active investors. However, there are material differences in the features of responses between economies with and without the active investor sector. First, there are few cyclical properties in the responses of the main variables in the economy without the sector. Second, for this economy, the responses are much smaller. The first point is particularly important because it suggests that the behaviour of active investors is a source of the business cycle. In order to confirm this suggestion, we will examine whether the cyclical property is due to any changes in the probability of default, that is, the presence of the variable X_t.

Hence, using our base model, we set $\overline{X}$ as 0.07 instead of 0.20 and perform the simulation with housing preference shocks[28]. As the result indicates, and as is shown in Figure 3, setting a lower value for $\overline{X}$ does not eliminate the cyclical property from the economy. Furthermore, if we perform the same simulation after setting the values of $\overline{X}$ = 0.001 and $\rho_x = 5$, we find that this setting enhances the cyclical property, although the fluctuation is attenuated[29].

The above simulation results indicate that the presence of the variable X_t may not necessarily produce the cyclical feature in the economy. Consequently, based on the comparison of responses as well as the simulation results, we can conclude that the active investor sector not only serves as a source of the business cycle but also amplifies fluctuations[30].

(2) Positive shocks to policy rates

Without the active investor sector, the economy indicates typical responses to positive shocks to monetary policy. Hence, the main variables—real output, real consumption, housing stock, house prices and inflation rates—decrease; consequently, the supply and demand of long-term bonds as well as term premia on the bond drop. Comparing this situation with the corresponding responses in the economy with the active investor sector, the results imply that active investors might impede the propagation of monetary policy effects.

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[28] Since the parameter setting of $\overline{X}$ = 0 and $\rho_x \leq 0.7$ does not fulfil the Blanchard and Kahn [1980] condition, we retain $\rho_x = 0.7130$ and set the value of $\overline{X}$ as low as possible.

[29] For the model with $\overline{X}$ = 0.07, the influences of shocks to house prices are much larger than those for the base model, whereas the influences of housing preference are smaller. In contrast, the model with $\overline{X}$ = 0.001 shows almost the same influences of shocks as the base model.

[30] Adrian, Moench and Shin [2010b] and Verona, Martins and Drumond [2011] present models with the active investor sector and calculate the impulse responses of macroeconomic variables to various shocks to the economy. However, the responses in their studies indicate very few cyclical properties as compared to the responses in this study. This may be because our base model specifies the relationship between the active investor sector and other sectors in the economy more elaborately than their models.

Finally, as in the case of housing preference shocks, the responses do not show any cyclical property.

3. Effects of changes in the active investors' behaviour (changes in the degree of financial regulation)

The parameter $v_{1,I}$ represents the degree of self-imposed liquidity requirement for active investors investing in long-term bonds. Therefore, lower values of $v_{1,I}$ indicate that the active investors are risk lovers, while higher values of $v_{1,I}$ point to risk-averse investors. From the viewpoint of financial regulation, this can be interpreted as suggesting the former's association with less strict regulation and the latter's association with more rigorous regulation. In order to examine the effects of changes in the degree of financial regulation, let us compare the responses of the main variables between economies with lower and higher $v_{1,I}$, where the responses are derived from positive housing preference shocks.

The results are presented in Figure 4. When $v_{1,I}$ is set as 0.01 instead of 0.5008, the change has little influence on the impulse responses with the original parameter value of 0.5008[31].

In contrast, when the value of $v_{1,I}$ is set higher at 5.999, which fulfils the condition $0 < v_{1,I} < 6$ in proposition 2, the higher $v_{1,I}$ enhances the cyclical property of the responses, while having little impact on the fluctuations. We can infer the following reasons for the enhanced cyclical property. One reason is that the higher $v_{1,I}$ does not inhibit active investors from investing in long-term bonds. Instead, it prompts faster adjustment of their holdings of long-term bonds when the ratio of reserves to long-term bond holdings moves away from an optimal value. The second reason is that, according to the results of the variance decomposition of shocks, the influences of housing preference shocks become much lower than those in our base case, while the influences of shocks to house prices are much larger. Hence, impacts on house prices through shocks to housing preference would induce cyclical variation of the economy.

The simulation results suggest that while more rigorous financial regulation for active investors could prevent the further growth of asset price bubbles, it might not necessarily usher in economic stability.

V Concluding remarks

In this study, we developed a new-Keynesian DSGE model in which market-based

[31] The influences of the other shocks are also virtually the same as in the case of the original value.

intermediaries or active investors have an interactive relationship with the ultimate borrowers and lenders. The study also sought to assess the effects of the presence of active investors on the business cycle.

We could derive four propositions from the theoretical analysis. First, the collateral constraint on end-user borrowers is likely to push down housing stock in a steady state. This implies that the constraint controls residential investment. The second proposition is that, under a probable condition on active investors' investment, the marginal increases in capital would raise the expected net profits. This proposition indicates that, when an increase in the active investor's asset size induces a rise in capital, it pushes up the expected net profits. The third proposition states that the active investor's holdings of risky assets are affected by the slope of the yield curve; hence, the steeper the yield curve, the larger is its asset size. This proposition, together with the second one, suggests that a steeper yield curve is likely to yield larger net profits for active investors. Finally, the fourth proposition posits that the term structure of interest rates is endogenously shifted by the modified relative amounts of money and each bond that is outstanding. This proposition is consistent with Tobin's [1969] view.

Based on the developed model and the U.S. quarterly data from 1990:Q1 to 2010:Q3, this study performed empirical analyses as follows. First, it examined the dynamics of the linearized model by using impulse responses, while focusing on the active investor sector. The examination empirically proved the second, third and fourth propositions and revealed that the period of impulse responses of the main variables elicited by positive shocks to house prices is much shorter than that elicited by positive shocks to housing preference.

Second, the study compared the impulse responses of the main variables between models with and without the active investor sector. In consequence, two points were brought to light: one, the active investor sector is not only one of the sources of the business cycle but also a fluctuation amplifier; two, active investors might impede the propagation of monetary policy effects.

Third, by changing the parameter of active investors, which represents a degree of self-imposed liquidity requirement along with their investment in long-term bonds, the study investigated whether and how financial regulation for active investors affects the economy. The results revealed that while more rigorous financial regulation could forestall asset price bubbles, it may not necessarily usher in economic stability.

The results from the theoretical as well as empirical analyses indicate that the active investor sector has significant effects on the business cycle, which supports the view of Adrian, Moench and Shin [2010b].

Although our model highlighted the material role played by the active investor sector on the business cycle, it did not further explore the matter. An important undertaking for future studies would be to incorporate the government sector, and particularly government bonds, into the model and to explore the effects of central bank purchases of government bonds from active investors in order to stabilize the economy.

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Appendix 1: Derivation of $\hat{\lambda}_{6,t}$, β and φ

Let us consider the agents that comprise the end-user borrowers and passive investors. We assume that they have the following period utility function in stable times:

$$U_p\left(C_t,\ C_{t-1}, N_t^s, S_{h,t}, e_{h,t}\right) = \frac{1}{1-\sigma_p}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma_p} + \frac{S_{h,t}^{1-\chi}}{1-\chi} e_{h,t} - \frac{\left(N_t^s\right)^{1+\varphi_p}}{1+\varphi_p}, \tag{1.1}$$

and the following period utility function in times of financial distress:

$$U_d\left(C_t,\ C_{t-1}, N_t^s\right) = \frac{1}{1-\sigma_d}\left(\frac{C_t}{C_{t-1}^h}\right)^{1-\sigma_d} - \frac{\left(N_t^s\right)^{1+\varphi_d}}{1+\varphi_d}, \tag{1.2}$$

where $C_t = C_{1,t} + C_{2,t}$, $N_t^s = N_{1,t}^s + N_{2,t}^s$ and all parameters are defined in the same manner as in the main body of the text.

Following this, we can derive the constraints on the agents' behaviour from the setup illustrated in the main text. Hence, in stable times, the constraints are

$$\begin{aligned}\frac{1}{P_t}&\left(W_t N_t^s + Q_{h,t} S_{h,t-1} + B_{l,t}^{*s} + H_{s,t-1} B_{s,t-1}^d + R_{t-1} M_{t-1} + T_t\right) \\ &= C_t + \frac{1}{P_t}\left\{Q_{h,t} S_{h,t} + H_{l,t-1} B_{l,t-1}^{*s} + B_{s,t}^d + M_t\left(1 + AC_{m,t}\right)\right\},\end{aligned} \tag{1.3}$$

and

$$\frac{1}{P_t} R_t B_{l,t}^{*s} \le \frac{1}{P_t} k_h Q_{h,t} S_{h,t}, \tag{1.4}$$

where $B_{l,t}^{*s} \equiv B_{l,t}^s - B_{2,l,t}^d$. On the other hand, in financial distress, the constraint in period t is given by

$$\frac{1}{P_t}\left(W_t N_t^s + k_{bs}\gamma B_{l,t-1}^{*s} + R_{t-1} M_{t-1} + T_t\right) = C_t + \frac{1}{P_t} M_t, \tag{1.5}$$

where it is assumed that $B_{l,t}^{*s}$ is equal to $B_{l,t}^{*d}$, denoting long-term bond holdings by active investors. In period $t+i$ ($i \ge 1$), this equation becomes

$$\frac{1}{P_{t+i}}\left(W_{t+i} N_{t+i}^s + R_{t+i-1} M_{t+i-1} + T_{t+i}\right) = C_{t+i} + \frac{1}{P_{t+i}} M_{t+i}. \tag{1.6}$$

Based on the above setup, the agent solves the optimizing problem. Supposing $\sigma_p = \sigma_d = \sigma$ and $\varphi_p = \varphi_d = \varphi$, the first-order conditions on the optimizing problem suggest that

$$\Lambda_{6,t} = \frac{C_t^{-\sigma}}{C_{t-1}^{h(1-\sigma)}} - \beta h E_t\left[\frac{C_{t+1}^{1-\sigma}}{C_t^{h(1-\sigma)+1}}\right] \tag{1.7}$$

and

$$\left(N_t^s\right)^{\varphi} = \Lambda_{6,t} W_t', \tag{1.8}$$

where $\beta \in (0, 1)$ is a discount factor related to the agent's utility, and $\Lambda_{6,t}$ represents the Lagrange multiplier for the budget constraint.

1. The derivation of $\hat{\lambda}_{6,t}$

In a steady state, equations (11), (55) and (1.7) become, respectively,

$$\overline{\Lambda}_1 = \left(1-\beta_1 h_1\right)\overline{C}_1^{\{(1-h_1)(1-\sigma_1)-1\}}, \tag{1.9}$$

$$\overline{\Lambda}_5 = \left(1-\beta_2 h_2\right)\overline{C}_2^{\{(1-h_2)(1-\sigma_2)-1\}}, \tag{1.10}$$

$$\overline{\Lambda}_6 = \left(1-\beta h\right)\overline{C}^{\{(1-h)(1-\sigma)-1\}}. \tag{1.11}$$

Accordingly, around the steady state, it can be presupposed that

$$\hat{\lambda}_{1,t} \approx \{(1-h_1)(1-\sigma_1)-1\}\hat{c}_{1,t}, \tag{1.12}$$

$$\hat{\lambda}_{5,t} \approx \{(1-h_2)(1-\sigma_2)-1\}\hat{c}_{2,t}, \tag{1.13}$$

$$\hat{\lambda}_{6,t} \approx \{(1-h)(1-\sigma)-1\}\hat{c}_{t}, \tag{1.14}$$

Substituting equations (1.12) to (1.14) into the market-clearing condition on consumption goods,

$$\frac{\overline{C}}{(1-h)(1-\sigma)-1}\hat{\lambda}_{6,t} \approx \frac{\overline{C}_1}{(1-h_1)(1-\sigma_1)-1}\hat{\lambda}_{1,t} + \frac{\overline{C}_2}{(1-h_2)(1-\sigma_2)-1}\hat{\lambda}_{5,t}. \tag{1.15}$$

Assuming that

$$(1-h)(1-\sigma) = (1-h_1)(1-\sigma_1) = (1-h_2)(1-\sigma_2), \tag{1.16}$$

equation (1.15) becomes

$$\hat{\lambda}_{6,t} \approx \frac{\overline{C}_1}{\overline{C}}\hat{\lambda}_{1,t} + \frac{\overline{C}_2}{\overline{C}}\hat{\lambda}_{5,t}. \tag{1.17}$$

2. The derivation of β

Equations (1.9) to (1.11) lead to the following equations:

$$\bar{\lambda}_1 - \{(1-h_1)(1-\sigma_1)-1\}\bar{c}_1 = \ln(1-\beta_1 h_1), \tag{1.18}$$

$$\bar{\lambda}_5 - \{(1-h_2)(1-\sigma_2)-1\}\bar{c}_2 = \ln(1-\beta_2 h_2), \tag{1.19}$$

$$\bar{\lambda}_6 - \{(1-h)(1-\sigma)-1\}\bar{c} = \ln(1-\beta h), \tag{1.20}$$

Calculating $\left(\frac{\overline{C}_1}{\overline{C}}\right)$ × (1.18) + $\left(\frac{\overline{C}_2}{\overline{C}}\right)$ × (1.19) – (1.20) for both sides of these equations under the assumption in (1.16), and using the relationship applied to $\bar{\lambda}_1$, $\bar{\lambda}_5$ and $\bar{\lambda}_6$ in (1.17), we obtain

$$\ln(1-\beta h) \approx \left(\frac{\overline{C}_1}{\overline{C}}\right)\ln(1-\beta_1 h_1) + \left(\frac{\overline{C}_2}{\overline{C}}\right)\ln(1-\beta_2 h_2). \tag{1.21}$$

Presuming not only that βh, $\beta_1 h_1$ and $\beta_2 h_2$ are all small but also that h_1 and h_2 are around h, equation (1.21) indicates that

$$\beta \approx \left(\frac{\overline{C}_1}{\overline{C}}\right)\beta_1 + \left(\frac{\overline{C}_2}{\overline{C}}\right)\beta_2. \tag{1.22}$$

3. The derivation of φ

Log-linearizing equations (12), (56) and (1.8) around the steady state, we obtain the following equations, respectively:

$$\varphi_1 \hat{n}^s_{1,t} = \hat{\lambda}_{1,t} + \hat{w}_t, \tag{1.23}$$

$$\varphi_2 \hat{n}^s_{2,t} = \hat{\lambda}_{5,t} + \hat{w}_t, \tag{1.24}$$

$$\varphi \hat{n}^s_t = \hat{\lambda}_{6,t} + \hat{w}_t. \tag{1.25}$$

Next, calculating $\left(\frac{\overline{C}_1}{\overline{C}}\right)$ × (1.23) + $\left(\frac{\overline{C}_2}{\overline{C}}\right)$ × (1.24) – (1.25) and using the relationship applied to $\bar{\lambda}_1$, $\bar{\lambda}_5$ and $\bar{\lambda}_6$ in (1.17), we obtain

$$\varphi \hat{n}^s_t = \left(\frac{\overline{C}_1}{\overline{C}}\right)\varphi_1 \hat{n}^s_{1,t} + \left(\frac{\overline{C}_2}{\overline{C}}\right)\varphi_2 \hat{n}^s_{2,t}. \tag{1.26}$$

On the other hand, since $N_t^s = N_{1,t}^s + N_{2,t}^s$,

$$\hat{n}_t^s = \frac{\overline{N}_1^s}{\overline{N}^s}\hat{n}_{1,t}^s + \frac{\overline{N}_2^s}{\overline{N}^s}\hat{n}_{2,t}^s. \tag{1.27}$$

Substituting equation (1.27) into equation (1.26) and rearranging the resultant equation, we get

$$\left(\frac{\overline{N}_1^s}{\overline{N}^s}\varphi - \frac{\overline{C}_1}{\overline{C}}\varphi_1\right)\hat{n}_{1,t}^s + \left(\frac{\overline{N}_2^s}{\overline{N}^s}\varphi - \frac{\overline{C}_2}{\overline{C}}\varphi_2\right)\hat{n}_{2,t}^s = 0. \tag{1.28}$$

In order to maintain the relationship in (1.28) for all t,

$\frac{\overline{N}_1^s}{\overline{N}^s}\varphi = \frac{\overline{C}_1}{\overline{C}}\varphi_1$ and $\frac{\overline{N}_2^s}{\overline{N}^s}\varphi = \frac{\overline{C}_2}{\overline{C}}\varphi_2$.

As a result,

$$\varphi = \left(\frac{\overline{N}^s}{\overline{N}_1^s}\right)\left(\frac{\overline{C}_1}{\overline{C}}\right)\varphi_1 \left(= \left(\frac{\overline{N}^s}{\overline{N}_2^s}\right)\left(\frac{\overline{C}_2}{\overline{C}}\right)\varphi_2\right). \tag{1.29}$$

Appendix 2: Data and data sources

Real consumption: Real personal consumption expenditures, seasonally adjusted annual rate, billions of chained 2005 dollars, Bureau of Economic Analysis (BEA)

Hours in business sector: Hours put in by all persons in the business sector, seasonally adjusted (s.a.), index (2005 = 100), Bureau of Labor Statistics (BLS)

Compensation per hour in business sector: Compensation per hour in the business sector, s.a., index (2005 = 100), BLS

House price: House price index for the United States, not seasonally adjusted (n.s.a), index (1980:Q1 = 100), Federal Housing Finance Agency

Housing stock: Quarterly estimates of the total housing inventory for the United States, all housing, n.s.a., numbers in thousands, U.S. Census Bureau

Real residential investment: Real private residential fixed investment, seasonally adjusted annual rate, billions of chained 2005 dollars, BEA

Consumer price: Personal consumption expenditures: chain-type price index, s.a., index (2005 =100), BEA

Money supply: M2 money stock, s.a., billions of dollars, Board of Governors of the Federal Reserve System (FRB)

One-period interest rate: Quarterly average of federal funds rate, FRB

One-period holding returns on the long-term bond: 10-year zero-coupon yield ($r_{l,t}$), n.s.a., FRB. $H_{l,t}$ is calculated by $H_{l,t} = 10\left\{\ln\left(\frac{1}{1+r_{l,t+1}}\right) - \ln\left(\frac{1}{1+r_{l,t}}\right)\right\}$.

Short-term bond outstanding: Active investor liability sum of federal funds, security repurchase agreements and open market paper, n.s.a., amounts outstanding at the end of the quarter, billions of dollars, FRB

Long-term bond outstanding: Sum of corporate and foreign bonds and agency- and GSE-backed securities, amounts outstanding at the end of the quarter, billions of dollars, FRB. B_l is calculated based on the total liabilities of long-term bond outstandings; $B_{1,l}$, based on the active investor liability of long-term bond outstandings; and $B_{2,l}$, based on $B_l - B_{1,l}$.

Reserve: Active investor asset sum of reserves in the federal reserves, vault cash, checkable deposits and currency, amounts outstanding at the end of the quarter, n.s.a., billions of dollars, FRB

Central bank loans to financial institutions: Loans to domestic banks by the monetary authority, amounts outstanding at the end of the quarter, n.s.a., billions of dollars, FRB

Table 1. Calibrated parameters

Parameter	Value	Parameter	Value
σ_1, σ_2	2	ω	0.6
β_1	0.97	η	0.85
h_1, h_2	0.9	Θ	0.2
φ_1, φ_2, φ	1	ρ_r	0.8
χ	1	ρ_π	1.5
k_h	0.925	ρ_y	0.3
k_{bs}	0.3	ι_h	0.016
β_2	0.99	τ_1	0.35
α	0.36	τ_2	0.037
β	0.983	$\overline{X}$	0.2

Table 2. Prior and posterior distribution of the structural parameters

Parameter	Prior distribution			Posterior distribution		
	distribution	Mean	Std. dev.	Mean	2.5%	97.5%
γ	Beta	0.5	0.01	0.4951	0.4802	0.5110
v_{1l}	Gamma	0.5	0.01	0.5008	0.4845	0.5171
v_{2s}	Gamma	0.2	0.01	0.2006	0.1840	0.2170
v_{2l}	Gamma	0.2	0.01	0.1987	0.1826	0.2128
ρ_μ	Gamma	0.3	0.1	0.1466	0.1033	0.1933
ρ_{qh1}	Beta	0.8	0.1	0.9070	0.8533	0.9602
ρ_{qh2}	Beta	0.03	0.01	0.0174	0.0088	0.0265
ρ_x	Gamma	0.5	0.1	0.7130	0.5647	0.8526
ρ_h	Beta	0.8	0.1	0.9596	0.9245	0.9969
ρ_a	Beta	0.8	0.1	0.9959	0.9930	0.9989
ρ_R	Beta	0.8	0.1	0.8569	0.7716	0.9521
σ_h	Inv. gamma	0.05	0.05	0.2762	0.2289	0.3212
σ_{qh}	Inv. gamma	0.05	0.05	0.0106	0.0093	0.0120
σ_a	Inv. gamma	0.05	0.05	0.0330	0.0277	0.0379
σ_R	Inv. gamma	0.05	0.05	0.0083	0.0078	0.0088

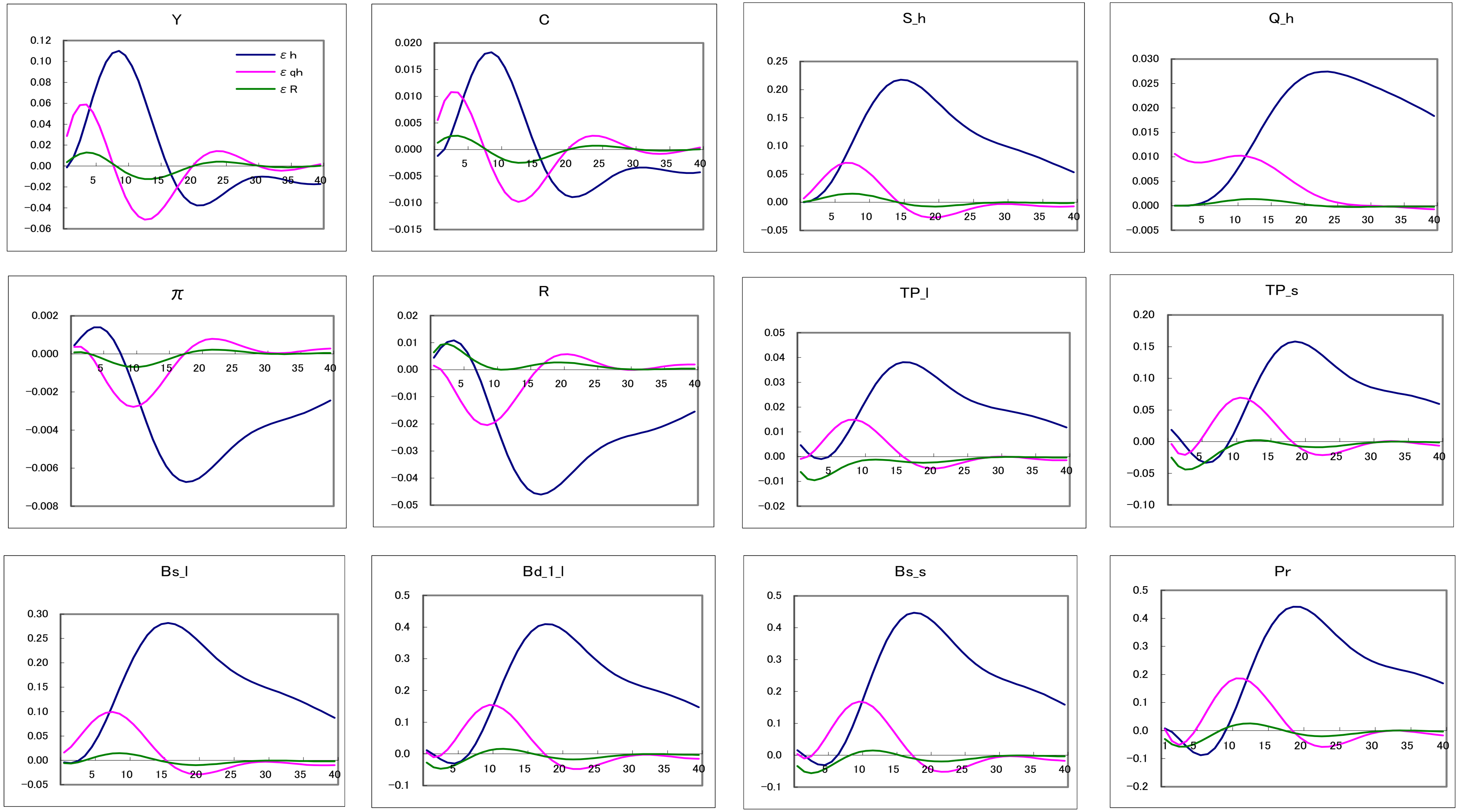
Table 3. Forecast error variance decomposition

(%)

Main variables	ε_h	ε_{qh}	ε_a	ε_R
Y	39.01	12.81	47.48	0.70
C	3.57	1.30	95.05	0.08
S_h (S_h)	86.45	4.47	8.86	0.22
Q_h (Q_h)	87.71	7.38	4.84	0.08
π	2.37	0.18	97.43	0.01
R	25.18	2.25	72.23	0.34
TP_l (TP_l)	47.05	3.34	48.61	1.00
TP_s (TP_s)	57.79	5.32	35.30	1.58
B_l^s (Bs_l)	80.36	4.56	14.96	0.11
$B^{d}{}_{1,l}$ (Bd_1_l)	77.89	5.11	16.64	0.35
B_s^s (Bs_s)	75.66	4.95	18.99	0.40
Pr	77.91	6.62	14.94	0.53

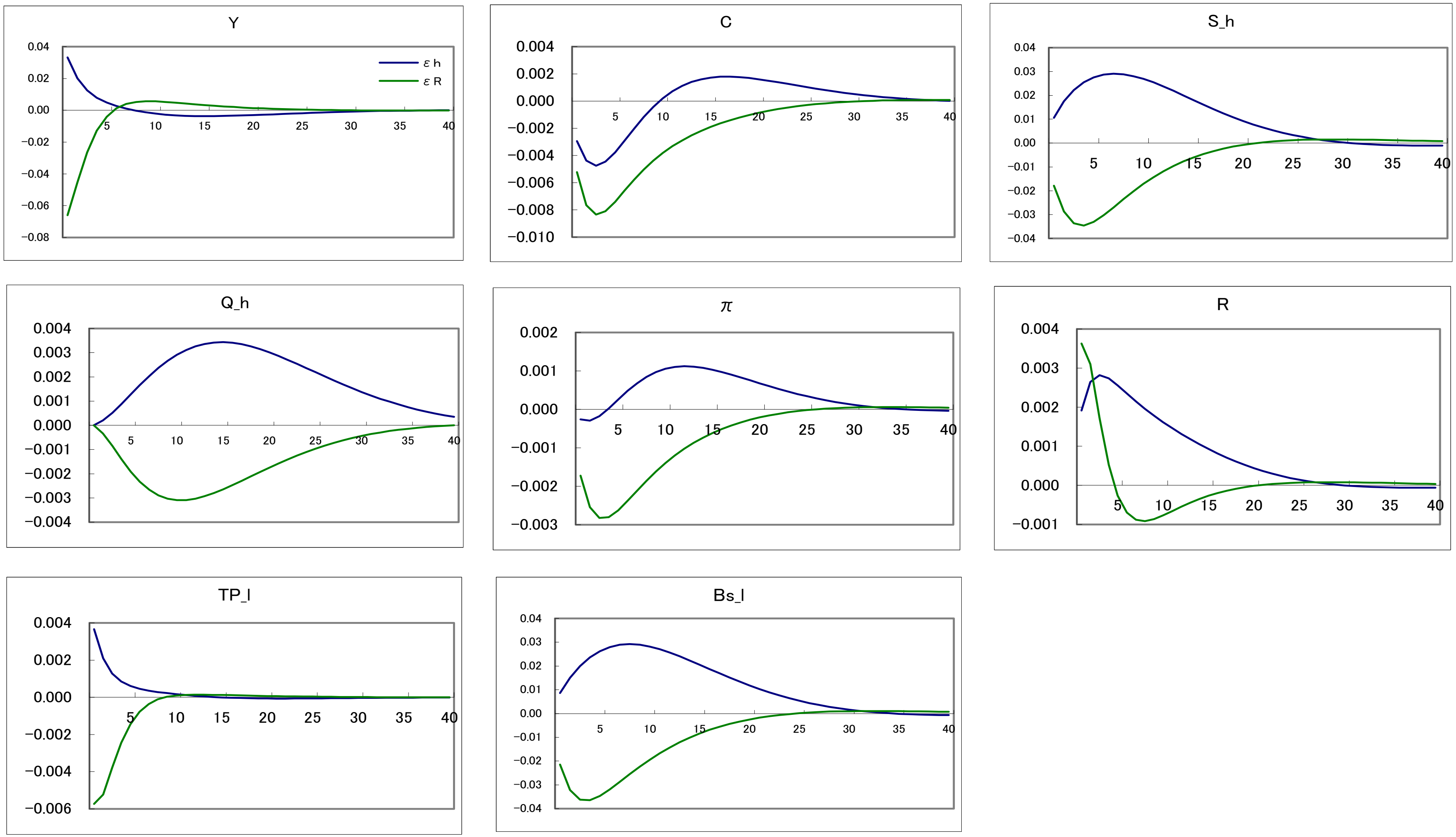
Note: The notations used in Figures 1 to 4 are shown in parentheses. 'Pr' stands for profits of active investors.

Figure1. Impulse responses of main variables in the model with the active investor sector



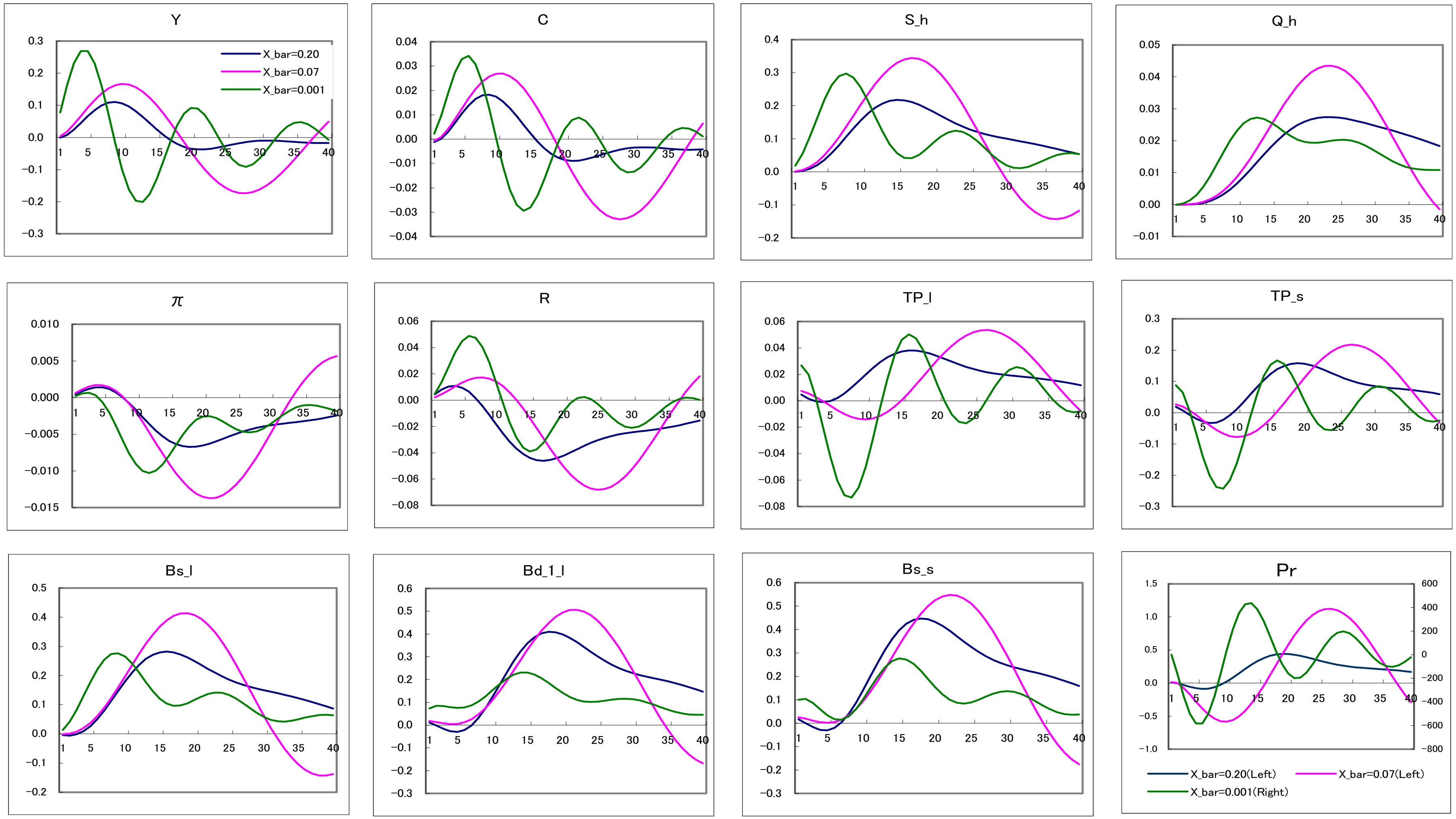
Note: The unit of horizontal axes is a quarter. Blue lines show impulse responses to housing preference shocks; red lines, to house price shocks; and green lines, to policy rate shocks.

Figure2. Impulse responses of main variables in the model without the active investor sector



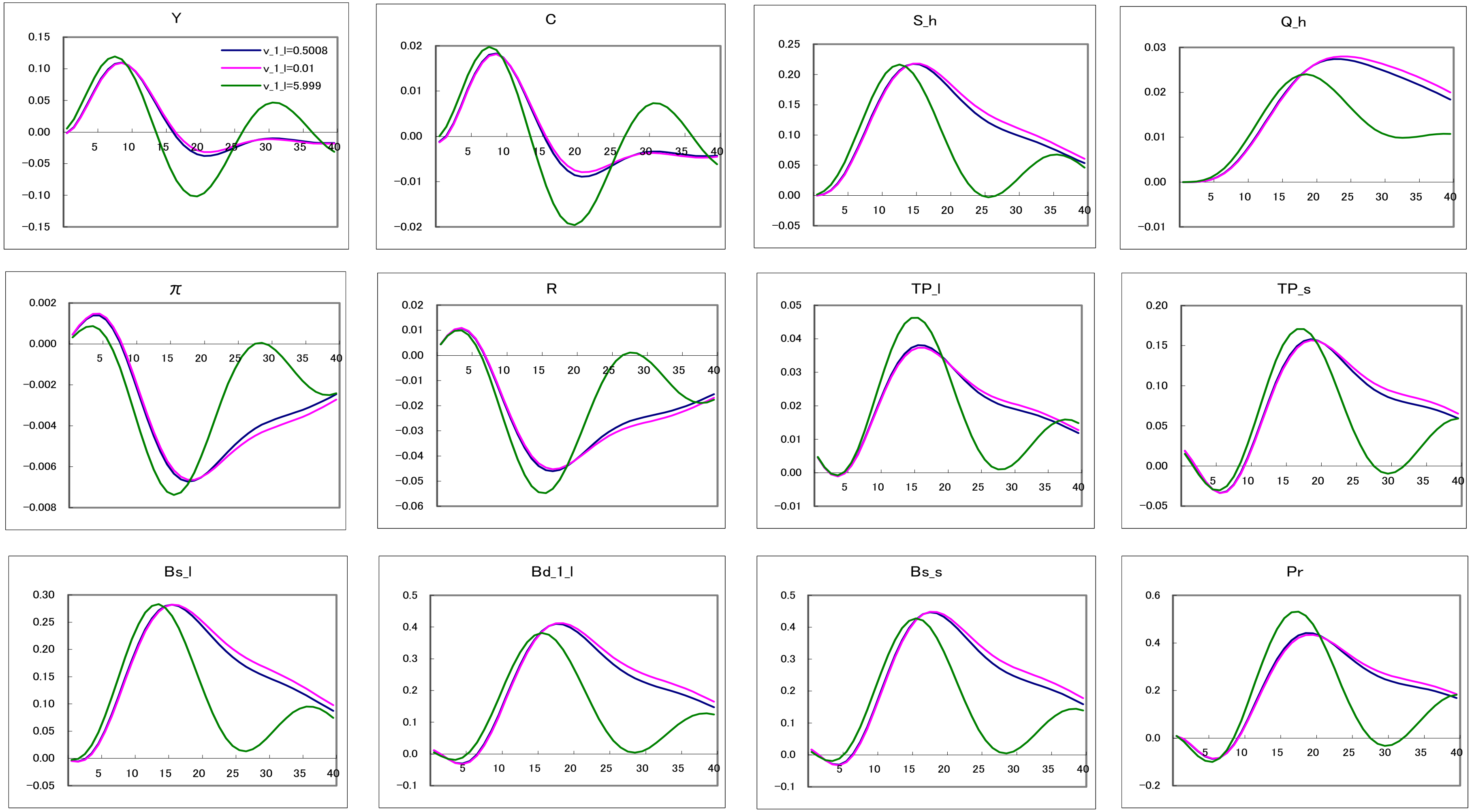
Note: The unit of horizontal axes is a quarter. Blue lines show impulse responses to housing preference shocks, and green lines, to policy rate shocks.

Figure3. Impulse responses of main variables in the model with very low probability of default



Note: The unit of horizontal axes is a quarter. Responses are to housing preference shocks. The model includes the active investor sector. Blue lines show impulse responses in the case of $\overline{X}$ = 0.20 and ρ_x = 0.7130 (base); red lines, in the case of $\overline{X}$ = 0.07 and ρ_x = 0.7130; and green lines, in the case of $\overline{X}$ = 0.001 and ρ_x = 5.

Figure4. Impulse responses of main variables in the model with various degree of financial regulation



Note: The unit of horizontal axes is a quarter. Responses are to housing preference shocks. The model includes the active investor sector. Blue lines show impulse responses in the case of $v_{1,l} = 0.5008$ (base); red lines, in the case of $v_{1,l} = 0.01$; and green lines, in the case of $v_{1,l} = 5.999$.